

## CORRESPONDENCE

To the Editor of *The Mathematical Gazette*

DEAR SIR,—Mr. Leigh Silver ends his article (*The Mathematical Gazette*, No. 363, p. 1) by remarking that musical theory and mathematics have gone hand in hand from the earliest times, but that music itself has not been influenced appreciably. Mr. Leigh Silver shows in his article that he himself could be described as being similarly mathematical; he shows considerable grasp of musical theory, but rather less of music. I found that, of his “curiosities”, the most curious was his statement (p. 8) that “A small group of atonalists have (used) a 12-note scale, but their compositions can hardly be classified as music.”

By Mr. Leigh Silver’s own criterion, it would seem that a mathematical journal might contain items on musical theory but not on music; in any case I would not have thought that the pages of the *Gazette* were a suitable place for this sort of comment.

Yours faithfully, B. L. MEEK

*Dept. of Mathematics,  
The University,  
Leeds 2.*

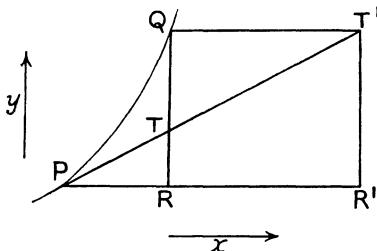
DEAR SIR,—

(i) A Paradox—Hugh Thurston;

(ii) Units and Symbols in mechanics—H. V. Lowry  
(*The Mathematical Gazette*, Feb. 1964).

(i) I wonder how seriously Hugh Thurston intends his paradox about differentials to be taken; it is certainly not to be derived from the texts he cites. May I add to these de la Vallée Poussin’s *Cours d’Analyse* and refer him to the remark at the end of section 51 of Vol. I (1923)?

A little naive geometry will clarify the analysis:



$PTT'$  is the tangent at  $P(x, y)$  to the curve  $PQ$  with equation  $y = f(x)$  or  $x = g(y)$ . Then

$$\Delta x = PR, \quad \Delta y = RQ = R'T'$$

$$df = f'(x) \cdot \Delta x = RT,$$

$$dg = g'(y) \cdot \Delta y = PR'.$$

It is clear that, if  $\Delta x$ ,  $\Delta y$  are corresponding increments,

$$df \neq \Delta y, \quad dg \neq \Delta x.$$

Thurston's paradox arises from his use of the same symbols,  $dy$  and  $dx$  respectively, for these pairs of unequal quantities. In doing so he disregards the fact that the equations he uses (equivalent to each other)

$$dy = f'(x) dx$$

$$dx = g'(y) dy$$

are *homogeneous* in  $dx$ ,  $dy$  and thus specify only their ratio. The value of *either*  $dx$  or  $dy$ , but not of both simultaneously, is arbitrary. The relation  $dx = \Delta x$  (in either equation) gives  $dy = df$ ;  $dy = \Delta y$  similarly gives  $dx = dg$ . In short,  $x$  and  $y$  cannot both vary independently at the same time.

Thurston's further troubles would seem to be due not to differentials in themselves but to the homogeneity of all relations between them. Does he find the same difficulties with homogeneous co-ordinates?

(ii) The superiority of first differentials over derivatives is that the former are invariant for a change of variable while the latter are not. In his article on "Units and symbols in Mechanics" Mr. Lowry (p. 24) makes a wistful plea for the mathematical representation of physical quantities by symbols which "include the units". Such symbols bear to the corresponding measures a relation which is analogous to that of the differential of a function to its derivative. In fact the quantity-symbol is invariant for a change of units while the measure is not. This invariance enables the mathematician to meet slugs, poundals and whatever else the engineers and physicists may devise, with complete equanimity. May I, even more wistfully, add my plea to Mr. Lowry's?

Yours faithfully, W. G. L. SUTTON

*The College of Technology,*  
*Leeds 1.*

[Note: Dr. Thurston's article has (not unexpectedly) caused some comment. If space allows, it may be possible to carry the discussion a little further later. E.A.M.]

DEAR SIR,—We should dearly like to interest your members in teaching at this College. We are at this moment short of teachers in elementary arithmetic and in first and second year mathematics—this covering the syllabus of the R.S.A.

Perhaps I should let you know something of the history of the College. It was opened originally in 1854 and is, I believe, the oldest adult education centre in England. Amongst its first teachers were Ruskin, Lowes Dickinson and Rossetti. Its aim has always been to place a liberal education within the reach of those who have been denied it. Thus we try to create a better understanding in the community by bringing together those who have been lucky enough to receive a university background with others less fortunate. Our teachers have therefore remained largely unpaid. Now, 100 years after our foundation, we continue our work and rejoice in a thousand or so pupils and