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OBITUARY

IRVINE NOEL BAKER 1932–2001

1. *Life*

Noel Baker was born on 10 August 1932 and died, of a heart attack, on 20 May 2001. He was the only child of a farming family living near the township of Virginia north of Adelaide in South Australia, and a fourth-generation Australian. From 1938 to 1944 he attended the local school, winning a scholarship to King's College, Adelaide, and from there another scholarship to Prince Alfred College, Adelaide. Here he was inspired by an enthusiastic mathematics teacher, Mr Williams, who expected pupils to practise mathematics assiduously, even on Saturday mornings. Prince Alfred College still aims to foster a love of mathematics today, and it has recently launched a centre for excellence in mathematics, named after Noel.

In 1948, Noel won the top leaving honours award in South Australia's public examinations. He studied Mathematics, Physics and Chemistry at the University of Adelaide, and was awarded a BSc in 1952. Noel then took an MSc by thesis; this was awarded in 1954. At this time he became a student of George Szekeres, later professor of mathematics at the University of New South Wales, Sydney.

The mathematical influence of Szekeres was profound, influencing much of Noel's subsequent research in the theory of iteration. They also became close friends, with a shared love of chamber music as well as mathematics. Noel had taken piano lessons at school, inspired by trying his grandfather's piano, and he developed his talent further by studying at the University of Adelaide's Elder Conservatorium. Szekeres was a violinist, and they often played together.

Noel's musical and mathematical interests encouraged him to learn German, and in 1955 he won a German government scholarship to the University of Tübingen, where he worked under Hellmuth Kneser and obtained his doctorate in 1957. Tübingen was then a sleepy, romantic university town on the banks of the Neckar, where the mathematics department included three well-known K's: Kneser, Knopp and Kamke. As an Australian, Noel was regarded as slightly unusual. Indeed, several years later at Oberwolfach the elderly H. Cremer, when thanking Noel for a talk, spoke of him as a remarkable young mathematician from far-off Australia who must be an *Autodidakt*.

In Tübingen, Noel met Gillian Hawkins, a language student from London and another classical pianist. They married in 1958, and their marriage was enduring and happy. They shared a deep love of music, attending many concerts and playing piano, both together *vierhändig* (a repertoire in which the works of Schubert were of supreme importance), and also with family and friends. Their other interests included walking, bird-watching and visiting historic places. They always maintained contacts in Australia, and used to offer Australian red wines to visitors to the family house in South London long before these wines became well known. Noel and Gillian's two sons Stephen and Michael both inherited a love of music, one a pianist and the other a violinist, as well as many of Noel's character traits, his love of the outdoors and his delight in the changing seasons.

From 1957 to 1959, Noel taught mathematics at the University of Alberta in Edmonton, Canada. In 1958, he spoke at the ICM in Edinburgh, where he was spotted by Walter Hayman, professor of pure mathematics at Imperial College, London. Hayman was founding what was to become a major international group in complex analysis, and Noel moved to London in 1959. He spent more than forty years in the Department of Mathematics at Imperial College, and made huge contributions to the College, both academically and socially. His colleagues remember him as reserved but warm, with a quiet yet incisive sense of humour ('blink and you'll miss it'), and he had a formal but popular lecturing style. His research was profound, though its significance was not fully appreciated in the early years, and as a research supervisor he was effective and approachable. Noel was also an efficient administrator, acting as Assistant Director of the Department for many years, dealing with the organisation of lectures and examinations, and as a joint organiser of several major conferences. A more lighthearted contribution to departmental life was playing the piano, often with Gillian, at occasional 'Musical Evenings'. Later, he was for over fifteen years the organiser of the Imperial College 'Lunch Hour Concerts', held every Thursday in the winter terms and featuring professional chamber musicians. Outside the Department, he played a leading role in the LMS publications, as a member of the Editorial Board (1973–86) and as Joint Editor-in-Chief of the *LMS Journal* (1989–94).

In his research, Noel worked on many problems in complex analysis, and had a wide range of collaborators, but iteration theory, his great love, was for many years a lone interest. However, when the subject was reborn around 1980, partly as a result of the advent of accessible computer graphics, it became clear to the new adherents that Noel had for many years been quietly and carefully completing the foundations begun earlier in the century by the French mathematicians Pierre Fatou and Gaston Julia. He had also pointed the way towards many future developments, both by proving new results, and by posing challenging problems. In the explosion of research on iteration theory that took place in the subsequent twenty years, very many of the papers published on iteration made reference to Noel's work, and he received a great many invitations to speak at international conferences on iteration. At these he would often appear reserved, much preferring to let others speak about the latest work, even though he was the acknowledged authority on very many matters, and the person whose judgement about the validity of a new proof was always sought. He became a reader in 1968 and then a professor in 1990, retiring from Imperial College in 1997. During that time he had supervised two students doing an MSc by research thesis, Joyce Whittington and Valerie Eke, and nine PhD students, Prodipeswar Bhattacharyya, Robert Goldstein, Lennox Liverpool, Jim Langley, Jonathan Weinreich, Gwyneth Stallard, Ramez Maalouf, Matthew Herring and Patricia Domínguez.

Noel continued to work vigorously on iteration in his semi-retirement, and became more relaxed, particularly enjoying the freedom to travel at any time, both for research visits and for leisure. One of his last papers was dedicated to George Szekeres, on the occasion of the latter's 90th birthday, and at the time of Noel's death other papers were in preparation; these have appeared posthumously. In May 2002, a memorial Lunch Hour Concert was given at Imperial College by the Maggini Quartet. They concluded with the slow movement of Haydn's Opus 33, No. 6.

2. *Functional equations and iteration of entire functions*

For his MSc at the University of Adelaide, Baker was encouraged by Szekeres to work on the functional equation

$$f(f(z)) = F(z), \tag{2.1}$$

where f and F are analytic functions. In 1950, Kneser had shown that the equation $f(f(x)) = e^x$ has a solution f that is real-analytic on the whole real line. In his first mathematical paper [**1**], Baker attacked the complex analytic version of this problem using the theory of iteration of analytic functions, which had been developed principally by Fatou and Julia, and which was not well known at that time. He used this theory to show, amongst other things, that if F belongs to a certain class of entire functions, which includes the exponential function, then equation (2.1) has no entire solution. This first paper also draws on Wiman's result, often used later, that an entire function of order less than $\frac{1}{2}$ must tend to ∞ along a sequence of expanding circles. Then, in order to construct examples, he used the sophisticated Wiman–Valiron method, which relates the maximum modulus of an entire function to the maximum term of its Taylor series. Throughout his career, Baker was to find ever more techniques from classical complex analysis that can usefully be applied to iteration theory.

Baker's doctoral thesis, published in [**2**], continued his study of functional equations. He first showed that if f and g are both transcendental entire functions with order less than $\frac{1}{2}$, then there is an expanding sequence of Jordan curves Γ_n

such that

$$|f(g(z))| > \exp(\max\{|z| : z \in \Gamma_n\}), \quad \text{for } z \in \Gamma_n,$$

and that this result is best possible. He then considered the problem of determining, for a given transcendental entire function f, the set $P(f)$ of transcendental entire functions g such that the equation

$$f(g(z)) = g(f(z)), \tag{2.2}$$

holds; that is, f and g commute, or are *permutable*. He showed that if g is a non-constant polynomial in $P(f)$, then $g(z) = ze^{2\pi im/n} + b$, where m and n are positive integers, and he gave a complete description of $P(f)$ for functions of the form $f(z) = ae^{bz} + c$. Finally, he returned to the functional equation (2.1) and showed, amongst other things, that if $F(z) = e^z - 1$, then there are no solutions f analytic at 0.

This work led Baker to consider problems about periodic points. Let f be a rational or entire function. The sequence of iterates f^n is defined by

$$\begin{aligned} f^1(z) &= f(z), \\ f^{n+1}(z) &= f(f^n(z)), \qquad n = 1, 2, \ldots. \end{aligned}$$

A *fixed point* ζ of f is a solution of the equation $f(z) = z$, and ζ is classified as *attracting*, *indifferent* or *repelling* according to whether the *multiplier* $f'(\zeta)$ satisfies $|f'(\zeta)| < 1$, $|f'(\zeta)| = 1$ or $|f'(\zeta)| > 1$. More generally, ζ is a *periodic point* of f of *period* p if ζ is a fixed point of f^p, and a similar classification of such ζ is made. The *exact period* p is the smallest period. Periodic points play a great role in the theory, both in relation to the local behaviour of iterates near periodic points of the various types, and in relation to the global behaviour of iterates, described later.

It was already known that for an entire function there must be infinitely many periodic points of period p, for all $p \geqslant 2$, but Baker considered the unsolved problem of the existence of periodic points of a given *exact* period. After obtaining partial results in [**3**] and [**5**], he showed in [**6**] that for all non-linear entire functions there exist periodic points of exact period p, for all p with at most one exception; for example, $f(z) = z + e^z$ has no fixed points. In a later paper [**13**], Baker showed that for a polynomial the only possible exceptional value in this result is $p = 2$, the corresponding exceptional functions being $f(z) = z^2 - z$ and other quadratics 'similar' to this one. He also conjectured that for a transcendental entire function the only possible exceptional value is $p = 1$, and this was shown to be true by Bergweiler $\langle \mathbf{3} \rangle$.

Baker studied the case $p = 2$ in greater detail in [**16**]. Here, he conjectured that if f is a transcendental entire function and ℓ is a line in $\mathbb{C}$, then there exist solutions of $f^2(z) = z$ not contained in ℓ. He proved that this is the case for functions f of order less than $\frac{1}{2}$, and the full conjecture was shown to be true by Bergweiler, Clunie and Langley in $\langle \mathbf{5} \rangle$.

Baker also built on his work in [**2**] to make a major study [**8**] of solutions of the functional equation (2.2) in the case when f has a fixed point of multiplier 1. Following Hadamard and Szekeres, he considered f and g as formal power series:

$$f(z) = z + a_{m+1}z^{m+1} + \ldots, \qquad a_{m+1} \neq 0,$$

and

$$g(z) = b_1 z + b_2 z^2 + \ldots.$$

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He showed that it is sufficient to take $b_1 = 1$, and that in this case (2.2) has a family of formal power series solutions $g = f_\lambda$, $\lambda \in \mathbb{C}$, of the form

$$f_\lambda(z) = z + \lambda a_{m+1} z^{m+1} + \sum_{n=m+2}^{\infty} b_n(\lambda) z^n,$$

where $b_n(\lambda)$ is a polynomial of degree at most $n - m$. For a given power series f, the family f_λ, $\lambda \neq 0$, may or may not include solutions that are convergent near 0. Baker established the striking result that the set

$$\mathcal{R}(f) = \{\lambda : f_\lambda(z) \text{ is convergent near } 0\}$$

must have one of the following simple forms:

(a) $\{0\}$;

(b) $\{n\lambda_0 : n \in \mathbb{Z}\}$, where $\lambda_0 \neq 0$;

(c) $\{m\lambda_0 + n\lambda_1 : m, n \in \mathbb{Z}\}$, where $\lambda_0, \lambda_1 \neq 0$ and λ_1/λ_0 is not real;

(d) $\mathbb{C}$.

Baker gave examples of cases (a), (b) and (d), and it was later shown by Écalle ⟨**9**⟩, and independently by Baker's student Liverpool ⟨**25**⟩, that case (c) cannot occur. In case (d), the function f is said to be *embeddable* in a continuous group of analytic iterates, and Baker showed in [**20**] (see also [**58**]) that many functions, including most algebraic functions, are not embeddable in this sense.

It was also shown in [**8**] that the values of λ in $\mathcal{R}(f)$ for which f_λ is entire, form a discrete and hence countable set. This result was used by Baker to show that if f is a transcendental entire function, exactly one of whose fixed points has multiplier 1, then the set $P(f)$ of entire functions that commute with f forms a countable set. In [**7**] he generalised this somewhat, but a completely general result was to follow eight years later. To describe this and much other work, we need to set out the basic elements of the global theory of iteration, or 'complex dynamics' as it has become known, as developed by Fatou and Julia. We use notation and language in common current use, but the reader is warned that there have been significant changes in these over the years.

Let f be a rational function of degree at least 2, or a transcendental entire function. The set of points near which the sequence of iterates f^n forms a normal family is called the *Fatou set* $F(f)$, and its complement is called the *Julia set* $J(f)$. The sets $F(f)$ and $J(f)$ have the following fundamental properties, first established for rational functions in ⟨**20**⟩ and ⟨**14**⟩, and for transcendental entire functions in ⟨**15**⟩. (A good modern source is ⟨**4**⟩.)

(a) $F(f)$ and $J(f)$ are both completely invariant sets; that is, $z \in F(f)$ if and only if $f(z) \in F(f)$.

(b) For $n \geqslant 2$, we have $F(f^n) = F(f)$ and $J(f^n) = J(f)$.

(c) $J(f)$ is a non-empty perfect set.

(d) There are at most two exceptional points α with the property that the set $\{z : f^n(z) = \alpha, \text{ for some } n\}$ is finite.

(e) Given any open disc D that meets $J(f)$, and any compact set K that contains no exceptional points, we have $f^n(D) \supset K$, for all sufficiently large n.

In ⟨**15**⟩, Fatou studied the iteration of transcendental entire functions in some detail, giving several examples that pointed to significant differences from the theory that had been developed for rational functions. He was led to ask the following fundamental questions about a general transcendental entire function f.

1. Are the repelling periodic points of f dense in $J(f)$?
2. Are there examples where $J(f) = \mathbb{C}$? In particular, is this true for $f(z) = e^z$?
3. Can $J(f)$ be totally disconnected?
4. Must $J(f)$ contain infinitely many analytic curves, at each point of which $f^n \to \infty$?

Question 1 is of great theoretical importance, and it had been answered in the affirmative for rational functions by both Fatou and Julia. Fatou had also given an example of a rational function f for which $J(f)$ is totally disconnected, and Lattès ⟨**23**⟩ had provided an example for which $J(f) = \mathbb{C}$. Most of Fatou's questions were answered by Baker during the decade 1965–1975, as we now indicate.

The first question was answered in the affirmative in the paper [**22**], which is of fundamental importance in complex dynamics, and is appropriately dedicated to Hellmuth Kneser. Here, Baker called on a deep covering theorem due to Ahlfors; see ⟨**18**, p. 148⟩. He used this theorem to prove that arbitrarily close to each point of $J(f)$, there is an open disc D that is mapped univalently by some iterate of f onto a superset of D. Thus D contains a repelling periodic point of f, as required. From this, Baker deduced the general result, mentioned earlier, that if f is any non-linear entire function, then the set of entire functions that commute with f is countable. Many authors have tried to simplify the proof given in [**22**] that the repelling periodic points are dense in $J(f)$, in order to avoid the deep theorem of Ahlfors. Eventually, elementary proofs based on a renormalisation technique were given by Bargmann ⟨**2**⟩, and by Berteloot and Duval ⟨**6**⟩.

Two years later [**25**], Baker answered the first part of Fatou's second question by showing that there is a function of the form $f(z) = kze^z$, where $k > 0$, such that $J(f) = \mathbb{C}$. To do this, he established a beautiful connection between the limiting behaviour of the iterates of a transcendental entire function f in components of $F(f)$ and the set $S(f)$ of *inverse function singularities* of f, which consists of the critical values and the asymptotic values of f. Taking $P(f) = \bigcup_{n=0}^{\infty} f^n(S(f))$ to be the *post-singular set* of f, Baker proved that if $\overline{P(f)}$ has empty interior and connected complement, then any limit function of a subsequence of f^n in a component of $F(f)$ must be a constant lying in $\overline{P(f)} \cup \{\infty\}$. He then chose $k > 0$ in such a way that for $f(z) = kze^z$, the set $P(f)$ is rather simple and it is possible to exclude all such constant limit functions. A proof was given ten years later by Misiurewicz ⟨**27**⟩ that if $f(z) = e^z$, then we have $J(f) = \mathbb{C}$.

Baker answered Fatou's third question in the negative in [**32**]. If $J(f)$ is totally disconnected, then $F(f)$ must have a single unbounded multiply connected component. Baker had already constructed in [**9**] an example of a transcendental entire function for which $F(f)$ has at least one multiply connected component. This function was of the form

$$f(z) = Cz^2 \prod_{n=1}^{\infty} \left(1 + \frac{z}{r_n}\right), \tag{2.3}$$

in which the positive constants $r_1 < r_2 < \dots$ have the property that

$$f(A_n) \subset A_{n+1}, \qquad \text{where } A_n = \left\{z : r_n^2 < |z| < r_{n+1}^{1/2}\right\}.$$

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However, [**9**] did not determine whether $F(f)$ has a single unbounded multiply connected component or a sequence of bounded multiply connected components. In [**33**], Baker used Schottky's theorem ⟨**18**, p. 169⟩, yet another result from classical complex analysis, to show that the latter must be the case. This solved another important problem in complex dynamics, open since the work of Fatou and Julia, by showing that the function (2.3) has a sequence of *wandering domains* – that is, distinct components U_n of $F(f)$ such that $f(U_n) \subset U_{n+1}$, for $n = 1, 2, \ldots$. In contrast, Sullivan ⟨**34**⟩ showed that rational functions do not have wandering domains. The paper [**32**], written later but published earlier than [**33**], used Schottky's theorem once again to show that a transcendental entire function cannot have an unbounded multiply connected component of $F(f)$, thus proving that $J(f)$ can never be totally disconnected.

The results in [**32**] and [**33**] led to much further work. In [**53**], Baker showed that wandering domains for transcendental entire functions may be infinitely connected, and it has recently been shown by Kisaka and Shishikura that they can have any given finite connectivity. The result in [**32**] shows that if f is a transcendental entire function, then $J(f)$ must contain a continuum, so its Hausdorff dimension $\dim_H J(f)$ is at least 1. Later, McMullen ⟨**26**⟩ gave examples for which $\dim_H J(f) = 2$, and Baker's student Stallard ⟨**33**⟩ proved that $\dim_H J(f)$ can take any value between 1 and 2. It remains an open question whether $\dim_H J(f) = 1$ is possible.

Fatou's fourth question was not stated quite precisely. He had observed that for the examples $f(z) = z + 1 + e^{-z}$ and $f(z) = h\sin(z)$, $0 < h < 1$, there exist infinitely many analytic curves in $J(f)$ on which $f^n \to \infty$, and he asked if this property holds more generally. In Baker's example (2.3), there are certainly no curves in $J(f)$ tending to infinity on which $f^n \to \infty$. A related question is: 'Does the set $I(f) = \{z : f^n(z) \to \infty\}$ contain curves tending to ∞, or more generally unbounded continua?' The first detailed study of $I(f)$ was made by Eremenko ⟨**10**⟩, who showed that $I(f) \neq \emptyset$, $\partial I(f) = J(f)$ and $\overline{I(f)}$ has no bounded components, and it was shown in ⟨**29**⟩ that $I(f)$ must always have at least one unbounded component. However, the properties of $I(f)$ are still not completely understood in general; for example, it remains an open question whether *all* components of $I(f)$ are unbounded.

Sullivan's remarkable result ⟨**34**⟩, that rational functions do not have wandering domains, was proved using new techniques based on quasiconformal conjugacy, and led to many major developments. Baker quickly saw that these new techniques would also apply to various families of transcendental entire functions, and a proof that exponential functions of the form $f_c(z) = e^{cz}$, $c \in \mathbb{C}$, have no wandering domains appeared in [**49**]. This was one of a number of papers at that time which established many of the basic dynamical properties of the exponential family and began the description of the corresponding parameter space, the 'exponential Mandelbrot set', which has since been the subject of much study. In [**50**], Baker went further by showing that entire functions in a certain much larger family, including those of the form

$$f(z) = \int^z P(t)e^{Q(t)}\,dt \quad \text{and} \quad f(z) = P(e^z),$$

where P and Q are polynomials, do not have wandering domains. He also constructed various examples of transcendental entire functions with simply connected wandering domains, including examples of such functions of all orders $\rho \geqslant 1$.

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In [**41**], Baker initiated another major development by showing that if a transcendental entire function f has order of growth at most $\frac{1}{2}$, minimal type, then $F(f)$ has no unbounded invariant components, and he also gave a more restrictive condition on the maximum modulus of f, which forces every component of $F(f)$ to be bounded. The question of whether the latter conclusion follows from order at most $\frac{1}{2}$, minimal type, remains open; see ⟨**32**⟩ and ⟨**1**⟩ for results in this direction. A key step in Baker's proof is to exclude unbounded invariant components of $F(f)$ in which $f^n \to \infty$. (An example of a transcendental entire function with such a component had been given by Fatou in ⟨**15**⟩.) By using Schottky's theorem again, he was able to show that in any such component there is a path Γ which tends to ∞, and on which $|f(z)| = O(|z|^k)$, for some $k > 0$, and this contradicts the given hypotheses on f. In a later paper [**57**], Baker used properties of the hyperbolic metric to improve this bound along a path to $|f(z)| = O(|z|)$ in the case of a simply connected domain. In recognition of his work, Eremenko and Lyubich used the name *Baker domain* in ⟨**13**⟩ for an unbounded invariant component of $F(f)$ in which $f^n \to \infty$. Baker himself had called such a component *essentially parabolic* or a *domain at* ∞, but the new name found instant favour with the complex dynamics community, and eventually Baker himself began reluctantly to use it. In their fundamental paper ⟨**13**⟩, Eremenko and Lyubich showed that if the set $S(f)$ of inverse function singularities of a transcendental entire function f is finite, then f has no wandering domains (see also ⟨**16**⟩ for this result), and if $S(f)$ is bounded, then f has no Baker domains.

Yet another fundamental contribution to the iteration of transcendental entire functions came in the papers [**65**], [**73**] and [**74**]. Once again, an unbounded invariant component U of $F(f)$ was considered, but now the aim was to describe the nature of the boundary of U. Some special cases had been investigated by other authors, following the appearance of computer pictures of Julia sets, but Baker and his students Weinreich and Domínguez attacked the general case. In [**65**], it was shown that:

(a) if U is not a Baker domain (that is, if U is an attracting basin, parabolic basin, or a Siegel disc), then ∂U is sufficiently complicated that ∞ belongs to the impression of every prime end of U; and

(b) if ∂U is a Jordan curve in the extended complex plane $\hat{\mathbb{C}}$ (and such U do exist), then not only must U be a Baker domain, but f must be univalent in U.

The key tool introduced in this work arises from the fact that if Ψ is a conformal map from the unit disc D onto U, then $\Psi^{-1} \circ f \circ \Psi$ is an *inner function* – that is, an analytic self-map of D whose angular limits have modulus 1 almost everywhere on ∂D. The paper [**65**] initiated a version of the Fatou–Julia theory for inner functions, a topic now of interest in its own right, and this theory was taken further in [**73**]. Here, more precise information about such unbounded invariant components U was obtained, by studying the set

$$\Theta = \{e^{i\theta} : \Psi(re^{i\theta}) \to \infty \text{ as } r \to 1\}.$$

Generalising work of Kisaka ⟨**21**⟩, Baker and Domínguez proved in [**73**] that:

(a) if U is not a Baker domain and ∞ is accessible in U, then $\bar{\Theta} = \partial D$; and

(b) if U is a Baker domain and f is not univalent in U, then $\bar{\Theta}$ contains a perfect set.

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It had been shown in [**41**] that ∞ is accessible in any Baker domain of a transcendental entire function f, but it is an open question as to whether this is true in other types of unbounded invariant components of $F(f)$. In both the above cases, it follows that ∂U and $J(f)$ are disconnected, and in [**74**] it was further deduced that in these cases $J(f)$ has uncountably many components. The latter paper also considered the situation where f is a transcendental entire function with a *completely invariant* component U of $F(f)$; that is, $z \in U$ if and only if $f(z) \in U$. (Baker had proved in [**24**] that there can be at most one such completely invariant component.) In [**74**] it was shown that in this case $J(f)$ is so 'hairy' that it is locally connected at no point, once again establishing in general what had previously only been known to be true for special cases.

3. *Iteration of meromorphic functions*

The iteration of transcendental meromorphic functions is a natural object of study because such iterations arise, for example, in the Newton–Raphson method. Baker played a key role in extending the Fatou–Julia theory to these functions, and to even more general classes of functions.

If f is a transcendental meromorphic function, then the iterates f^n, $n = 1, 2, \ldots$, are defined only on the complement of the set $B = \bigcup_{n=0}^{\infty} f^{-n}(A)$, where A is the set of poles of f. By Picard's theorem, B is infinite except in the special case when A is a singleton $\{\alpha\}$ and f is a self-map of the punctured plane $\mathbb{C} \setminus \{\alpha\}$. The basic Fatou–Julia theory of this special case was established by Rådström ⟨**28**⟩ and Baker's student Bhattacharyya ⟨**7**⟩. In [**54**], Baker showed that for an analytic self-map f of the punctured plane, at most one component of the Fatou set $F(f)$ is multiply connected, and if such a component occurs, then it must be doubly connected and invariant. Thus any wandering domains are simply connected, and Baker showed that such wandering domains can indeed occur. In [**72**], Baker and Domínguez studied the Julia sets of such functions, and established many of their connectedness properties, which are somewhat different from those of transcendental entire functions.

The Fatou–Julia theory of the iteration of *general* transcendental meromorphic functions was established in the fundamental papers [**62**], [**63**], [**64**] and [**66**] by Baker, Kotus and Lü. The Fatou set $F(f)$ is here taken to be the set of points near which the iterates f^n are defined and form a normal family, and then $J(f) = \hat{\mathbb{C}} \setminus F(f)$. Many of the basic results turn out to be similar to those for rational and entire functions, but there are some striking differences. For example, in [**62**] the authors showed that $J(f)$ is once again the closure of the repelling periodic points of f, and this fact is used to give a complete classification of those transcendental meromorphic functions, such as $f(z) = \tan z$, for which $J(f)$ is a subset of the real line. Then, in [**63**], they used techniques from approximation theory, pioneered by Eremenko and Lyubich ⟨**12**⟩, to construct transcendental meromorphic functions with wandering domains of all possible connectivities. The question of non-wandering components was taken up in [**64**], where they showed that precisely five possible types can arise for a transcendental meromorphic function, namely, attracting basins, parabolic basins, Siegel discs, Herman rings and Baker domains. Moreover, any invariant components of $F(f)$ must be simply connected, doubly connected, or infinitely connected. But perhaps the most striking result here was the construction, using the technique of quasiconformal surgery

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introduced by Shishikura ⟨**31**⟩, of a transcendental meromorphic function f with a preperiodic component of $F(f)$ of any given finite connectivity. This is in sharp contrast to a transcendental entire function, for which any multiply connected component of $F(f)$ must be wandering [**50**]. Finally, in [**66**], Sullivan's method of quasiconformal conjugacy was adapted to show that a transcendental meromorphic function with only finitely many inverse function singularities has no wandering domains. These four papers opened a new and fruitful area of research, made even more accessible by the excellent survey article ⟨**4**⟩, which appeared soon after.

However, it turns out that Fatou–Julia theory can be developed in a yet more general, and also more natural, context. For a transcendental meromorphic function f, the iterates f^n need not be meromorphic. It is desirable, however, to have a closed system of iterates, so that we can consider, for example, the Fatou set of f^n, for $n \geqslant 2$. To obtain such a system, Baker's student Herring ⟨**19**⟩, and independently Bolsch ⟨**8**⟩, developed Fatou–Julia theory for functions such as $f(z) = e^{\tan z}$, which are meromorphic outside certain compact totally disconnected subsets of $\hat{\mathbb{C}}$. Much of this theory, and its subsequent developments, is expounded in the last papers [**75**], [**77**], [**78**] and [**79**]. This elegant general theory builds on many of the ideas and techniques that Baker introduced to complex dynamics, and which will continue to be used by all those who work in this field.

4. *Other topics in complex analysis*

Baker worked on many problems outside iteration theory, some of which we describe here.

Edrei asked under which circumstances an entire function can have all but a finite number of the roots of the equation $f(z) = a$ on a line. There are many examples of such f with all their zeros on a line, and the function $f(z) = e^z$ has all solutions of $f(z) = a$ distributed on lines. However, Baker proved [**14**] that:

(a) if f has order less than 1, and two values have their solutions distributed on lines, then they must lie on the same line, and the set of such values of a forms a line segment; and

(b) if f has all values linearly distributed, then f is either a polynomial of degree at most 2 or $f(z) = c + de^{bz}$.

As described in Section 2, Baker had already worked on functional equations in his thesis [**2**], and he kept this interest throughout his career; see [**4**, **17**, **21**, **23**, **31,** **34**, **35**, **47**, **71**]. For example, [**17**] is related to *Fermat's theorem*. Here the aim is to find non-constant functions F and G such that

$$F^n(z) + G^n(z) = 1, \qquad \text{where } n \geqslant 2.$$

This can never happen for polynomials, but for $n = 2$ we can have rational, entire and meromorphic solutions, and these had already been classified by Gross ⟨**17**⟩. For $n = 3$ there are no entire or rational solutions, but there are meromorphic solutions, and Baker showed in [**17**] that these solutions can all be expressed in a particular form, in terms of elliptic functions, thus verifying a conjecture of Gross. Another example is the paper [**71**], in which he studied the functional equation

$$f(p(z)) = f(q(z)),$$

where f is a transcendental meromorphic function and p and q are polynomials.

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Baker gave a complete description of all solutions of this equation, thus completing earlier work with Gross [**21**] for the case when f is entire.

A *Picard set* is a set outside which every transcendental entire function takes all finite values with at most one exception infinitely often. In [**27**] and [**28**], Baker and his student Liverpool proved several results about these. For example, suppose that a complex sequence a_n and a positive sequence ρ_n satisfy

$$\left|\frac{a_{n+1}}{a_n}\right| > q > 1, \qquad \text{for } n = 1, 2, \ldots,$$

and

$$\log \frac{1}{\rho_n} > K \left(\log |a_n|\right)^2, \qquad \text{for } n = 1, 2, \ldots.$$

They showed that if

$$K = 4(q+1)/((q-1)\log q),$$

then the union of the discs $\{z : |z - a_n| < \rho_n\}$ forms a Picard set, but this is not necessarily the case if

$$K < 1/(2\log q).$$

In [**29**], Baker considered the question of whether a positive sequence a_n such that $a_{n+1} \geqslant (1+1/n)^\lambda a_n$, for $n > n_0$, where $\lambda > 0$, must form a Picard set. The answer is 'no' if $\lambda = 2$, by an elementary example, and Baker showed by a more complicated example that it is 'no' for all $\lambda > 2$. In [**42**], the authors found analogues of Picard sets for the equation $f^k(z)f'(z) = a$, and their results were strengthened by Baker's student Langley in $\langle$**22**$\rangle$.

Finally, in [**70**] Baker and Stallard made a substantial advance in a difficult problem of Littlewood. Let f be a polynomial of degree N. What is the growth of

$$\phi(N) = \sup_f \int_{\{z:|z|<1\}} \rho_f(z)\, dx\, dy,$$

where

$$\rho_f(z) = \frac{|f'(z)|}{1 + |f(z)|^2}$$

is the *spherical derivative* of f? Lewis and Wu $\langle$**24**$\rangle$ proved that

$$\phi(N) < CN^{1/2-\varepsilon}, \qquad \text{where } \varepsilon = 2^{-264}.$$

In the other direction, Eremenko $\langle$**11**$\rangle$ showed that there exist positive constants c and α such that $\phi(N) > cN^\alpha$, and Baker and Stallard obtained this lower bound with the constant $\alpha = 1.11 \times 10^{-6}$. Eremenko's proof developed an argument of Ruelle $\langle$**30**$\rangle$ concerning an estimate for the Hausdorff dimension of the Julia sets of functions of the form $f(z) = z^q + \lambda$, where $q \geqslant 2$, and the paper [**70**] made this difficult estimate explicit, a nice illustration of the way in which complex dynamics can repay some of its debt to complex analysis.

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