ACCRETION DISK EVOLUTION

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ABSTRACT. Three aspects of mass transfer instability models of dwarf novae are examined. The hydrodynamic development of instabilities in the secondary are examined within Roche geometry and shown to extend at least a few degrees away from the line of centres. The form of the outburst light curves observed in SS Cygni are shown to be a natural consequence of mass transfer bursts with a duration either less than, or greater than, the disk viscous timescale. Finally the two-dimensional structure of the disc in the plane of the orbit is studied. As with α-disks the viscous evolution time following a burst of mass transfer determines the size of viscosity within the disk. Significant deviations from axial symmetry are found to be present.

1. STELLAR STABILITY IN ROCHE POTENTIALS

Analysis of the stability of stars whose photospheric surface is close to the gravitational saddle point at the inner Lagrangian point has been examined by Bath (1975), Papaloizou and Bath (1975), Wood (1977) and Edwards (1984). For spherical models, with the effect of the gravitational saddle point spread over the whole surface of the star, the envelope is found to be dynamically unstable and to give rise to bursts of mass transfer. Two main limitations of these models are the fact that they ignore, or treat in a simplified way, the role of time-dependent convection in the unstable envelope, and that apart from the linear analysis of Papaloizou and Bath (1975) they ignore the three-dimensional geometrical structure. Gilliland (1985) has made some progress with this latter problem and we report here work in various two-dimensional Roche geometries, confined to the plane of the orbit.

The technique employed was based on an initial model constructed with spherically symmetric equations. In order to construct the on-axis and off-axis models, the radial position of a given potential was calculated and the appropriate dependent variables set at these points. Thus the Roche geometry was approximated by a set of one-dimensional models (of different initial radii), with dependent

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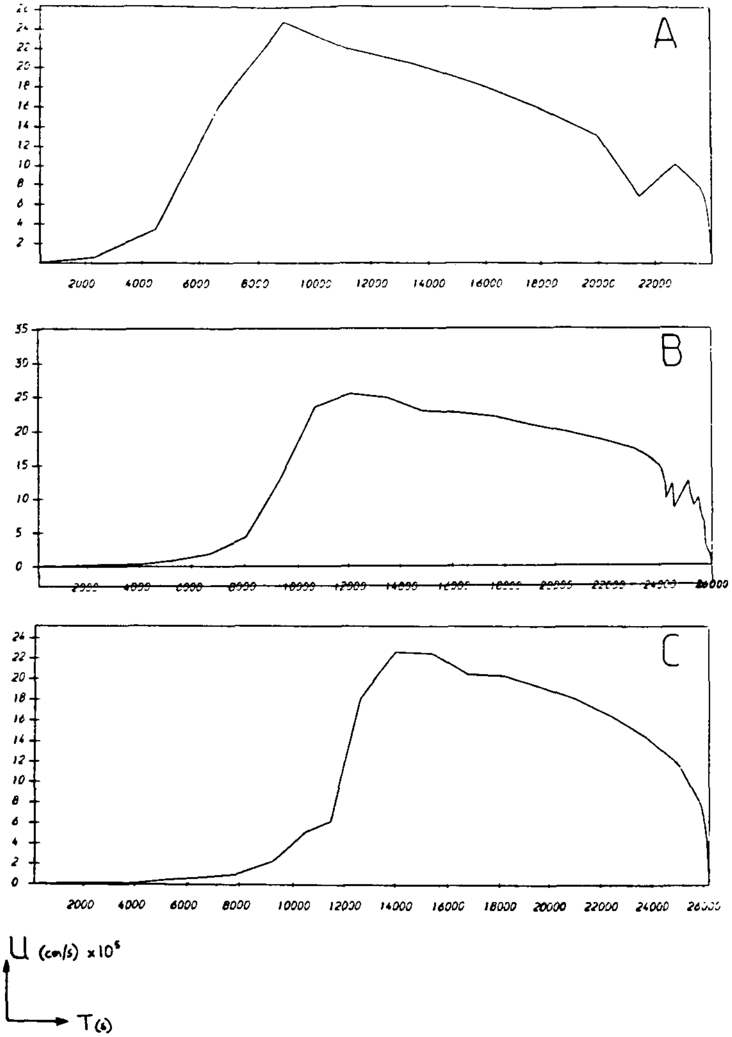

Figure 1 Growth and decay of the surface velocities along the line-of-centres (A), and at angles 1.0° (B) and 1.2° (C) to the line-of-centres.

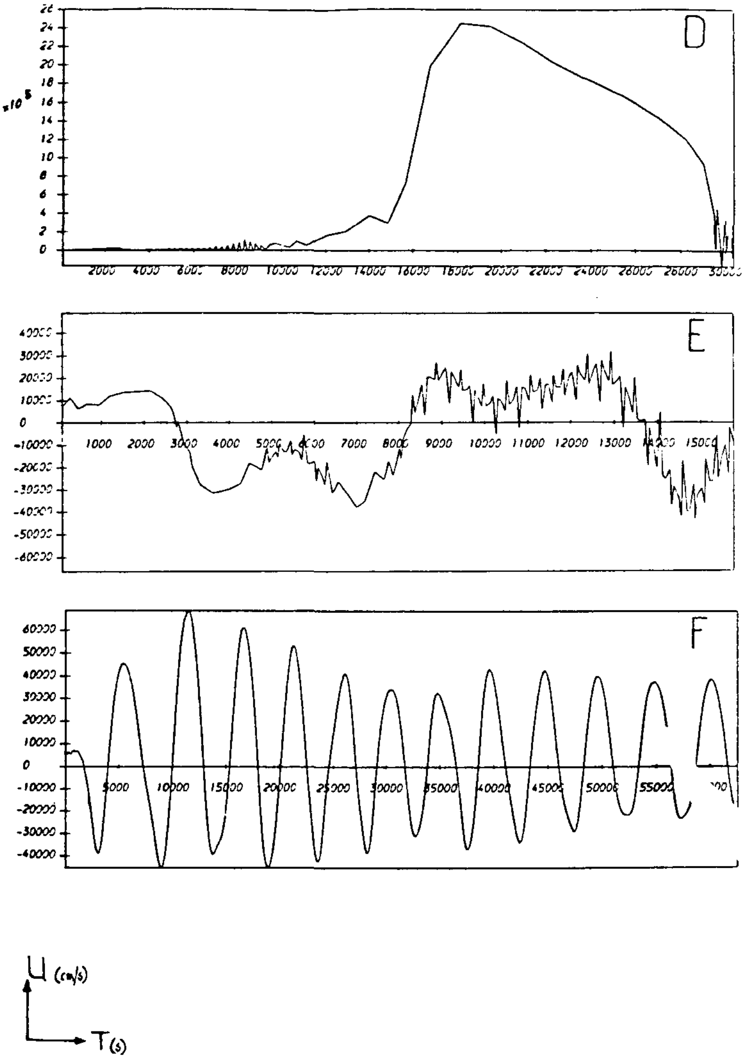

Figure 2 Evolution of the surface velocity at angles 1.4^0 (D), 1.5^0 (E) and 5.0^0 (F) to the line-of-centres.

variables set constant on equipotential surfaces. This requirement is a necessary condition to remove azimuthal flows along the potential surfaces within the initial model.

Four geometric correction factors must be introduced to insure that the initial models are static with respect to radial flows. These must be calculated independently for the equations of continuity, hydrostatic equilibrium, radiative diffusion and the requirement that the blackbody luminosity is the same at different points on the surface.

In one series of models the initial model was constructed along the line of centres, and the off-axis corrections calculated for seven spokes out to an angle of 1.0^0 from the line of centres. In a second series of models the initial spherically symmetric model was chosen to be the off-axis radial spoke for which the gravitational attraction of the primary star was perpendicular to this radial spoke at the surface of the secondary star. The structure along, and close to the line-of-centres was constructed by sweeping the dependent variables along the equipotential surfaces, with the separation between the stars set such that the surface of the on axis model was one half of a scale height from the inner Lagrangian point.

Figures 1 and 2 show the growth and later decline in the outermost velocity at angular displacements from the line-of-centres of 0.0^0 (A), 1.0^0 (B), 1.2^0 (C), 1.4^0 (D) 1.5^0 (E) and 5.0^0 (F). These results are for the second class of model. The first class are essentially similar.

The figures clearly demonstrate that the instability extends to an angular size of almost 1.5^0, which corresponds to a radius of about 3 scale heights. The maximum mass transfer rate was 7.8×10^{17} g s^{-1} and lasted 3.0×10^4 s. Such a pulse of mass transfer will lead to an enhanced accretion luminosity from the disc. We examine the dependence of the outburst light curve on the properties of such a pulse in the next section.

2. THE LIGHT CURVE PROPERTIES OF MASS TRANSFER EVENTS

We have examined the response of the disc to a range of variations in the mass transfer event profile. The disc evolution models in this section are α-disc models with α=2.0 (Bath and Pringle 1981, Bath, Edwards and Mantle 1983). We assume a white dwarf mass $1M_0$, inner disc radius $R_{IN} = 5 \times 10^8$ cm, outer disc radius $R_{OUT} = 5 \times 10^{10}$cm and circularization radius $R_K = 1.5 \times 10^{10}$cm. These parameters are appropriate for the longer period cataclysmic variables such as SS Cygni.

The typical observed forms of the outbursts of SS Cygni have been classified by Campbell (1934) (see also Bath, Clarke and Mantle 1986). They are divided into four distinct classes depending on the speed of the rise from quiescence to outburst maximum; A, very rapid rise; B, somewhat slower; C. moderately slow and D, slow and often irregular. The properties of the outburst vary with the outburst class. When the outburst rises rapidly (cases A and B) the light curve decays with a constant, minimum timescale. Both narrow and wide outbursts occur as

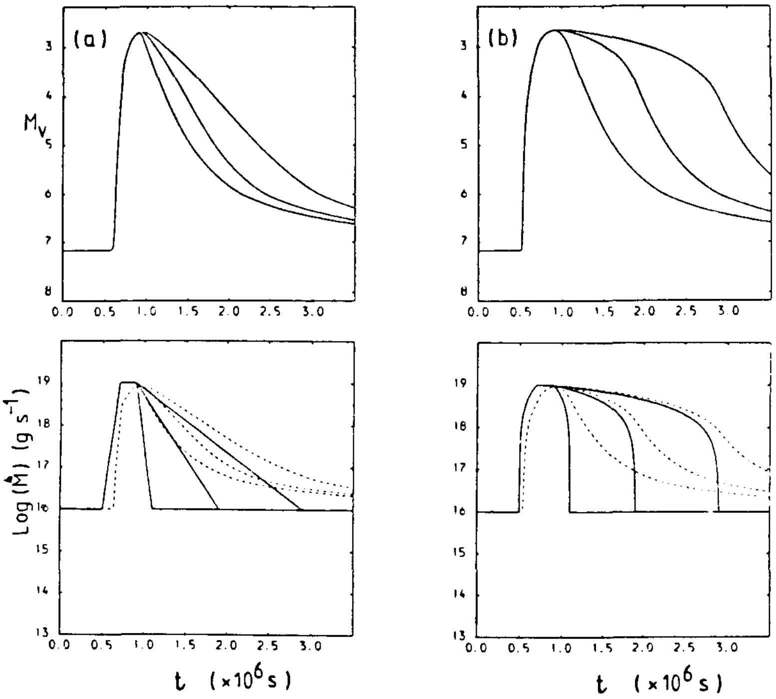

Figure 3 The upper figures show the visual light curves produced by the mass transfer rate profiles below. The full lines in the lower figures show the mass input by the mass transfer stream whilst the broken lines show the resulting accretion rate on the white dwarf.

distinct classes but both have the same characteristic decay timescale. When the outburst rises slowly, in particular on a timescale equal to or exceeding the decay time of classes A and B then the decay is often extended and irregular. Thus class D shows a wide variety of light curve forms, with class C showing intermediate behaviour.

Disc evolution models show the same behaviour in response to mass-transfer bursts by the red component. If the overall timescale for the mass-transfer rate to rise and fall is shorter than the disc response time due to viscous stresses then the outburst light curve shows a rapid rise and declines on the viscous timescale. If, alternatively, the rate of decline (or rise) is longer than the viscous timescale then the light curve decays (or rises) on the timescale imposed by the mass transfer variations themselves. Irregularities in the mass transfer flow show directly in the light curves and the form of class D outbursts is then obtained. On the other hand fast mass transfer changes produce light curves similar to cases A and B.

In figure 3 we show the disc response to a variety of mass transfer inputs with varying rates of decline. Decay times of 2 x 10^5s, 10^6, and 2 x 10^6s, are considered, varying exponentially (left hand side) and linearly (right hand side). The most notable feature of the light curves is the existence of a minimum decay time for an outburst. The disc is unable to follow changes in the mass-transfer rate which occur on timescales shorter than this minimum time. The first light curve in each case corresponds to this minimum time. The second curve in the exponential case is produced by a burst which decays on the minimum disc response timescale. The rate of decline is the same but changed from slightly concave to convex. The third curve in the exponential case corresponds to a longer timescale burst decay. The behaviour provides a natural explanation of the D-type maxima. The last two light curves in the linear case show initially slow decline as the disc follows the initially slow decrease in the mass-transfer rate which is then followed by a decay on the viscous timescale as the mass-transfer rate drops rapidly back to the quiescent level. These curves bear a strong resemblance to some of the C-type light curves.

3. TWO DIMENSIONAL MODELS

An independent analysis of the disc evolution problem has been carried out by Whitehurst, who has developed a two dimensional numerical model of the accretion flow in close binaries. Existing models (Lin & Pringle 1976, Hensler 1982), have not studied the properties of time-varying accretion flow, but because of computational limitations have covered only steady state solutions of the disc structure. Therefore it was impossible for these models to have a satisfactorily calibrated viscosity, that is to say a viscosity which can be shown to be equivalent to dwarf nova discs in terms of decay timescales.

In order to follow rapidly evolving flows it was necessary to chose a method suited to high speed computation. That chosen was due to Larson (1978) and employs a purely particle based model of the flow. Unlike other techniques, such as Particle-in-Cell codes, no grid is required upon which to interpolate the fluid forces. Instead all the parameters of the flow; mass, momentum and energy, are carried by the particles and their interactions represent the viscous and pressure forces. The interactions are easy to calculate. They are a combination of simple collisions and pressure forces, (Larson 1978). This ensures conservation of momentum and energy within each collision. Taking advantage of the two-dimensional nature of the model, it is assumed that instead of incrementing the particles internal energy, dissipated kinetic energy is instantaneously radiated away in the vertical direction. Hence the quantities found most directly are density and bolometric luminosity, which are presented in Figures 4 to 7.

In the sample run taken to illustrate some points of the model

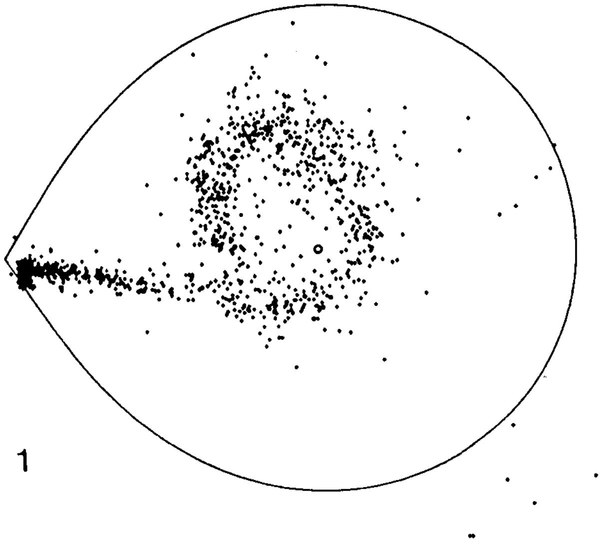

Figure 4 Plot of particle positions within primary's Roche lobe. The distribution at the early stages of the rise follows the free-fall trajectory.

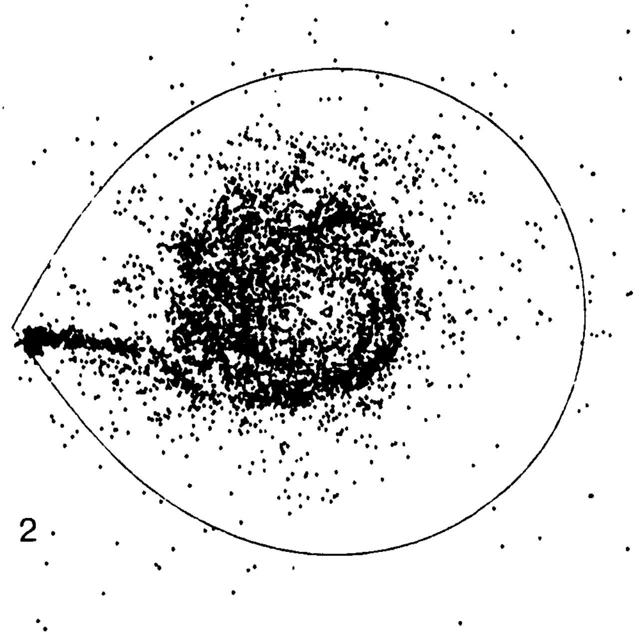

Figure 5 The particle distribution at maximum light still follows the stream trajectory but is viscously broadened.

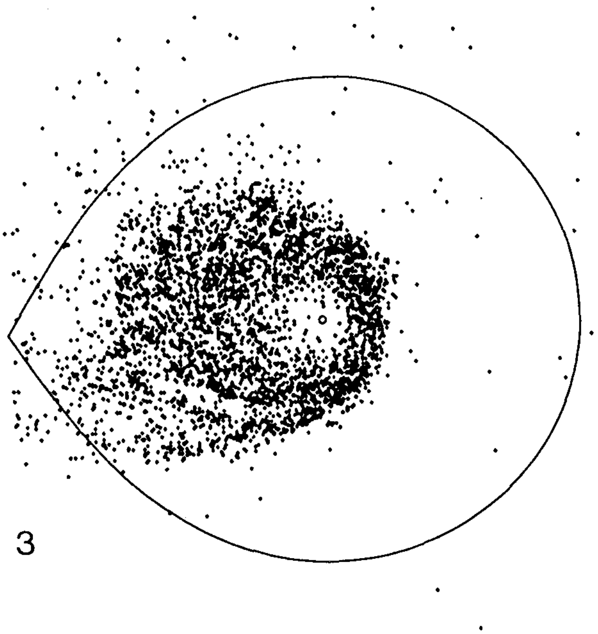

Figure 6 On the decline the particle distribution forms a set of nested eclipses along the line-of-centres. The radial component of motion is not entirely dissipated. This behaviour persists into quiescence.

properties physical parameters, (mass ratio, binary separation & period), have been chosen to correspond to those determined observationally for SS Cygni. The model's parameters governing viscosity have been selected to give an appropriate decay time for a speed class 'A' outburst. (Campbell 1934)

The light curve generated in Figure 7 shows a rise of some 5 magnitudes in luminosity on a timescale of about 10^5s. This is in reponse to a Gaussian mass transfer burst of 2.5 x 10^{23}g with half-width 7 x 10^4s, (maximum mass transfer rate 6 x 10^{18} gs^{-1} at 5 x 10^4s). The development of the density structure of the disc is shown in Figures 4 to 6, taken at the times indicated on the light curve.

Initially the primary's Roche lobe was empty, and by a time of 3 x 10^4s into the rise, (Figure 4), most of the mass is distributed along the stream trajectory. Luminous dissipation is already occuring most strongly near the white dwarf, although the stream impact region is also quite bright. By the time of maximum light, (Figure 5), the stream is declining towards its quiescent rate, but is still dominating the density distribution. Here the particles have spread out from their

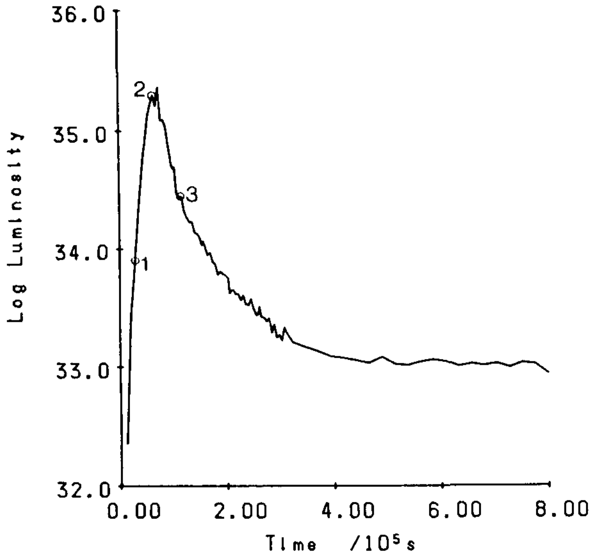

Figure 7 Luminosity evolution through an outburst following a 5 x 10^4s mass transfer burst. 1, 2 and 3 indicate the times at which figures 4, 5 and 6 were taken.

original orbits and now almost all the luminosity is arising near the white dwarf. Finally, in Figure 6, taken on the decline, the stream is now insignificant and the particles are distributed in their equilibrium configuration; an ellipse with major axis aligned with the line of centres of the stars. Effectively all the light now originates from the central portion of the disc, where it is clear that accretion on to the white dwarf maintains a relative void of particles. As the disc drains away towards the end of the outburst, the size and shape of the disc remains constant but the luminosity of the stream disc impact region gradually becomes more important.

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