1
Overview and overture

Einstein's theory of the classical relativistic dynamics of gravity is remarkable, both in its simple elegance and in its profound statement about the nature of spacetime. Before we rush into the diverse matters which concern and motivate the search which leads to string theory and beyond, such as the nature of the quantum theory, the unification with other forces, etc., let us remind ourselves of some of the salient features of the classical theory. This will usefully foreshadow many of the concepts which we will encounter later.

1.1 The classical dynamics of geometry

Spacetime is of course a landscape of 'events', the points which make it up, and as such it is a classical (but of course relativistic) concept. Intuition from quantum mechanics points to a modification of this picture, and there are many concrete mechanisms in string theory which support this expectation and show that spacetime is at best a derived object or effective description. We shall see some of these mechanisms in the sequel. However, since string theory (as currently understood), seems to be devoid of a complete definition that does not require us to refer to spacetime, the language and concepts we will employ will have much in common with those used by professional practitioners of General Relativity, and of classical and quantum Field Theory. In fact, it will become clear to the newcomer that success in the physics of string theory is greatly aided by having technical facility in both of those fields. It is instructive to tour a little of the foundations of our modern approach to classical gravity and observe how the Relativist's and the Field Theorist's perspective are muddled together. String theory makes good and productive use of this sort of conflation.

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It is useful to equip a description of spacetime with a set of coordinates x^μ, $\mu = 0, 1, \ldots, D-1$, where $x^0 \equiv t$ (the time) and we shall remain open-minded and work in D dimensions for much of the discussion. The metric, with components $g_{\mu\nu}(x)$, is a function of the coordinates which allows for a local measure of the distance between points separated by an interval dx^μ:

$$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu.$$

The metric is a tensor field since under an arbitrary change of variables $x^\mu \rightarrow x'^\mu(x)$ it transforms as

$$g_{\mu\nu} \longrightarrow g'_{\mu\nu} = g_{\alpha\beta} \frac{\partial x^\alpha}{\partial x'^\mu} \frac{\partial x^\beta}{\partial x'^\nu}. \qquad (1.1)$$

Of course, 'distance' here means the more generalised Special Relativistic interval characterising how two events are separated, and it is negative, zero or positive, giving us timelike, null or spacelike separations, according to whether if it possible to connect the events by causal subluminal motion (appropriate to a massive particle), or by moving at the speed of light (massless particles), or not. This of course defines the signature of our metric as being 'mostly plus': $\{- + + + \cdots\}$ henceforth.

As a particle moves it sweeps out a path or 'world-line' $x^\mu(\tau)$ in spacetime (see figure 1.1), which is parametrised by τ. The wonderful thing is that what we would have said in pre-Einstein times was 'a particle moving under the influence of the gravitational force' is simply replaced by the statement 'a particle following a geodesic', a path which is determined by the metric in terms of the second order geodesic equation:

$$\frac{d^2 x^\lambda}{d\tau^2} = -\Gamma^\lambda_{\mu\nu}(g) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} \ , \qquad (1.2)$$

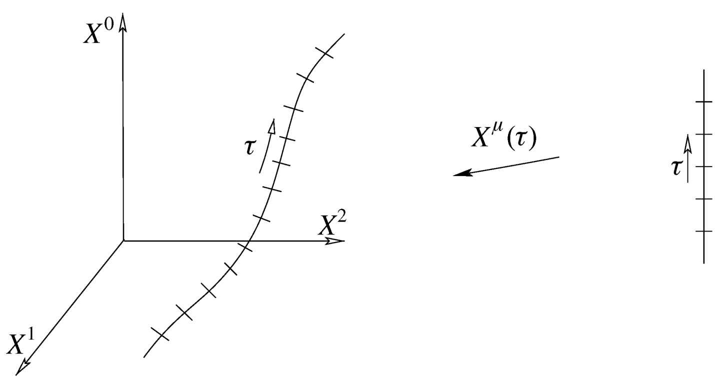

Fig. 1.1. A particle's world-line. The function $x^\mu(\tau)$ embeds the world-line, parametrised by τ, into spacetime, coordinatised by x^μ.

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where the affine connection $\Gamma(g)$ is made out of first derivatives of the metric:

$$\Gamma^{\lambda}_{\mu\nu}(g) = \frac{1}{2} g^{\lambda\kappa} \left(\partial_\mu g_{\kappa\nu} + \partial_\nu g_{\kappa\mu} - \partial_\kappa g_{\mu\nu}\right).$$

Here and everywhere else, when working with curved spacetime we lower and raise indices with the metric and its inverse, (which has components $g^{\mu\nu}$ such that $g_{\mu\lambda} g^{\mu\alpha} = \delta^{\alpha}_{\lambda}$). Also note that $\partial_\mu \equiv \partial/\partial x^\mu$.

Switching language again we see that since the term on the left hand side of the equation (1.2) is what we think of as the 'acceleration', our Newtonian intuition determines the right hand side to be the 'applied force', attributed to gravity. In such language, $g_{\mu\nu}(x)$ is interpreted as a potential for the gravitational field.

In the purely geometrical language, there are no forces. There is only geometry, and the particle simply moves along geodesics. The above statement in equation (1.2) about how a particle moves in response to the metric is derivable from a simple action principle, which says that the motion minimises (more properly, extremises) the total path length that its motion sweeps out:

$$S = -m \int (-g_{\mu\nu}(x) dx^\mu dx^\nu)^{1/2} = -m \int_{\tau_{\rm i}}^{\tau_{\rm f}} (-g_{\mu\nu}(x) \dot{x}^\mu \dot{x}^\nu)^{1/2} d\tau \ , \quad (1.3)$$

where a dot denotes a derivative with respect to τ. (The reader might consider checking this by application of the Euler–Lagrange equations or by direct variation.)

The only question (which is of course one of the biggest) remaining is the nature of what determines the metric itself. This turns out to be governed by the distribution of stress-energy-momentum, and we must write field equations which determine how the one sources the other, just as we would in any field theory like Maxwell's electromagnetism (see insert 1.1).

The stress-energy-momentum contained in the matter is captured in the elegant package that is the tensor $T^{\mu\nu}(x)$, a second rank, symmetric, divergence-free tensor which for an observer with four-velocity $\mathbf{u}$, encodes the energy density as $T_{\mu\nu} u^\mu u^\nu$, the momentum density as $-T_{\mu\nu} u^\mu x^\nu$, and shear pressures (stresses) as $T_{\mu\nu} x^\mu y^\nu$, where the unit vectors $\mathbf{x}$ and $\mathbf{y}$ are orthogonal to $\mathbf{u}$.

Einstein's field equations are:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_{\rm N} T_{\mu\nu} \ , \quad (1.6)$$

where $G_{\rm N}$ is Newton's constant. As one would expect, the quantity on the left hand side is made up of the metric and its first and second derivatives,

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Insert 1.1. A reminder of Maxwell's field equations

'Maxwell's equations' are second order partial differential equations for the electromagnetic potentials $\vec{A}\,(\vec{x},t), \phi(\vec{x},t)$ from which the magnetic $(\vec{B}\,(\vec{x},t))$ and electric $(\vec{E}\,(\vec{x},t))$ fields can be derived:

$$\begin{aligned}\vec{E}\,(\vec{x},t) &= -\,\vec{\nabla}\,\phi(\vec{x}\cdot t) - \frac{\partial\vec{A}\,(\vec{x},t)}{\partial t}\\ \vec{B}\,(\vec{x},t) &= \vec{\nabla}\times\vec{A}\,(\vec{x},t).\end{aligned} \tag{1.4}$$

In terms of the fields, Maxwell's equations are:

$$\begin{aligned}\vec{\nabla}\cdot\vec{E} &= 4\pi\rho\\ \vec{\nabla}\cdot\vec{B} &= 0\\ \vec{\nabla}\times\vec{E}+\frac{\partial\vec{B}}{\partial t} &= 0\\ \vec{\nabla}\times\vec{B}-\frac{\partial\vec{E}}{\partial t} &= 4\pi\,\vec{J}\,.\end{aligned} \tag{1.5}$$

Here, the functions $\vec{J}(\vec{x},t)$ and $\rho(\vec{x},t)$, the *current density* and the *charge density* are the *'sources'* in the field equations.

We have written the equations with the sources on the right hand side and the expression for the derivatives of the resulting fields (to which the sources give rise) on the left hand side. We will write these much more covariantly in insert 1.3.

where the Ricci scalar and tensor,

$$R \equiv g^{\mu\nu}R_{\mu\nu}, \qquad R_{\mu\nu} \equiv g^{\kappa\rho}g_{\lambda\rho}R^{\lambda}_{\mu\kappa\nu}, \tag{1.7}$$

are the only two contentful contractions of the Riemann tensor:

$$R^{\lambda}_{\mu\kappa\nu} \equiv \partial_\mu\Gamma^{\lambda}_{\kappa\nu} - \partial_\nu\Gamma^{\lambda}_{\kappa\mu} + \Gamma^{\rho}_{\kappa\mu}\Gamma^{\lambda}_{\rho\nu} - \Gamma^{\rho}_{\kappa\nu}\Gamma^{\lambda}_{\rho\mu}. \tag{1.8}$$

Except for the metric itself, the quantity on the left hand side of equation (1.6) is the unique rank two, divergenceless and symmetric tensor made from the metric (and its first and second derivatives), and hence can be allowed to be equated to the stress tensor.

When the stress tensor is zero, i.e. when there is no matter to act as a source, the vanishing of the left hand side is equivalent to the vanishing

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$R_{\mu\nu} = 0$, and solutions of this equation are said to be 'Ricci-flat'. This includes highly non-trivial spacetimes such as Schwarzschild black holes, which follows from the non-linearity of the left hand side, representing the fact that the stress-energy in the gravitational field itself can act as its own source ('gravity gravitates').

The physical foundation behind the geometric approach is of course the Principle of Equivalence, which begins by observing that gravity is indistinguishable from acceleration, and tells one how to find a locally inertial frame: one must simply 'fall' under the influence of gravity (i.e. just follow a geodesic) and one does not feel one's own weight, and so one is in an inertial frame where the Laws of Special Relativity hold. See insert 1.2 for a reminder of this in equations. The sourceless field equations then follow from the recasting of the relative motion observed between frames on neighbouring geodesics in terms of an apparent 'tidal' force.

The full statement of the field equations to include sources is also guided by covariance, which means that it is a physical equation between tensors of the same type, and with the same divergenceless property (which is a physical statement of continuity). The equations are therefore true in all coordinate systems obtained by an arbitrary change of variables $x^\mu \to x'^\mu(x)$, since they transform as tensors in a way generalising the transformation of the metric in equation (1.1).

Note that the statement of divergencelessness is a covariant one too, i.e. $\nabla_\mu T^{\mu\nu} = 0$ uses the covariant derivative*, which is designed to yield a tensor after acting on one, say V:

$$\nabla_\kappa V^{\mu\cdots}_{\nu\cdots} \equiv \partial_\kappa V^{\mu\cdots}_{\nu\cdots} + \Gamma^\mu_{\lambda\kappa} V^{\lambda\cdots}_{\nu\cdots} + \cdots - \Gamma^\lambda_{\kappa\nu} V^{\mu\cdots}_{\lambda\cdots} - \cdots . \tag{1.9}$$

Finally, note that the field equations themselves may be derived from an action principle, the extremising of the Einstein–Hilbert action coupled to matter:

$$\begin{aligned} S &= S_{\mathrm{M}} + S_{\mathrm{EH}} \\ S_{\mathrm{EH}} &= \frac{1}{16\pi G_{\mathrm{N}}} \int d^D x \sqrt{-g}\, R \\ T^{\mu\nu} &\equiv -\frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{M}}}{\delta g_{\mu\nu}}, \end{aligned} \tag{1.10}$$

where g is the determinant of the metric.

* In fact, this (not entirely unambiguous) procedure of replacing the ordinary derivative by the covariant derivative, together with the replacement of the Minkowski metric $\eta_{\mu\nu}$ by the curved spacetime metric $g_{\mu\nu}(x)$ is often called the principle of 'minimal coupling' as a procedure for how to generalise Special Relativistic quantities to curved spacetime.

Insert 1.2. Finding an inertial frame by freely falling

In order to find an inertial frame, we must find coordinates so that at least locally, at a point $x_{\rm o}^{\nu}$, say, we can can do special relativity. This means that we perform a change of coordinates $x^{\mu} \to x'^{\mu}(x)$ so that when the metric changes, according to (1.1), the result is

$$g_{\mu\nu}(x_{\rm o}^{\nu}) = \eta_{\mu\nu},$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $\mathrm{diag}(-1, +1, \ldots,)$. How accurately can we achieve this? In our coordinate transformation, we have in the neighbourhood of $x_{\rm o}^{\nu}$:

$$\begin{aligned} x^{\mu}(x^{\nu}) = {} & x^{\mu}(x_{\rm o}^{\nu}) + \frac{\partial x^{\mu}}{\partial x'^{\nu}}(x'^{\nu} - x'^{\nu}_{\rm o}) \\ & + \frac{1}{2}\frac{\partial^2 x^{\mu}}{\partial x'^{\nu}\partial x'^{\kappa}}(x'^{\nu} - x'^{\nu}_{\rm o})(x'^{\kappa} - x'^{\kappa}_{\rm o}) \\ & + \frac{1}{6}\frac{\partial^3 x^{\mu}}{\partial x'^{\nu}\partial x'^{\kappa}\partial x'^{\lambda}}(x'^{\nu} - x'^{\nu}_{\rm o})(x'^{\kappa} - x'^{\kappa}_{\rm o})(x'^{\lambda} - x'^{\lambda}_{\rm o})\ldots \end{aligned}$$

so we have, at first order, D^2 coefficients to adjust. Since $g'_{\mu\nu}$ has $D(D+1)/2$ components, we are left with

$$D^2 - \frac{D(D+1)}{2} = \frac{D(D-1)}{2}$$

transformations at our disposal. Happily, this is precisely the dimension of the Lorentz group, $SO(D-1, 1)$ of rotations and boosts available in our inertial frame. At second order, we have $D^2(D+1)/2$ coefficients to adjust, which is precisely the same number of first derivatives $\partial g'_{\mu\nu}/\partial x'^{\kappa}$ of the metric that we need to adjust to zero, cancelling all of the 'forces' in the geodesic equation (1.2). At third order, we have $D^2(D+1)(D+2)/6$ coefficients to adjust, while there are $D^2(D+1)^2/4$ second derivatives of the metric, $\partial^2 g'_{\mu\nu}/\partial x'^{\kappa}\partial x'^{\lambda}$, to adjust, which is rather more. In fact, this failure to adjust

$$\frac{D^2(D+1)^2}{4} - \frac{D^2(D+1)(D+2)}{6} = \frac{D^2(D^2-1)}{12}$$

second derivatives is of course a statement of physics. This is precisely the number of independent components of the Riemann tensor $R^{\lambda}_{\kappa\mu\nu}$, which appears in the field equations determining the metric. So everything fits together rather nicely.

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A favourite example of a stress tensor for a matter system is the Maxwell system of electromagnetism. Combining the electric potential ϕ and vector potential $\vec{A}$ into a four-vector $\mathbf{A}(\mathbf{x}) = (\phi, \vec{A})$, with components A_μ, the magnetic induction $\vec{B}$ and electric field $\vec{E}$ are captured in the rank two antisymmetric tensor field strength:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu,$$

and an observer with four-velocity $\mathbf{u}$ reads the fields as:

$$E_\mu = F_{\mu\nu}u^\nu, \qquad B_\mu = \epsilon_{\mu\nu}{}^{\kappa\lambda}F_{\kappa\lambda}u^\nu, \tag{1.11}$$

where $\epsilon_{\mu\nu\kappa\lambda}$ is the totally antisymmetric tensor in four dimensions, with $\epsilon_{0123} = -1$. (See insert 1.3 for more on this covariant presentation of electromagnetism.) The action is:

$$S_{\mathrm{M}} = \int d^D x \mathcal{L} = -\frac{1}{16\pi}\int (-g)^{1/2} F_{\mu\nu}F^{\mu\nu} d^D x, \tag{1.12}$$

and so it is easily verified that the Euler–Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial A_\mu} - \frac{\partial}{\partial x^\nu}\left(\frac{\partial \mathcal{L}}{\partial(\partial_\nu A_\mu)}\right) = 0,$$

give the field equations

$$\nabla_\nu F^{\mu\nu} = 0,$$

where we have used a very useful identity which is easily derived:

$$\delta(-g)^{1/2} = \tfrac{1}{2}(-g)^{1/2} g^{\mu\nu}\delta g_{\mu\nu}. \tag{1.13}$$

On the other hand, since

$$\frac{\partial \mathcal{L}}{\partial g_{\mu\nu}} = -\frac{(-g)^{1/2}}{8\pi}\left(g_{\lambda\beta}F^{\mu\lambda}F^{\nu\beta} - \tfrac{1}{4}g^{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}\right) \tag{1.14}$$

the stress tensor is

$$T^{\mu\nu} = \frac{1}{4\pi}\left(g_{\lambda\beta}F^{\mu\lambda}F^{\nu\beta} - \tfrac{1}{4}g^{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}\right). \tag{1.15}$$

1.2 Gravitons and photons

The quantum Field Theorist's most sacred tool is the idea of associating a particle to every sort of field, whether it be matter or force. So a force is

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Insert 1.3. Maxwell written covariantly

Probably most familiar is the flat space writing:

$$F^{\mu\nu} = \begin{pmatrix} 0 & E_1 & E_2 & E_3 \\ -E_1 & 0 & B_3 & -B_2 \\ -E_2 & -B_3 & 0 & B_1 \\ -E_3 & B_2 & -B_1 & 0 \end{pmatrix} \tag{1.16}$$

for the Maxwell tensor. In addition to the four-vector $\mathbf{A}(\mathbf{x}) = (\phi, \vec{A})$, one in general will have a four-current for the source, which combines the current and electric charge density: $\mathbf{J}(\mathbf{x}) = (\rho, \vec{J})$. With these definitions, Maxwell's equations take on a particularly simple covariant form:

$$\nabla_\nu F^{\mu\nu} = -4\pi J^\mu, \qquad \partial_\mu F_{\nu\kappa} + \partial_\nu F_{\kappa\mu} + \partial_\kappa F_{\mu\nu} = 0, \tag{1.17}$$

for the equations with sources, and the source-free equations (Bianchi identity). The energy-momentum tensor for electromagnetism is given in terms of $\mathbf{F}$ in equation (1.15), and is subject to the conservation equation (when the sources $J^\mu = 0$): $\nabla_\mu T^{\mu\nu} = 0$. This contains familiar physics. Specialising to flat space:

$$T_{00} = \frac{1}{8\pi}((\vec{E})^2 + (\vec{B})^2), \qquad T_{0i} = -\frac{1}{4\pi}(\vec{E} \times \vec{B}),$$

which is the familiar expression for the energy density and the momentum density (Poynting vector) of the electromagnetic field

mediated by a particle which propagates along in spacetime between objects carrying the charges of that interaction. There is great temptation to do this for gravity (by allowing all sources of stress-energy-momentum to emit and absorb appropriate quanta), but we immediately run into a conceptual log jam. On the one hand, we have just reminded ourselves of the beautiful picture that gravity is associated to the dynamics of spacetime itself, while on the other hand we would like to think of the gravitational force as mediated by gravitons which propagate on a spacetime background. A technical way of separating out this problem into manageable pieces (up to a point) is to study the linearised theory.

The idea is to treat the metric as split between the background which is say, flat spacetime given by the Minkowski metric $\eta_{\mu\nu}$, diag$(-1, +1, \ldots,)$,

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and some position dependent fluctuation $h_{\mu\nu}(x)$ which is to be small $h_{\mu\nu}(x) \ll 1$. Then the equations determining $h_{\mu\nu}(x)$ are derived from Einstein's equations (1.6) by substituting this ansatz:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x),$$

and keeping only terms linear in $h_{\mu\nu}$.

Let us carry this out. We will raise and lower indices with $\eta_{\mu\nu}$, and note that $g^{\mu\nu}$ will continue to be the inverse metric, which is distinct from $\eta^{\mu\alpha}\eta^{\nu\beta}g_{\alpha\beta}$. Note also that $g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu}$, to the accuracy to which we are working. The affine connection becomes:

$$\Gamma^{\rho}_{\mu\nu} = \tfrac{1}{2}\eta^{\rho\alpha}\left(\partial_\mu h_{\nu\alpha} + \partial_\nu h_{\mu\alpha} - \partial_\alpha h_{\mu\nu}\right), \tag{1.18}$$

and to this order, the Ricci tensor and scalar are:

$$\begin{aligned} R_{\mu\nu} &= \partial^\alpha \partial_{(\nu} h_{\mu)\alpha} - \tfrac{1}{2}\partial^\alpha\partial_\alpha h_{\mu\nu} - \tfrac{1}{2}\partial^\mu \partial_\nu h + O(h^2), \\ R &= \partial^\alpha\partial^\beta h_{\alpha\beta} - \partial^\alpha\partial_\alpha h + O(h^2), \end{aligned} \tag{1.19}$$

where $h = h^\mu_\mu$. Thus we learn that

$$\begin{aligned} &R_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}R = \partial^\alpha\partial_{(\nu}h_{\mu)\alpha} - \tfrac{1}{2}\partial^\alpha\partial_\alpha h_{\mu\nu} - \tfrac{1}{2}\partial^\mu\partial_\nu h \\ &-\tfrac{1}{2}\eta_{\mu\nu}\left(\partial^\alpha\partial^\beta h_{\alpha\beta} - \partial^\alpha\partial_\alpha h\right) + O(h^2). \end{aligned}$$

Defining $\bar{\gamma}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$, we find our linearised field equations:

$$-\tfrac{1}{2}\partial^\alpha\partial_\alpha \bar{h}_{\mu\nu} + \partial^\alpha\partial_{(\mu}\bar{h}_{\mu)\alpha} - \tfrac{1}{2}\eta_{\mu\nu}\partial^\alpha\partial^\beta \bar{h}_{\beta\gamma} = 8\pi G_{\mathrm{N}} T_{\mu\nu}. \tag{1.20}$$

There is an explicit gauge degree of freedom (recognisable from equation (1.1) as an infinitesimal coordinate transformation)

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu, \tag{1.21}$$

for arbitrary an arbitrary vector ξ_μ. Using this freedom, we *choose the gauge* $\partial^\nu \bar{h}_{\mu\nu} = 0$ (using a gauge transformation satisfying $\partial^\nu\partial_\nu\xi_\mu + \partial^\nu \bar{h}_{\mu\nu} = 0$), which implies

$$\partial^\alpha\partial_\alpha \bar{h}_{\mu\nu} = -16\pi G_{\mathrm{N}} T_{\mu\nu}. \tag{1.22}$$

This is highly suggestive. Consider the system of electromagnetism, with equations of motion (1.17). The equations are invariant under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \Lambda,$$

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where Λ is an arbitrary scalar. We can use this freedom to choose a gauge $\partial_\mu A^\mu = 0$, (with a parameter satisfying $\partial_\mu \partial^\mu \Lambda + \partial^\nu A_\nu = 0$), which gives the simple equation

$$\partial_\mu \partial^\mu A_\nu = -4\pi J_\nu.$$

This is of a very similar form to what we achieved in equation (1.22) for the system of linearised gravity. The analogy is clear. The Maxwell system has yielded a field equation for a vector (spin one) particle (the photon $A_\mu(x)$) sourced by a vector current ($J_\mu(x)$), while the gravitational system yields the precisely analogous equation for a spin two particle (the graviton $h_{\mu\nu}(x)$) sourced by the stress tensor $T_{\mu\nu}(x)$.

This is the starting point for treating gravity on the same footing as field theory, and in many places later we will have cause to use the word or idea 'graviton', and it is in this sense (a spin two particle propagating on a reference background) that we will mean it. We have seen how to make the delicate journey from the Relativist's geometrical understanding of gravity to a perturbative Field Theorist's. To make the return journey, reconstructing a picture of, say the non-trivial spacetime metric due to a star by starting from the graviton picture is a bit harder, but roughly it is conceptually similar to the same problem in electromagnetism. How does one go from the picture of the photon moving along in spacetime to building up a picture of the strong magnetic fields around a pair of Helmholtz coils? Words and phrases which are offered include 'coherent state of photons', or 'condensation of photons', and these should invoke the idea that the coils' field cannot be constructed using only the perturbative photon picture. One can instead use the photon description to describe processes *in the background* of the Helmholtz field, and we can similarly do the same thing for gravity, describing the propagation of gravitons in the background fields produced by a star. In this way, we see that there is a possibility that there are situations where the conceptual separation between particle quanta and background *in principle* needs be no more dangerous in gravitation than it is in electromagnetism.

Eventually, however, we would like to compute beyond tree level, and the celebrated problems of the theory of gravity treated as a quantum theory will be encountered. Then, the linearised Einstein–Hilbert action

$$S = \frac{1}{16\pi G_\mathrm{N}} \int d^D x \left(\partial^\alpha \partial^\beta h_{\alpha\beta} - \partial^\alpha \partial_\alpha h \right), \tag{1.23}$$

will eventually reveal itself to be non-renormalisable once we add interactions coming from the next order above linear. In particular, the process of recursively adding counterterms to the bare action in order to define physically measurable quantities does not terminate. As Field Theorists

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(and perhaps as Relativists) we would have cause to be discouraged, and it is a much celebrated statement that as String Theorists, we won't be.

1.3 Beyond classical gravity: perturbative strings

A reason for dwelling on some of the previous points is that it is customary to do a lot of moving back and forth between the picture of quanta moving on a flat background and other pictures, for example ones involving considerably curved background fields. This is not because string theorists have a clever collection of new technological tools for seeing how to move from one to the other (although as we shall see with the aid of supersymmetry, in some cases we can often keep track of many properties of objects in moving between pictures) but because as was said before, string theory is a developing subject which has borrowed and hybridised intuition from the Relativist's and the (perturbative and non-perturbative) quantum Field Theorist's worlds.

This borrowing is not to be taken as a sign of intellectual bankruptcy, but quite the opposite. The adoption of terminology and concepts from a wide range of other fields is as a result of the richness of genuinely novel physical phenomena, with (as a whole) no precise precedent or analogue, which the theory appears to be revealing. This is very similar to what happened almost precisely a century ago. The treatment of quanta in a context dependent manner either as a wave or as a particle, an understanding still called 'Wave–Particle Duality' by many, grew out of the attempt to grasp a new physical phenomenon – Quantum Mechanics – by reference to established physical concepts from the century before.

In the next chapter we will review how one proceeds to describe the relativistic string propagating in a flat background. There are two very broad categories, open strings which have end-points, and closed strings which do not. The basic input parameter is the mass per unit length of the string, its tension:

$$T = \frac{1}{2\pi\alpha'} \equiv \frac{1}{2\pi\ell_{\rm s}^2}.$$

As is well known, the characteristic length scale of the string, $\ell_{\rm s}$, is traditionally very small compared to scales on which we do current-day physics. This means that string excitations will have a good description as point-particle-like states on scales much longer than $\ell_{\rm s}$. After quantisation, it rapidly becomes clear that the spectrum of string theory is rather rich and demands application. Since finite masses in the spectrum are set by the inverse of $\ell_{\rm s}$, the infinite tower of massive excitations of the string (see figure 1.2) will be very inaccessible at low energy (long distance, or infra-red (IR)). The tower is of course crucial to the properties of the

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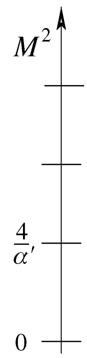

Fig. 1.2. The string spectrum has a massless sector separated by a gap (set by the tension) after which there is an infinite tower of massive states.

high energy (short distance, or ultra-violet (UV)) physics of the string. It is the massless part of the spectrum which is accessible at low energy and hence relevant to phenomenology.

For example, closed string theories describe a massless spin two particle which is identified with the graviton. The questions of non-renormalisability which arose in quantum field theory turn out to be circumvented by the remarkable ultra-violet properties of string theory, which give rise to an extremely well-behaved perturbative description of multi-loop processes involving gravitons†. The simple fact is that string theory is very unlike field theory at short distances, since it assembles together an infinity of increasingly massive excitations (in a particular way) which all play a role in the UV. The theory's supplying a satisfactory perturbative quantum theory of gravity is just the beginning of the many phenomena which arise from its properties as an extended object, as we shall see.

Other massless fields which arise in string spectra are Abelian and non-Abelian gauge fields, and various fermions and scalars, some of which one might expect give rise to the observed gauge interactions and matter fields. There is also a family of higher rank antisymmetric tensor fields generalising the photon on which we will focus in some detail. Remarkably, the value of one scalar excitation of interest, the dilaton Φ, determines the strength of the string self-interaction, $g_{\rm s} = e^{\Phi}$, and hence (since closed strings excitations can be gravitons) the value of $G_{\rm N}$. It is a striking fact that string theory dynamically determines its own coupling strength. (See figure 1.3.)

† Sadly, lack of space will prevent us from describing this here, and we refer the reader to a textbook on this[1, 5].

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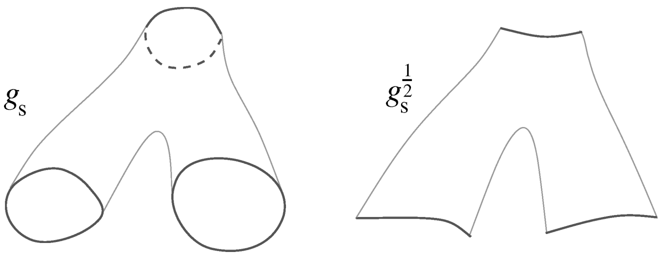

Fig. 1.3. The basic three-string interaction for closed strings, and its analogue for open strings. Its strength, g_s, along with the string tension, determines Newton's gravitational constant G_N.

Just as with the particle, it is straightforward to generalise the treatment of the string to motion in a curved background with metric $g_{\mu\nu}(x)$, and one can derive the analogue of classical geodesic equations of motion (if desired) for the string.

The string sweeps out a 'world-sheet' with coordinates $(\sigma^1, \sigma^2) \equiv (\tau, \sigma)$. The string's path in spacetime is described by $X^\mu(\tau, \sigma)$, giving the shape of the string's world-sheet in target spacetime (see figure 1.4). There is an 'induced metric' on the world-sheet given by ($\partial_a \equiv \partial/\partial\sigma^a$):

$$h_{ab} = \partial_a X^\mu \partial_b X^\nu g_{\mu\nu}, \tag{1.24}$$

with which we can perform meaningful measurements on the world-sheet as an object embedded in spacetime. Using this, we can define an action analogous to the one we thought of first for the particle, by asking that

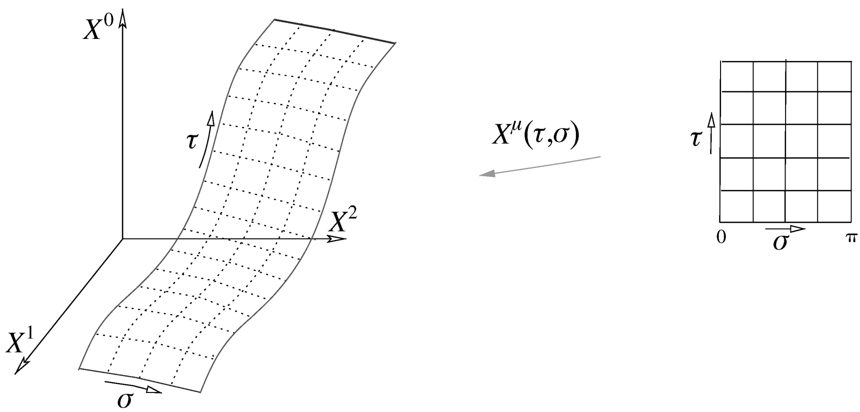

Fig. 1.4. A string's world-sheet. The function $X^\mu(\tau, \sigma)$ embeds the world-sheet, parametrised by (τ, σ), into spacetime, coordinatised by X^μ.

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we extremise the area of the world-sheet:

$$S = -T\int dA = -T\int d\tau d\sigma\,(-\mathrm{det}h_{ab})^{1/2} \equiv \int d\tau d\sigma\;\mathcal{L}(\dot{X}, X'; \sigma, \tau). \tag{1.25}$$

Expanded, this is

$$\begin{aligned} S &= -T\int d\tau d\sigma\left[\left(\frac{\partial X^\mu}{\partial\sigma}\frac{\partial X^\mu}{\partial\tau}\right)^2 - \left(\frac{\partial X^\mu}{\partial\sigma}\right)^2\left(\frac{\partial X_\mu}{\partial\tau}\right)^2\right]^{1/2} \\ &= -T\int d\tau d\sigma\left[(X'\cdot\dot{X})^2 - X'^2\dot{X}^2\right]^{1/2}, \end{aligned} \tag{1.26}$$

where X' means $\partial X/\partial\sigma$.

This is very analogous to the case of the particle, and we will analyse it further in the next chapter. However, there is much more to the story than this. The thorny question arises concerning what dynamics govern the allowed metrics, and it is a riddle of considerable depth: the string has revealed itself as generating the basic quantum of gravity as one of its modes of oscillation. Our experience from before allows us to trust that there ought to be a manner in which one can treat the graviton (and hence the string that carries it) as a small disturbance on a fixed background, but there is an additional problem which we did not have last time. Since the string is also the source of gravity, and if it dynamically generates the strength of the coupling, it ought to also determine gravitational dynamics. How does it go about determining the gravitational background in which it is supposed to propagate? In the terms we used previously, where do the field equations governing the background come from?

The surprise turns out to be that internal quantum mechanical consistency of the string theory *does* make certain demands on the properties of spacetime, in ways that no previous theory has managed before. First of all, it requires that it only propagates in spacetimes of certain dimensionality (for example, 26 for bosonic strings, 10 for superstrings). Secondly, it demands that at low energy the background metric satisfies Einstein's equations (sourced by the stress tensor due to the other massless fields)! This should be contrasted with the case of a particle where the issue of how it propagates in a metric is completely divorced from whether the metric satisfies Einstein's equations.

Somehow, the simple generalisation of a particle to a string has captured something very new. Is there an analogue of the Equivalence Principle at work which gives Einstein's equations at low energy and then new physics[‡]

[‡] It is hoped that this new physics will cure a number of problems in strongly coupled gravity, like the loss of predictability of relativistic physics at spacetime singularities such as in black holes or at the Big Bang.

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at high energy? Even though this remarkable fact is relatively old by now, there is no simple thought experiment which explains why a generalisation from a particle to a string quantum-mechanically demands the solution of field equations for which the underlying principle is covariance and equivalence.

1.4 Beyond perturbative strings: branes

The reader may have noticed that the word 'perturbative' was used a lot in the last section, even when describing the remarkable successes of string theory in the arena of quantum gravity. The Second Superstring Revolution gets its name from the remarkable change of perspective which occurred with breakthroughs in understanding of this very issue, and the resulting flow of ideas and results. A great deal of quite surprising insight was gained about the supersymmetric string theories (whose existence and consistency followed from discoveries in the First Superstring Revolution) in the limit of very strong coupling, much of which we will cover later.

The big question which arose time and again in string theory over the years before the revolution was the issue of its description beyond perturbation theory. Actually, there were possibly two problems and not just one, however they usually are discussed together, although they may be logically distinct. Motivated by analogy with field theory, string theorists sought for something like a field theory of strings, which would allow for the non-perturbative exploration of the landscape in which vacua lie, in a way which is familiar in field theory, allowing the study of important phenomena like tunnelling, instantons, solitons, etc. The idea was that there would be a 'string field' Σ whose role was to create and destroy a string in a particular configuration. This begins by being conceptually on a par with the successful ordinary field theory concept about the role of a field in creating and destroying particle quanta, but this view soon changes when one remembers that the string is like an infinite number of particles from the point of view of field theory. The ideally next simplest step would be to find a simple way of writing a kinetic energy and potential $V(\Sigma)$, which would allow a study of dynamics and hence 'second quantised' strings (to use another old misnomer). See figure 1.5. In principle, some type of field theory is not an altogether crazy thing to want to find. Given the success of the field theory framework, it would be an understatement to say that it would have been neglectful if the possibility had not been explored. There is another problem, however, into which experience with field theory seems to offer little insight. This is 'background independence'. In ordinary quantum field theory, a Lagrangian for the theory is defined with reference to a spacetime background. This

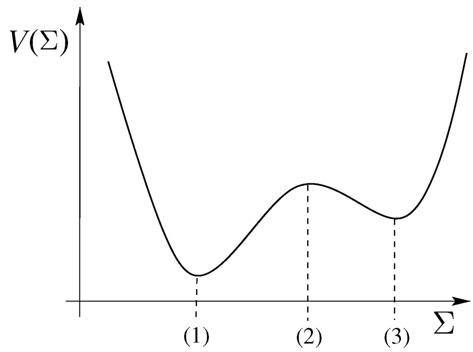

Fig. 1.5. A fanciful view of a slice through the infinite dimensional landscape of non-perturbatively accessible string vacua. Σ represents the entire field content of a string theory, and $V(\Sigma)$ is a potential. Locations (1) and (3) represent perturbatively stable vacua, while (2) is unstable. Important physics may be found in the non-perturbative effects relating these vacua.

is of course fine, since the fields are supposed to propagate on this background. However, it is not clear that this luxury should be available to us in the string theory, since it is supposed to determine the background upon which it is propagating, given that it generates gravity and the value of $G_{\rm N}$.

The search for string field theories were not entirely unsuccessful, but since they are very difficult to work with, at the time of writing, it is not clear what they have taught us. It is a remarkable achievement in itself that one *could* define a string field Σ, *and* find a sensible Lagrangian. Both the kinetic and potential are on the face of it, written in such a way that there is a chance of background independence since the 'derivative' and the means of multiplying together string fields do not seem to directly refer to spacetime. Sadly, the means of unpacking the Lagrangian to perform a computation require one to make reference to objects which originally were defined with perturbative intuition about backgrounds again, and so background independence is still not apparent.

This is not really a failure, if one reduces ones expectations about what a string field theory is supposed to do for us. It is possible to imagine that such a theory can tell us interesting physics involving various types of string vacua, and how they are inter-related, without ever addressing the background independence issue.

This possibility was regarded as unsatisfactory for a long time, since it made string theory seem logically incomplete, with no physical principle or mechanism to appeal to, given that it was supposed to be the theory

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of everything. Happily, the Second Revolution happened, and now we have a new possibility. String theory is not a theory of strings after all. There are two clear signs of this (which we will discuss later in detail). One is that there are extended objects in the theory ('D-branes') which carry[265] the basic charges of a special class of higher rank antisymmetric fields which the string theory necessarily describes, but cannot itself source. Coupled with this fact is that at arbitrarily strong coupling, these objects can become arbitrarily light (see insert 1.4), indeed lighter that the string itself, and so their behaviour dominates the low energy physics, undermining the fundamental role of the strings. A second sign is that some string theories are directly related at strong coupling (sometimes by a condensation of a tower of increasingly light D-particles) to a field theory – at low energy – which includes gravity. The short-distance completion of this gravitational theory does not seem to involve the dynamics of strings, and the new degrees of freedom are unknown. This unknown theory, whose existence is strongly suggested by the intricate web of strong/weak coupling dualities between the superstrings in diverse situations[151, 152, 153], is often called 'M-theory', and it seems that all of the superstring theories that we know of may be obtained as a limit of it. In this sense, we see that string theory is itself an *effective theory*, albeit a remarkably interesting one. All of the various string theories that we know are perturbative corners of a larger coupling space. See figure 1.6. From this new picture (in which in some cases the extended objects which become light at strong coupling are weakly coupled strings of an

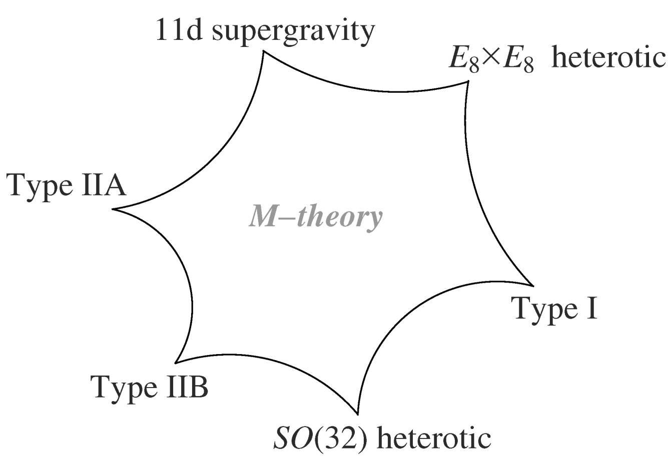

Fig. 1.6. A schematic diagram of the statement that all superstring theories, and eleven dimensional supergravity, are effective descriptions of certain dynamical corners of a larger theory, called 'M-theory'.

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Insert 1.4. Soliton properties and the kink solution

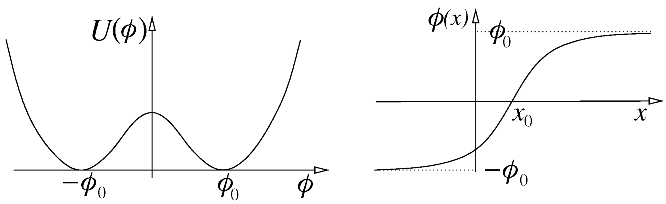
Everybody's favourite soliton example is the kink solution of ϕ^4 theory in 1+1 dimensions. The mass m and the coupling λ combine into a dimensionless coupling $g = \lambda/m^2$, and we write:

$$\mathcal{L} = -\frac{1}{2}\partial^\mu\phi\partial_\mu\phi - U(\phi), \qquad U(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2.$$

The kink (or anti-kink) solution is

$$\phi_\pm(x) = \pm\frac{1}{\sqrt{g}}\tanh\left(\frac{m(x-x_0)}{\sqrt{2}}\right),$$

and so it is clearly an *interpolating solution* between the two vacua (located at $\pm\phi_0 = \pm 1/\sqrt{g}$) of the double well potential.



The parameter x_0 is a constant, corresponding to the ability to translate the solution. The configuration's mass-energy is:

$$E = \int_{-\infty}^{\infty}\left(\frac{1}{2}\left(\frac{\partial\phi_\pm}{\partial x}\right)^2 + U(\phi_\pm)\right)dx = \frac{2\sqrt{2}}{3}\frac{m}{g},$$

which is inversely proportional to the dimensionless coupling. So at weak coupling, this is a very heavy localised lump of energy. If we could trust this formula at strong coupling (and for various types of soliton in e.g. supersymmetric theories, we can), it is clear that for large g this solution becomes a light, sharply localised particle. In fact, it has a conserved charge, due to the existence of the topological current $j_\mu = (\sqrt{g}/2)\epsilon_{\mu\nu}\partial^\nu\phi$, which is:

$$Q = \int_{-\infty}^{\infty} j_0 dx = \frac{\sqrt{g}}{2}\left(\phi(+\infty) - \phi(-\infty)\right) = \pm 1.$$

All of these properties will appear for solitons of theories which we shall study. The validity of the mass formula at strong coupling will allow various 'dualities' of supersymmetric theories to be uncovered.

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entirely different type from the starting theory, giving a 'string–string' strong/weak coupling duality), it is clear that the string field theory approach would have had to produce a completely unlooked-for phenomenon, and convert the world-sheet expansion of one type of string (say a closed one) into the completely different type of world-sheet expansion of another type of string (say an open one). It would also have to point to new directions in which there is a perturbation theory not involving strings at all. Lastly, it would also have to be background independent.

Of course, this may yet happen (but we might not call it a field theory any more!), but another possibility is that string field theory (at least in the intuitive form in which it was conceived) will be useful as an effective theory (arising from M-theory) useful for the study of a restricted but important set of non-perturbative effects.

1.5 The quantum dynamics of geometry

The issue of background independence may be tied up with matters which the theory is only really still just touching on, and so it may have been premature to worry about it previously. This is the fact that there are dynamical signs that clearly show that string theory avoids a definite picture of some of the properties of spacetime which we would have thought were fixed, if we were field theorists.

Scattering of strings seems to show that attempts to confine the string to a small domain of spacetime are defeated by the strings' tendency to increasingly extend itself and spread out. From T-duality[14] (to be first encountered in chapter 4, but probably in every chapter beyond that), we learn that when a string theory is compactified on a circle, there is an ambiguity in the spectrum about whether the propagation is on a circle of radius R or radius ℓ_s^2/R. The standard 'momentum' states with energy in multiples of $1/R$ are joined by 'winding' states whose energy is in multiples of R/ℓ_s^2, coming from winding around the circle. The 'duality' exchanges these two types of mode. This is remarkable, especially if one considers the limit that $R \to 0$, since it says that an arbitrarily small circle compactification (reducing an effective spacetime dimension) is physically equivalent to having an arbitrarily large dimension (restoring an effective dimension). The outcome of this reasoning is that there appears to be an effective minimum distance arising in the dynamics of (perturbative) strings, of order the string scale ℓ_s. This is qualitatively just the sort of granularity of spacetime which one might have anticipated (and indeed it was) in thinking about expectations for a quantum theory of gravity. We can go even further, however.

As already mentioned, at strong coupling some string theories turn into something which at low energy is a field theory in one dimension *higher* than the target spacetime of the weakly coupled string. Since the string coupling is dynamically generated by the string itself, we arrive at the result that the dimension of spacetime itself is dynamical.

Also, the coordinates describing various objects like D-branes located in string theory's target space arise as not just numbers, but matrices[26]. For example, in superstring theory for N pointlike D-branes (known as D0-branes or D-particles), there are nine $N \times N$ matrices, $X^i(\tau)$, describing their world-lines parametrised by τ. When the D-branes are widely separated from each other, it is dynamically favourable for these matrices to be diagonal, and we have N copies of the usual coordinates x^μ describing the positions of N pointlike objects in nine spatial directions:

$$X^i(\tau) = \begin{pmatrix} x_1^i(\tau) & 0 & 0 & \cdots & \cdots \\ 0 & x_2^i(\tau) & 0 & \cdots & \cdots \\ 0 & 0 & x_3^i(\tau) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \ddots & \cdots \\ \cdots & \cdots & \cdots & \cdots & x_N^i(\tau) \end{pmatrix} . \tag{1.27}$$

When the branes are close together, there are dynamically favourable regimes when these matrices are non-commuting, and correspondingly, the spacetime coordinate interpretation is now in terms of a non-commutative picture. There is more here, actually. Since D0-branes turn out to be momentum modes, in a compact direction, of an eleven dimensional graviton, this picture turns out to be a sort of light cone formulation of the eleven dimensional theory. This is the beginning of the *Matrix Theory*[157] formulation of M-theory.

Spacetime is clearly a far more interesting place when the dynamics of string/M-theory are explored, and so it may be a while before we know even if we are asking the right sorts of questions about its nature. This includes the issue of background independence, and it may be that we have to wait for a complete formulation of M-theory (which may well have nothing to do with spacetime at all) before we get an answer.

1.6 Things to do in the meantime

While we wait for a complete formulation of M-theory to show up, there is a lot to do in the meantime. String theory's second revolution has provided us with a large number of tools to explore many regimes of fundamental physics, both old and new.

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Gauge theories arise in string theories in many different (and often interrelated) ways, for example by dimensional reduction and the Kaluza–Klein mechanism (described in section 4.1.1), or as the collective dynamics on the world-volume of branes (described in section 4.10), or from gauge fields intrinsic to the structure of a closed string theory (described in section 7.2). So string theory is an arena for studying gauge theories. The very geometrical way in which string theories treat gauge fields allows for many gauge theory phenomena to be usefully recast in geometrical terms. This also means that known gauge theory phenomena, correctly interpreted in this context, can also teach us new things about the geometry of string theories. Many of the applications of D-branes which we will discuss later in this book are concerned with this powerful dialogue.

In this way, useful tools can be extracted for application to very concrete and pragmatic questions in the dynamics of strongly coupled gauge theory, of great concern to us of course in the physics being explored or shortly to be explored in experiments.

Since string theory is also a theory of gravity, it is exciting to learn that there are regimes where much progress may be made in the study of situations where hard questions about quantum gravity arise. The most celebrated example of this is the precise statistical interpretation of Bekenstein's thermodynamical black hole entropy[262], for a large class of black holes. This thermodynamical quantity can arise as the inevitable conclusion of semi-classical treatments of quantum gravity, where quantum fields are studied in a classical black hole background (a useful conceptual and technical compromise alluded to earlier). Such a treatment led Hawking[261] to realise that there is thermal radiation (at a specific temperature) from a black hole, after other suggestive properties[289, 292] led Bekenstein to the realisation that there is an entropy associated to the area of the horizon. The universal Bekenstein–Hawking entropy for a black hole is:

$$S = \frac{A}{4G_{\mathrm{N}}}, \tag{1.28}$$

and is at the heart of the laws of black hole thermodynamics. This was a bit awkward, since there was no underling theory of quantum gravity to supply the 'statistical mechanics' which account for the precise relation between the entropy and the properties of the black hole. As we will describe in detail, for a large class of black holes, string theory provides the precise answer, in terms of D-brane constituents, and the gauge theories which describe them. In fact, (for a smaller class of black holes) the spacetime dynamics of individual D-branes conspires to provide a microscopic mechanism for the operation of the second law of thermodynamics as well[7].

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One of the most profound insights of the revolution which might have the furthest-reaching consequences, is the identification of tractable regimes where a duality between gravitation and gauge theory can be found. This grew out of the above results concerning black holes, and even the ideas concerning the translation of gauge theory phenomena into geometry, but it is in some sense logically distinct from those. There is a very striking and intricate dynamical duality between the two, which again crosses dimensionality and is indicative of a very rich underlying picture. The 'AdS/CFT correspondence'[270, 271, 272], the title under which the simplest examples are known, is also the sharpest known example of what is known as the 'Holographic Principle'[286, 287], which states (roughly) that there should be a lower dimensional non-gravitational representation of the degrees of freedom of any quantum theory of gravity. Matrix theory is another example[158].

The idea of the principle arises from the realisation that any high energy density scattering used to probe the short distance degrees of freedom in a theory including gravity will ultimately create black holes. Black holes seem to exhibit all of their degrees of freedom on their horizon, an object which is of one dimension fewer than the parent theory. This suggests (but of course does not supply a definite constructive tip for how to find it) that there is a more economical description of theories of D-dimensional gravity in terms of a theory in $D-1$ dimensions. The AdS/CFT correspondence manages this by relating a theory of gravity in an anti-de Sitter background (a highly symmetric spacetime with negative cosmological constant, reviewed in section 10.1.7) to a strongly coupled $SU(N)$ gauge theory (of large N) in one dimension fewer. This is remarkable, since theories of gravity and gauge theory are so very different in crucial dynamical respects, and we explore this in detail in chapter 18, showing how it arises from our study of D-branes, and exploring some of the consequences for new descriptions of strongly coupled gauge theory phenomena.

Exploring the correspondence in more complicated cases is of great interest, as it might give us insights and new tools which we can apply to more phenomenologically relevant gauge theories, and we spend some time discussing some examples of this.

1.7 On with the show

It is apparently an Irish saying that one will never plough a field by turning it over in one's mind, and so we should now begin the task of exploring things more carefully. In setting the scene, we have begun to unpack some of the more difficult concepts and some of the language which we will encounter many times as we go along. We will proceed by developing the basic language of string theory, uncovering many remarkable phenomena and vacua, using perturbation theory only. Certain perturbative hints of non-perturbative physics will appear from time to time, and with the help of D-branes and supersymmetry, we later uncover such physics using many 'duality' relations. Much later, we combine these techniques and ideas to probe and map out aspects of M-theory, and also to study certain aspects of duality in field theory. It will be an exciting journey.