PHASE MIXING OF PROPAGATING ALFVÉN WAVES

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Among MHD waves, Alfvén waves have been proved to be the best candidates to reach the solar corona and, eventually, to be responsible for the heating of this outer part of the solar atmosphere. The problem arises, however, about the mechanism able to transform the energy stored in the waves into heat.

Recently (Heyvaerts and Priest 1983, or HP), a simple and very appealing idea has been proposed for the dissipation of Alfvén waves in an inhomogeneous medium: Let the inhomogeneity be described by a space dependent Alfvén speed $v_A(x)$ and let the oscillations, whose velocity we denote by $\underline{v} = v(x,z,t)\hat{y}$, be polarized in the y-direction and propagating along the mean magnetic field in the z-direction (Fig. 1).

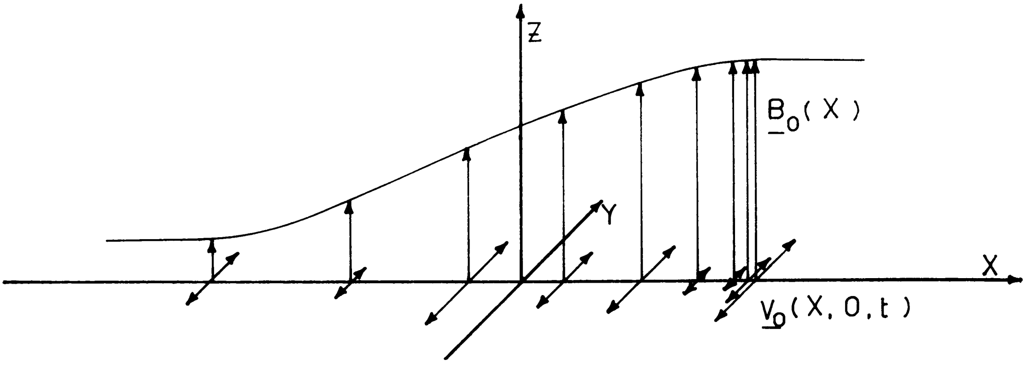

Fig. 1 The background magnetic field profile. Each field line is excited at the boundary z = 0 by an inhomogeneous disturbance $\underline{v}_0$

Due to the gradient in the phase speed $v_A(x)$, the oscillations of neighbouring field lines, that are in phase at some altitude, will become increasingly out of phase as the wave propagates outward, what

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Heyvaerts and Priest denoted by 'phase mixing'. In other words, at high altitudes, strong lateral gradients are expected to develop in the velocity field.

If viscosity and resistivity are taken into account, now, this mechanism leads to a considerable dissipation of energy. The heating rate is indeed conspicuous, provided the magnetic field is not very strong (which would lower the effective viscosity); hence the above mechanism seems to fail in active regions where, on the other hand, the heating demands are highest (see Priest, 1983).

In these regions, the magnetic field shows mainly a closed geometry, in which we expect to find standing waves. These waves (for which phase mixing occurs as time elapses) can, now, be shown to suffer Kelvin-Helmholtz and tearing-mode instability (see HP and Browning and Priest, 1983). If, on one hand, the appearance of instabilities increases the values of the viscosity and resistivity, it can possibly lead to the onset of a turbulent cascade, thus enhancing the heating rate and making of the phase mixing a good candidate for the heating of active regions too.

Generally speaking, both propagating and standing waves are governed by a modified wave equation taking into account wave propagation and energy diffusion as well

$$\left(\frac{\partial^2}{\partial t^2} - v_A^2(x)\,\frac{\partial^2}{\partial z^2}\right)v = \nu\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right)v. \tag{1}$$

Here ν represents the sum of the kinematic viscosity and of the magnetic diffusivity, both assumed to be small (i.e. large Reynolds and magnetic Reynolds numbers). The lateral diffusion of energy (x-directed) can be thought of as the excitation of new oscillations on neighbouring field lines. These new oscillations, of course, can propagate both outward and inward. We can get rid of this complication provided the phase mixing is not much developed (see Nocera et al. 1983, for a detailed discussion of the various orderings quoted here).

Let the oscillations be generated at $z=0$ at a frequency ω and let $k(x) = \omega/v_A(x)$ be their wavenumber. If we define a transverse scale length

$$a = |d\,\ell n\,k/dx|^{-1} \tag{2}$$

and a total Reynolds number

$$R_{TOT} = \omega a^2/\nu, \tag{3}$$

we find, in the long wave ($k << R_{TOT}/a$), large altitude ($z >> a$) limit (Nocera et al 1983)

$$v(x,z,t) \sim \exp[-(z/\Lambda)^2] \exp i(\omega t - kz) \cdot z^{-(R_{TOT}/8)^{\frac{1}{2}}} \quad (4)$$

$$\Lambda = 2^{3/4} (R_{TOT})^{1/4}/k. \quad (5)$$

This result is different from HP's one

$$v_{HP} \sim \exp[-(z/\Lambda_{HP})^3] \exp i(\omega t - kz), \quad (6)$$

$$\Lambda_{HP} = (6 R_{TOT})^{1/3}/k, \quad (7)$$

the difference arising from the relaxation of some oversimplifications introduced by HP.

The solution (6) seems to hold up to several solar radii where steep x-gradients unavoidably develop, thus contradicting the weak phase mixing hypotesis. In this circumstance the Alfven wave spreads out laterally, just like a light beam defocuses in an inhomogeneous medium.

A similar effect is found at small heights ($z \lesssim a$), where the main energy flux is due to outgoing waves from the nearby source and, again, we can neglect the ingoing waves. Thus, if we assume that the perturbation acts on a single line of force located at x = 0, say, and that the plasma is strongly magnetized, we find (Nocera et al. 1983)

$$v \sim \exp(i\omega t) [2\pi k(0) z]^{-\frac{1}{2}} \exp[-w^2(x)/z], \quad (8)$$

where

$$w(x) = (2\nu)^{-\frac{1}{2}} \int_0^x v_A^{\frac{1}{2}}(s)\, ds \quad (9)$$

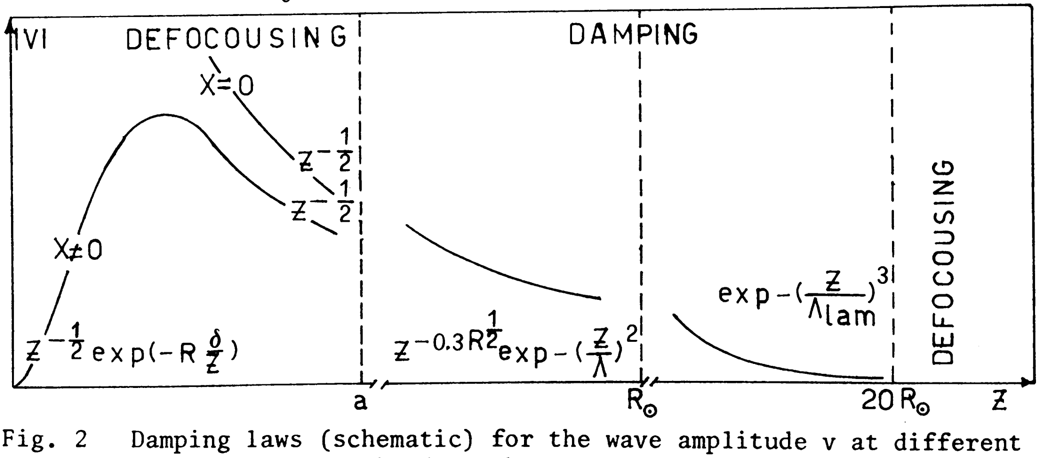

Fig. 2 Damping laws (schematic) for the wave amplitude v at different altitudes z, for a pointlike disturbance (according to Nocera et al. 1983)

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So far, we dealt with propagating waves. As concerns standing waves in closed structures of length ℓ, they vibrate with eigenfrequences $\Omega_n = 2\pi n v_A(x)/\ell$, and their main properties can be described by the Green function of the fundamental (n = 1) oscillator (see HP)

$$G(x,t) = \frac{\sin(\Omega_1 t)\exp[-(t/\tau_D)^3]}{\Omega_1} \tag{10}$$

where

$$\tau_D = (6R_{TOT})^{1/3} / \Omega_1 \tag{11}$$

is a damping time which, in typical coronal conditions equals about 20 wave periods.

A consistent discussion of dissipation processes cannot bypass the problem of Kelvin-Helmholtz (KH) and Tearing Mode (TM) stability, which is particularly important in a highly sheared medium. A simplified, local, stability analysis (HP) shows that propagating waves are stable whereas a standing wave, whose velocity amplitudes is u and whose transverse inhomogeneity scale length is d, shows instability with growing times

$$\tau_{KH} = d/u, \quad \tau_{TM} \simeq \left(\frac{ud}{\nu_m}\right)^{3/7} \frac{d}{u}\ . \tag{12}$$

Recently, the KH stability analysis has been performed by Browning and Priest (1983) in more realistic conditions, leading to the conclusion that the instability develops well before the phase mixing reaches its strong stage; the Alfvén wave may be significantly disrupted within a very few wave periods and it reaches a turbulent state well before the laminar damping becomes significant.

The heating rate E_H can be calculated according to some guess on the turbulent spectrum (Hollweg, 1983, uses a Kolmogo rov spectrum). It turns out to be of the same order (or even greater) of the input required for the heating of a coronal loop, $E_H \simeq 4.10^{-5} \div 1.8.10^{-3}$ erg cm $^{-3}$ s^{-1}, suggested by Hollweg (1983) (see tab. 1)

	Wave period(s)	E_H(erg cm^{-3} s^{-1})
Wave amplitude = 60 km/s	3	$7.2.10^{-2}$
inhomogeneity scale = 1000 km	30	$1.2.10^{-2}$
Table 1	300	$1.7.10^{-3}$

Turbulent heating rate for different periods of the laminar oscillations, in the solar corona (After Browning and Priest 1983).

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DISCUSSION

Ionson: In actual plasma systems, e.g. coronal loops, the field lines communicate phase primarily by $\nabla_{\perp}(B_{\parallel}B)$ rather than through the dissipative terms.

Heyvaerts: In the working hypothesis used here, there is no compressional effect. But it is true that if we go to cylindrical or toroidal geometry such effects will show up. Their effect should be to broaden the resonances, on the one hand, and also to introduce new collective modes, on the other. I expect the general effect to remain for the continuum of modes.

Migliuolo: The transverse structure B(x) dictates the form of eigenmodes of the systems. What is the relationship to the model presented here?

Nocera: The eigenvalue problem in the x-direction was not solved in the general form, since it is difficult to manage with three (x,z,t) coordinates.

At low altitudes, however, this has been done. The x- dominion is actually unbounded and the solution (equation 8) seems to vanish at $x \to \pm\infty$ for any model for $v_A(x)$ in equation (9) and for any value of the frequency ω: then, we have a continuous spectrum there.