Gaussian integers and the work of Galois. Topology, graph theory, combinatorics and algebraic curves involve new mathematical entities which will require their own logical underpinning.

The last third of the book is devoted to metamathematics—set theory, logic and computation. An extended discussion of the Zermelo-Fraenkel axioms, including the Axiom of Choice, leads to consequences for measure theory and analysis. A discussion of issues concerning consistency and completeness, where Kurt Gödel inevitably makes an appearance, to be joined later by Brouwer and König, when issues of consistency and completeness are interrogated. The final pages are about computability theory and the work of Turing and Post. I was pleased to see mention of a theorem which is not provable in Peano Arithmetic, as I knew both authors over forty years ago!

This is very tough reading indeed, but it is typical of the author of this book to make an attempt to explain it. This is, I think, a very special piece of work, and it will take me a long time to unpick more of its riches. I can thoroughly recommend it to readers of the *Gazette*.

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The nuts and bolts of proofs by Antonella Cupillari, pp 416, £45.95 (paper), ISBN 978-0-32399-020-2, Elsevier/Academic Press (2023)

This is a fifth edition, so this book must have found favour quite extensively. New are a chapter on 'more challenging mathematical material' (sets, groups, functions, limits and infinite sets), and a more extensive selection of exercises and solutions. So the targeted readership in the UK would be first-year undergraduates. Nevertheless, the book could be of use to sixth-formers of high ability thinking of university maths, and second-year undergraduates who may for whatever reason have missed some or all of these basic proof techniques.

Before Chapter 1 there is a page of symbols and lists of 'facts and properties' about numbers and functions. Chapter 1 itself, 'Getting Started', begins conventionally by discussing what proof is, its relationship with logic, and a paragraph each on terms commonly appearing in mathematical writing: statement, tautology, paradox, hypothesis, conclusion, definition, proof, theorem, lemma, corollary and example. These clarified, the chapter ends with a couple of examples of taking the first step toward proving something, which is to be quite clear about what is being claimed (the conclusion) and what assumptions (the hypotheses) justify it. All fairly trivial, you may think, but in my experience often necessary.

Many claims fit into the "If ... then ..." category, so chapter 2 covers basic techniques for proving such results, after recognising that often such claims are not expressed colloquially in exactly this way. The proof methods discussed and illustrated are direct proofs, negation, contrapositive and contradiction. The examples are mainly from integer, rational and real arithmetic and elementary real functions.

Chapter 3 follows the same pattern to cover 'special kinds of theorems', including if-and-only-if, existence theorems, uniqueness theorems, theorems with multiple hypotheses and/or conclusions, and proving that two numbers are equal, backed up by discussion of the logical truth tables involved.

Some idea of the thoroughness can be gained from a page count: chapters 1, 2 and 3 cover 6, 36 and 53 pages respectively. Chapter 4 is 150 pages of specific mathematical topics to illustrate and practice all the techniques previously covered, including set theory and indexed families, functions, relations, equivalence classes, groups (up to Cayley's Theorem), binary operations, modular arithmetic, further group theory, limits, and cardinality.

The final chapter, also about 150 pages, consists of review exercises, some exercises without solutions, and then a nice touch—twelve 'alleged proofs' which require the reader to examine each for its validity, improve the valid ones and locate the flaw in the rest. Extensive solutions are then provided for most of the exercises.

To my somewhat jaundiced eye, A-level maths has moved dangerously further along the route of 'here's the formula you need to apply to answer this sort of question', and 'this theorem needs to be applied here'. The questions of where such formulae come from and how such theorems are established are disappearing from the syllabus. This of course makes the gap between A-level and undergraduate mathematics even more of a chasm than it was in my day. This is happening at a time when the need for more good teachers of school mathematics is widely acknowledged; such people have become an endangered species.

How well does his book alleviate the problems? My answer is 'quite well', provided the reader already has a strong motivation other than pressure of exams. Other target readers, probably the majority, will be quite likely to find the treatment daunting or even boring, not because of its difficulty but because of the sheer bulk of the material. It is all very thorough and well-explained, but an important something is missing: this level of maths is hard but it is also very beautiful, just as a piece of music may be initially rather testing but repays the reward of effort by becoming beautiful. A poem, too, may express thoughts in strikingly neat and unexpected ways. Mathematics shares these features, but there is nothing in the book to highlight the fact, and without it, where is the motivation for the struggle?

Nor is there any 'entertainment', nothing to lighten the load with, say, historical snippets or ideas presented in a more light-heated way. It is perfectly possible to bring out both of these features at this level, but the opportunity is missed. I feel that the title rather gives it away—if you are building an elegant machine, the handbook should mention more than the details of the nuts and bolts used in its construction.

Very occasionally a proof is acknowledged to be particularly neat, but this is lost in the sheer number and density of the proofs presented, many of which are so apparently trivial that readers could certainly be excused for thinking, 'if this is what mathematicians spend their time doing, I don't want to know'.

I believe that this is rather a good book, which makes it disappointing that my few negative impressions will be the first, and therefore possibly the only ones, to strike readers who could benefit from it, and who would not then go on to find the good bits. If it were carefully but severely pruned, and some of the pruned material replaced by more motivational items, it would be much more attractive and useful.

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