PLANETARY AND EARTH FIGURE PERTURBATIONS IN THE LIBRATIONS OF THE MOON

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ABSTRACT. Comparisons of numeric, semi-analytic and analytic lunar libration models indicate that the planetary and Earth figure perturbations used to supplement the semi-analytic and analytic models are inadequate. Using the ELP-2000 of Chapront and Chapront-Touzé, an improved solution of these perturbations is developed. For the libration in longitude there are numerous terms with periods near that of the free libration; their amplitudes remain ill-defined.

THE PROBLEM

Comprehensive numeric and semi-analytic lunar libration solutions have been generated for dynamical models that take into account the direct and indirect effects of the Earth (including its flattening), the Sun, the planets, and elasticity and tidal dissipation of the Moon; the gravity field of the Moon is usually developed through a third or fourth degree spherical harmonic representation. Various independently derived solutions have been compared with one another as well as with data from lunar laser ranging measurements, differential VLBI from ALSEP transmitters, and artificial lunar satellites. There is excellent agreement between independent numeric solutions and very good agreement between independent semi-analytic and analytic solutions that correspond to the main problem of lunar theory (three body problem) but, because the planetary and Earth figure libration perturbations used are inadequate, there is only fair to good agreement between semi-analytic and numeric solutions.

Cappallo et al. (1981) have compared solutions of the two most widely used numerically integrated libration models: those of the MIT Planetary Ephemeris Program (PEP) and of the JPL LLB-5 with the LURE-2 Solution parameters for a rigid Moon (Williams, 1977). Using the method of least squares, they estimated three biases in the Cassini angles and six initial conditions of rotation so as to minimize the post-fit sum of the squared Euler-angle differences between the models. Over the six year span (Julian days 2440400-2442600) of the fit, the biases (PEP minus LLB-5) in τ, ρ and $I\sigma$ are, respectively, $0\overset{''}{.}287$, $0\overset{''}{.}085$ and $0\overset{''}{.}066$; the

O. Calame (ed.), High-Precision Earth Rotation and Earth-Moon Dynamics, 193–198.

corresponding route mean square scatters about the means are 0″.009, 0″.021 and 0″.019.

Cappallo and Eckhardt (1982) have compared solutions of the semi-analytic model of Eckhardt (1981) with those of PEP using the same technique, LURE-2 parameters and time span of fit as for the PEP - LLB-5 fit. The biases (PEP minus semi-analytic) in τ, ρ and $I\sigma$ are, respectively, -0″.467, -0″.126 and -0″.036; the corresponding route mean square scatters about the means are 0″.053, 0″.044 and 0″.088.

Migus (1981) and Moons (1982) have developed analytic libration models excluding planetary and Earth figure perturbations and have compared their results with the corresponding portion of the Eckhardt model. The discrepancies between the incomplete analytic and semi-analytic models are significantly smaller than the discrepancies between the complete numeric and semi-analytic models, so the latter discrepancies are likely due, in part, to inadequacies in the planetary terms of the ILE (Improved Lunar Ephemeris; Eckert, Jones and Clark, 1954) which was used to calculate the planetary libration terms.

Chapront-Touzé and Chapront (1980) compared their provisional ELP-1900 planetary perturbations of the Moon with those of the ILE. The planetary term with argument $-5T+3V+w_1$ (ILE Serial No. 1477) has a much smaller amplitude for ELP-1900 than for the ILE, so two near-monthly terms in ρ and $I\sigma$ generated from the ILE with amplitudes of 0″.033 and 0″.014 are much too large. There are various other differences between the ILE and ELP-1900 that impact on the derived libration terms.

A SOLUTION

The definitive ELP-2000 solution is now complete (Chapront and Chapront-Touzé, 1982). Its planetary and Earth figure perturbations have been used to calculate the consequent perturbations in the physical librations. The method (and nomenclature) used is that of Eckhardt (1981) except for three modifications: the perturbation in the direction cosine u_3 is determined by

$$\delta u_3 = -\delta p_1 - \delta B_{\leftmoon} - p_2(\delta s - \delta\tau) - (s-\tau)\delta p_2;$$

a programming error that, in effect, reversed the sign of the term 33909″.1 cos ℓ in the sine parallax cubed (Table IV, Eckhardt, 1981) has been corrected; and the effects of perturbations in parallax are dropped from the perturbation equations because they are insignificant. The solutions are supplemented by terms allowing for the rotation of the plane of the ecliptic and the direct effect of the Earth's figure (Eckhardt, 1981).

Table I is a tabulation of the perturbations in $\delta\tau$,

$$\delta q_1 = \delta p_1 \cos F - \delta p_2 \sin F = \delta[\sin\theta \sin(\tau-\sigma)] \text{ and}$$

$$\delta q_2 = \delta p_1 \sin F + \delta p_2 \cos F = \delta[\sin\theta \cos(\tau-\sigma)].$$

They are developed as Fourier series, with sine terms for $\delta\tau$ and δq_1, and cosine terms for δq_2. The series are truncated for any frequency at which none of the perturbation amplitudes attains the magnitude 0″.010. The trigonmetric arguments are tabulated in two forms. Symbolically, the arguments are given in terms of the mean longitudes of the planets – Q(Mercury), V(Venus), T(Earth), M(Mars), J(Jupiter) and S(Saturn) – referred to the fixed equinox 2000; the mean longitude of the Moon, L, referred to the mean equinox; and the Delaunay arguments, D, ℓ and F. Using the numerical rates recommended by Chapront and Chapront-Touzé (1982), the arguments are also given in the form (a+bt)x360° where t is measured in ephemeris days from 2000.

Table I. Planetary and Earth figure lunar libration perturbations. The period of each Fourier term is given in years.

Period	$\delta\tau$	δq_1	δq_2	a	b	
1912	-.579	-.015	.000	.23129	.0000014322	-10V+3T+ℓ+26°.69
1783	.936	.000	.000	.20155	.0000015351	-4T+8M-3J+254°.01
883	.175	.000	.000	.03867	.0000030995	-2J+5S+192°.23
650	-.018	.000	.000	.00941	.0000042109	-3T+7M+D-2ℓ+F+215°.56
302	-.033	.000	.000	.42399	.0000090526	3V-7T+4M+328°.23
273	14.403	.391	-.035	.33262	.0000100268	18V-16T-ℓ+26°.54
239	.233	.000	.000	.07089	.0000114591	8V-13T+235°.75
143	-.011	.000	.000	.31953	.0000192128	2T-3J+S+2D-2ℓ+1°.30
127	-.105	-.003	.001	.38326	.0000214859	26V-29T-ℓ+255°.00
104	-.064	-.010	-.010	.49768	.0000262903	2M+D-F+343°.72
95.8	.109	.000	.000	.39638	.0000285803	-3V+4T-D+ℓ+89°.66
72.8	.014	.000	.000	.23942	.0000376329	-3T+4M-D+ℓ+208°.75
70.9	-.015	-.001	.000	.18318	.0000386071	15V-12T-D+279°.70
58.1	.010	.000	.000	.03736	.0000471337	-24V+24T-2D+3ℓ+0°.57
54.7	.013	.000	.000	.28943	.0000500662	23V-25T-D+168°.18
49.8	-.053	-.014	-.014	.24597	.0000549871	5V-6T+2D-2F+92°.30
47.9	.066	.000	.000	.33043	.0000571606	-6V+8T-2D+2ℓ+12°.87
40.7	.035	.002	-.001	.18831	.0000671874	12V-8T-2D+ℓ+228°.51
40.4	-.026	.000	.000	.35429	.0000677235	8T-15M+335°.31
39.9	.043	.002	-.002	.14409	.0000685536	-20V+20T+D-ℓ+F+345°.97
39.3	-.029	-.001	.001	.18858	.0000697168	-3Q+T+2D-ℓ+263°.44
37.1	.320	.009	-.006	.29530	.0000737306	-2T+3J-2D+2ℓ+169°.96
36.4	-.023	.000	.000	.47926	.0000752657	-6T+8M-2D+2ℓ+57°.64
34.9	-.018	.000	.000	.13805	.0000785391	T-J+D-ℓ+180°.70
34.8	-.108	-.006	.004	.22460	.0000786465	20V-21T-2D+ℓ+91°.80
32.7	.026	.006	.006	.15241	.0000836087	T+J+D-F+75°.47
29.5	-.019	.000	.000	.36192	.0000929434	S+80°.21
27.9	.013	.000	.000	.29775	.0000982247	5T-6M+2D-2ℓ+331°.68
20.2	.002	.017	-.017	.31030	.0001357410	-8V+12T-D+F+206°.52
18.6	8.183	8.188	-8.095	.15265	.0001470938	F-L+180°.00
18.6	.739	7.484	-7.478	.38752	.0001472000	-T-D+F+84°.55
18.2	-.001	-.013	.013	.49329	.0001502996	-T-2J+5S-D+F+300°.94
17.4	.159	-.003	.001	.27247	.0001570782	2T-2J+2D-2ℓ+0°.09
15.8	.347	-.002	.002	.31184	.0001734904	-T+2M+221°.86

Period	$\delta\tau$	δq_1	δq_2	a	b	
13.5	-.019	-.004	-.004	.11244	.0002021872	5V-7T+D-F+349°.27
11.9	-.099	.000	.000	.39180	.0002308088	J+106°.70
10.1	-.027	.000	.000	.25219	.0002707408	-15V+13T+2D-ℓ+333°.68
9.75	-.023	.000	.000	.08432	.0002807677	3V-3T+2D-2ℓ+180°.04
9.31	.043	.015	-.015	.30531	.0002941876	2F-2L+0°.00
8.35	-.037	.000	.000	.12014	.0003279014	-21V+21T+ℓ+180°.06
8.10	.098	.000	.000	.42185	.0003379282	-3V+5T+195°.47
7.89	.109	.000	.000	.05739	.0003469808	-2T+5M+239°.86
7.84	.027	.000	.000	.49001	.0003493873	5V-8T+70°.24
5.93	.011	.000	.000	.06911	.0004616177	2J+316°.18
5.26	-.024	.000	.000	.33293	.0005204712	-3T+6M+88°.66
4.66	.011	.000	.000	.18259	.0005881947	5T-9M+242°.30
4.50	.016	.000	.000	.20449	.0006086691	-18V+18T+2D-ℓ+0°.11
4.50	-.019	.004	-.004	.06424	.0006088178	-T+2J-D+F+259°.47
3.98	-.750	.000	-.001	.42365	.0006873155	2V-3T+89°.95
3.68	-.013	.000	.000	.21535	.0007444761	-4V+5T-2D+2ℓ+268°.89
3.59	-.031	.000	.000	.45911	.0007616851	4T-7M+91°.45
2.97	-.011	.000	.000	.10100	.0009231229	18V-16T-3ℓ+2F+26°.54
2.93	-.847	.000	-.001	.23622	.0009351755	3T-5M+120°.80
2.89	-.274	.000	-.001	.12488	.0009465973	-21V+23T+2D-ℓ+15°.06
2.89	-.228	.000	.000	.32400	.0009466634	V-2T-D+2ℓ-F+256°.79
2.86	-.032	.000	.000	.15562	.0009566902	19V-18T-D+ℓ-F+102°.97
2.86	-.011	.000	.000	.22029	.0009580564	-13V+10T+2D-ℓ+259°.64
2.84	.019	.000	.000	.36667	.0009653744	4Q-3T-ℓ+279°.37
2.83	.037	.000	.000	.01320	.0009680832	5V-6T+2D-2ℓ+91°.88
2.67	-.269	.000	-.001	.26455	.0010252438	-V+2T+76°.28
2.54	.012	-.001	-.001	.04229	.0010802309	4V-4T+2D-2F+0°.02
2.47	.112	.000	.000	.01367	.0011086659	2T-3M+150°.29
2.23	-.015	.000	.000	.40743	.0012274310	4V-5T+D-F+76°.51
2.14	-.013	.000	.000	.29422	.0012821563	T-M+0°.88
1.99	-.023	.000	.000	.30092	.0013746311	4V-6T+343°.21
1.91	-.013	.000	.000	.14230	.0014317917	-2V+2T-2D+2ℓ+180°.03
1.60	.372	.000	.001	.22645	.0017125593	V-T+0°.01
1.20	.034	.000	.000	.42436	.0022761854	T-2J+121°.01
1.14	-.032	.000	.000	.15296	.0023998749	3V-4T+270°.99
1.09	-.110	.000	-.001	.18707	.0025069943	T-J+1°.229
1.07	.031	.000	.000	.08287	.0025643127	2T-2M+179°.77
.799	-.025	.000	.000	.45342	.0034251186	2V-2T+0°.20
.075	.000	.034	-.034	.26329	.0364388471	-T-D+ℓ+F+264°.87
.074	.000	.013	-.013	.26634	.0367858280	-3T+4M-D+ℓ+F+125°.16
.074	.000	-.015	.015	.05912	.0369239755	-3V+3T-2D+ℓ+2F+180°.01
.074	.000	-.015	.015	.37071	.0370476649	-2T+2J-2D+ℓ+2F+179°.88
.074	.000	-.012	.012	.25488	.0370575431	T+D-ℓ+F+95°.13

CAVEAT

There is a major disagreement between the monthly perturbations in $\delta(I\sigma) \approx \delta q_1 + I\delta\tau$ and $\delta\theta \approx \delta q_2$ in Table I and the corresponding table of Eckhardt (1981). The root sum square amplitude of all the monthly terms in δq_1 and δq_2 drops from 0".130 to 0".062. The sources of the changes are the program refinement and correction cited above and sizeable differences between ILE and ELP-2000 planetary perturbations in latitude.

Near the adopted 2.8912 year resonance for the free libration in longitude, there are two $\delta\tau$ terms in Table I with amplitudes 0".274 and 0".228 that are missing from the ILE derived table. These near resonance terms are very sensitive to the exact period of resonance, and the resonance period adopted depends on the parameters and theory used. For example, using the same LURE-2 parameters and the theory of Migus (2.8917 year resonance), the amplitudes of these two terms are 0".540 and 0".565. The tabulated amplitudes of longitude terms with periods near 2.89 years must, therefore, be considered as highly tentative. A list of all arguments found with periods near 2.89 years is given in Table II.

Table II. Arguments (except for phases) of libration perturbations with periods near that of the free libration in longitude. The periods are given in years.

Period		Period	
2.9276	3T-5M	2.8996	3V-4T-M
2.9233	-39V+39T+2D	2.8967	-11V+20T+2D-2ℓ
2.9218	17V-19T+6M-2D+ℓ	2.8923	-21V+23T+2D-ℓ
2.9202	-24V+30T-4M+2D-ℓ	2.8921	V-2T-D+2ℓ-F
2.9145	2M+D-2ℓ+F	2.8797	-3T+4M-D-ℓ+2F
2.9138	28V-32T+J-ℓ	2.8620	-3V+7T+2D-2ℓ
2.9074	-3V+4T-D-ℓ+2F	2.8618	19V-18T-D+ℓ-F
2.9069	27V-31T-2M-2D+ℓ	2.8577	-13V+10T+2D-ℓ
2.8997	-26V+29T+4M+2D-ℓ		

ACKNOWLEDGEMENT

I thank Jean Chapront and Michelle Chapront-Touzé for providing me with a copy of the magnetic tape ELP-2000/81 and the associated descriptive information.

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DISCUSSION

King : You may have left a wrong impression in stating that you can remove the wobble by adjusting the initial conditions. In fact, adjusting the initial conditions of a numerical integration always removes the free librations. The point is that the wobble signature is very pronounced and would be seen in the data. We are only seeing a part of the 80-year period, so it may be correlated with other parameters that we do not yet know about, but it is certainly real. Would Dr. Calame comment on the significance of the fact that R. Cappallo gets an amplitude of 8", compared to her 5".

Calame : I have not really explanation for the difference; only, we have models totally different.

Mulholland : It may be the result of using different time spans of data.