and analogues. Chapter 7 consists of further generalisations to the case of Toeplitz operators associated with a kernel $K(s, t)$, defined with reference to two measure spaces and a function $\phi(x, s)$ measurable with respect to their product measure. In the later part of the chapter some previous results are re-examined in the light of these generalisations—for instance, in § 7.6, there is an ingenious alternative proof of a theorem of § 5.2. If a certain limiting property holds, the class of Toeplitz matrices corresponding to a class of real-valued functions is said to be trace-complete ; the first part of chapter 8 concerns the properties of such classes of matrices. §§ 8.4, 8.5 concern Toeplitz matrices associated with certain special orthogonal systems, while the rest of the chapter examines two special kernels $K(s, t)$.

The first chapter (9) of the Applications section of the book makes use of the properties of the eigenvalues of Toeplitz forms to deduce several results on analytic functions regular within the unit circle. Some of the results in the final chapters (10, 11), which deal with the applications to probability theory and statistics, are recent, and there is a link between Toeplitz forms and the work of Kolmogorov and Wiener on stationary processes. Some of the work of Grenander on stochastic processes is incorporated into these chapters.

This is a stimulating book, designed primarily for the research worker who is anxious to have available, in a compact form, some of the recent work on the subject, together with a survey of older results. Its value is enhanced by the bibliography and by the Appendix which provides (with additional references) an annotated commentary on the text. An unusual amount of material is compressed into the 228 pages, mainly by the omission of tedious detail. Despite this, the printing is uncramped and the book remains, both in appearance and content, a pleasure to read.

DENNIS C. RUSSELL

CHURCHILL, R. V., *Operational Mathematics*, 2nd ed. (McGraw-Hill Book Co., New York, 1958), 306 pp., 41s.

The general arrangement of the book is unaltered from the first edition published in 1944 under the title *Modern Operational Mathematics in Engineering*. There is, however, an extensive revision of detail, the concept of the delta-function is introduced, the chapter on elementary applications now contains sections on electric circuits and servomechanisms, and many more examples for the reader to work are included. The bibliography extends to twenty-two titles including references to tables of Laplace and Fourier transforms.

Perhaps the most obvious change is in the title, implying that the book is now of interest to a wider class of reader than was suggested by the title of the first edition. The change is justified for this is a sound mathematical treatment of the theory of the Laplace transform and its application to problems of physical origin. Great care is taken to state clearly and precisely the conditions under which results are established. These conditions are usually simple and practical, the author having resisted the temptation to enshroud the theory in mathematical sophistication. The treatment should satisfy all with a reasonable outlook whether they incline to pure or applied mathematics.

Recent developments, applying operator methods to partial differential equations by the use of integral transforms with kernels of various types, are reflected in the chapters entitled " Sturm-Liouville Systems " and " Fourier Transforms ". The first of these has been radically revised providing a much more satisfying account of second order linear differential systems ; an addition of particular interest in the other chapter is the account of how the kernel appropriate to a given differential system may be determined.

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The impression was gained that this extensive revision of a well established book was carried out by an author still very much absorbed in his subject.

J. FULTON

KOLMOGOROV, A. N., AND FOMIN, S. V., *Elements of the Theory of Functions and Functional Analysis*, vol. i : *Metric and Normed Spaces*, translated by LEO F. BORON (Graylock Press, Rochester, N.Y., 1957), 129 pp., 32s.

This is an excellent introduction to the ideas and methods of functional analysis. The original version was based on lectures given by the author in Moscow, and the translator has produced a very readable English text. Although nothing beyond elementary analysis is presupposed, the principal abstract concepts are illustrated by an ample variety of examples, and the theory is elegantly applied to problems of considerable practical interest. The approach is essentially " classical " : all limits are sequential, and no use is made of any principle of transfinite induction. This is a weakness from the point of view of the serious student of modern analysis, but it makes substantial parts of the subject easily accessible to many others.

There are four chapters. The first is concerned with some elementary facts of abstract set theory (with no mention of the Axiom of Choice). Chapter II, the longest, is devoted to metric spaces, with a brief reference to general topological spaces. The main themes here are completeness and compactness. The fixed-point theorem for contraction mappings in a complete metric space is thoroughly exploited in a discussion of iterative methods for solving equations, including differential and integral equations, for which several existence theorems are proved. The fundamental properties of compact sets in metric spaces are established (the word " compact " is used in the relative sense, which is now unusual), and there is a careful account of the relations between compactness and equicontinuity in spaces of continuous functions on compacta. The chapter ends with a useful discussion of rectifiable curves. Questions of category are not considered, and there is no mention of local compactness.

Chapter III, on normed vector spaces, is concerned largely with continuous linear functionals. Conjugate spaces are determined in some simple cases, and their are brief discussions of reflexivity and weak convergence. A proof of the Hahn-Banach extension theorem, and an account of weak* compactness, are restricted to separable spaces (of necessity, since transfinite induction is not available). Continuous linear operators on Banach spaces are also considered, and there is a proof of Banach's theorem on the continuity of inverse operators. The Banach-Steinhaus theorem is, surprisingly, omitted. An addendum to this chapter gives some of the main facts about " generalized functions " (distributions).

In Chapter IV, the idea of the spectrum of a linear operator is introduced, and the main theorems on completely continuous operators are proved, giving the Fredholm theory of integral equations.

There is a good index, and the translator has added a bibliography. The authors promise further volumes, in which they propose to discuss, among other things, Lebesgue integration and Hilbert-space theory.

J. D. WESTON

PONTRYAGIN, L. S., *Foundations of Combinatorial Topology* (Graylock Press, Rochester, N.Y., 1952), 99 pp., $3.00.

This is a translation of the first (1947) Russian edition of a book which the author says is " essentially a semester course in combinatorial topology which I have given several times at Moscow National University ".

There are three chapters. In Chapter I the Betti (homology) groups are defined for polyhedra ; in Chapter II the topological invariance of the groups is proved ;

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