MAGNETIC FIELDS AND STELLAR EVOLUTION

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1. INTRODUCTION

Magnetic fields are now observed or inferred in a wide variety of stellar objects. The class of early-type stars with strong large-scale fields extends from types B to F, with effective fields from 300 gauss up to several x 10^4 gauss (Borra and Landstreet 1980). Fields between 4×10^6 and 10^8 gauss have been inferred in a small percentage of white dwarfs, and of over 10^{12}gauss in neutron stars. Some Cepheids show measurable fields. Evidence has built up of solar-type activity in late-type stars. The pioneering work by Wilson (1978) on Ca activity has shown convincingly the occurrence of periodicity reminiscent of the solar cycle in a number of G, K and M stars. Ca II emission appears to be a good predictor of simultaneous X-ray emission from hot coronae around cool stars (Vaiana 1979, Mewe and Zwaan 1980). Fields of some 2×10^3 gauss have been reported in two late-type main sequence stars (Robinson, Worden and Harvey 1980).

The positive correlation between Ca activity and stellar rotation has been demonstrated both for main sequence stars (Wilson 1966, Kraft 1967) and more recently for giants (Middelkoop and Zwaan 1980). Perhaps most spectacularly, the RSCVn close binary systems show what appears to be greatly enhanced solar activity, with strong chromospheric, radio and X-ray emission, and provide evidence that a large fraction of their surfaces are covered by dark spots (Hall, D.S.in Fitch (ed.) 1976; papers in Plavec et al (eds.) 1980). Theoretical developments in dynamo theory, in which rotation (both absolute and differential) is crucial, encourage the treatment of all late-type stars with extensive outer convective zones as locations of solar-type dynamos, with the rotation as a fundamental parameter of magnitude determined by the star's history. From a complementary point-of-view, there have been a number of studies on the gross interaction between large-scale stellar magnetic fields and the thermal-gravitational-dynamical field, in both radiative and convective zones. Much of this work is independent of whether the field is being maintained by contemporary dynamo action, or is instead a "fossil" - a slowly-decaying relic of either the galactic

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field pervading the gas from which the star formed, or of dynamo action in an earlier epoch in the star's life (e.g. the Hayashi phase). Dynamo action in the solar convection zone does not rule out a primeval field trapped within the solar radiative core, as postulated by several authors; nor do tentative arguments for a fossil theory for the observable fields of strongly magnetic early-type stars require the non-occurrence of dynamo action in their convective cores. When concerned with magnetic fields and stellar evolution, it is prudent at the present state of knowledge to keep options open.

2. STELLAR MAGNETIC FIELDS - GLOBAL CONSIDERATIONS

A strongly magnetic early-type star with a large-scale field of mean strength $\bar{B}$ over the radiative outer zone has a ratio of magnetic to gravitational energy of order

$$\varepsilon \equiv (\bar{B}^2/8\pi)(4\pi R^3/3)/(GM^2/R) \simeq F^2/(6\pi^2 GM^2), \tag{1}$$

where $F = \pi\bar{B}R^2$ is an estimate for the total flux. The parameter ε is a convenient measure of whether the field in a magnetic star is globally "strong" or "weak". Stellar fields can be observed directly only if the field-strength is $\simeq 10^3$ gauss over a large part of the disk. The star with the strongest observed surface field B_s is HD 215441, with $B_s \simeq 3.4 \times 10^4$ gauss. If we extrapolate this inwards, adopting the largest-scale, slowest decaying Cowling mode (Cowling 1945; Wright 1974), we find an internal field $\bar{B} \simeq 10^6$ gauss, yielding $\varepsilon \simeq 10^{-5}$. This immediately raises the important theoretical question of the actual relation between the strength of the field that emerges through the photosphere to yield an observable Zeeman effect, and the mean field $\bar{B}$ deep within the star. We return to this below; for the moment we note that even for this exceptional star one needs $\bar{B}$ to be 30 times larger for ε to approach 10^{-2}.

Similar conclusions hold for magnetic white dwarfs and neutron stars, as follows if we assume that during contraction to form a degenerate star, the central regions of the parent star conserve their magnetic flux, so that $\bar{B}$ increases like $\bar{\rho}^{2/3}$, and the analogue of ε stays constant. Then as the density increases from a main-sequence central value of $\simeq 10$ gm/cm^3 to a typical mean white dwarf value of 10^6, $\bar{B}$ would increase from a hypothetical value of 10^6 to well over 10^8 - at the upper limit of the values inferred for the surface fields of white dwarfs. The analogous argument would predict a neutron star field of 10^{13} - 10^{15}, again on the high side for both observation and theory. Thus there seems no prima facie reason to doubt that even the most strikingly magnetic stars are "weak". If anything, the flux-freezing argument suggests that 10^6 gauss would be exceptional for $\bar{B}$ within most main-sequence stars: 10^4 - 10^5 would fit in better with the estimates for white dwarfs and pulsars.

The evidence is now rather strong that rapid flux leakage occurs during the molecular cloud phase of star formation. With field freezing re-established during the opaque phase, e.g. at a density of 10^9 particles/cm^3, the galactic field would be amplified to $\simeq 5 \times 10^4$ gauss on the main sequence, so there is no difficulty in producing magnetically "weak" Type I stars. However, Type II (and a fortiori Type III) stars may have formed from a gas of weakly polluted hydrogen and helium at $\simeq 10^4$ °K (Hoyle 1953) which maintains a high level of ionization. Fragmentation can occur via preferential flow down the field-lines but would yield magnetically "strong" proto-stars (Mestel 1965a). It is known that even in sub-adiabatic stellar domains, magnetic fields of simple topology are subject to dynamical instabilities near their neutral points (Wright 1973; Markey and Tayler 1973, 1974); for under non-axisymmetric adiabatic motions that are nearly perpendicular to the gravitational field, the stabilizing effect of the sub-adiabaticity disappears, and magnetic energy is spontaneously released. To remove these obvious instabilities one needs to construct complex fields, with e.g. toroidal flux loops linking the poloidal loops. One wonders, however, whether this would be sufficient if the magnetic energy is high, or whether other dynamical instabilities inevitably arise unless the parameter ε is much below unity, whatever the structure of the field.

With dynamical stability supposed satisfied, there remains the question of secular stability. Even in a stable, sub-adiabatic radiative zone, individual flux-tubes tend to float to the surface via "magnetic buoyancy" (Parker 1979; Acheson 1979), at a rate determined by heat diffusion into the cool gas within the tube - an "Eddington-Sweet" effect (Sweet 1950). It may be that global fields with mutually linking poloidal and toroidal loops are subject to analogous instabilities, perhaps depending also on the finite resistivity which allows changes in field topology. Indefinite stabilization over a stellar lifetime may require a negative gradient of mean molecular weight μ.

The fossil theory of stellar magnetism is a theoretical possibility because a large-scale poloidal field $\underset{\sim}{B}_p$ is maintained by a toroidal current density $\underset{\sim}{j}_t = c\nabla \times \underset{\sim}{B}_p/4\pi$ flowing in the high conductivity interior, yielding a decay-time $\tau \simeq 4\pi\bar{\sigma}R^2/c^2$, where $\bar{\sigma}$ is a mean value. In the cooler outer regions the currents are much weaker, and the field in a large-scale Cowling mode is locally nearly curl-free. But the toroidal field $\underset{\sim}{B}_t$ linking $\underset{\sim}{B}_p$ (as required for stability) is maintained by poloidal currents $\underset{\sim}{j}_p$ that flow nearly parallel to $\underset{\sim}{B}_p$, since in a non-turbulent domain the total field must be nearly torque-free. Thus if a particular loop of $\underset{\sim}{B}_p$ passes through the surface regions, the currents flowing along it will suffer much greater dissipation, because the surface resistivity is some 10^3 higher than the mean. One therefore expects the surviving toroidal flux to be largely concentrated deep within the star, maintained by currents flowing primarily along poloidal loops that do not come too near the surface. Does this highly non-uniform distribution of $\underset{\sim}{B}_t$ have any serious effect on the dynamical stability of the field $\underset{\sim}{B}_p$? *Prima facie*, the stability near the neutral points should survive, as long as the toroidal flux is of the same order as the poloidal; but one would like reassurance that the effective Ohmic destruction time of the total field has not been substantially reduced.

The stability of stellar magnetic fields is clearly relevant to the question as to whether the fields of the strongly magnetic early-type stars are primeval, or require continual regeneration by a contemporary dynamo. The bewildering complexity of the observed parameters of the magnetic stars can be cited as an argument in favour of the flux being an extra parameter rather than one closely linked with the structure and the rotation of the star. Although our understanding of both kinematic and dynamical dynamos is very incomplete, and there remain the queries about instabilities possibly shortening the life-times of unregenerated fields, one can still claim that as yet there are no obvious astronomical advantages in insisting on a contemporary dynamo rather than a fossil explanation for the fields of the strongly magnetic stars; and likewise, a fossil field trapped within the radiative core of a late-type star remains a theoretical possibility.

3. MAGNETIC FIELDS AND STELLAR STRUCTURE

Since the ratio ε is apparently so small, the magnetic field should not have a great effect on the overall structure of the star. However, interesting effects can arise if stellar hydrodynamics leads to a local increase of field strength in regions of small scale, or into the low-density surface regions. And at least in non-convective domains, the field should be of paramount importance in its interaction with stellar rotation, as long as any mass motions have velocities well below the Alfvén speed. In particular, during the leisurely pace of normal stellar evolution, we can expect a contracting burnt-out core to be kept more or less corotating with an expanding envelope, as long as they remain magnetically linked. Thus there is no reason for surprise that even the most rapid pulsars are slow rotators at birth. Simple estimates show how essential is such redistribution of angular momentum. Suppose instead that in contracting from $\rho \sim 10$ to $\rho \sim 5 \times 10^{14}$ gm/cm^3, the central core of a main-sequence star with a rotation period of about 1 day had conserved its angular momentum: the period of a resulting neutron star would be 6×10^{-5} secs, with centrifugal forces eighty times gravity - clearly a *reductio ad absurdum*. Inverting the problem, we find that a pulsar will rotate at birth with centrifugal force just one percent of gravity if angular momentum conservation began when the core had reached a density near 10^4 gm/cm^3.

However, once contraction begins to be rapid, and approximate co-rotation is no longer maintained, important effects can follow from the conversion of the energy of non-uniform rotation - itself fed from the gravitational field - into toroidal magnetic energy. This local concentration of energy is crucial in some models of the dynamics of supernovae (LeBlanc and Wilson 1970; Kundt 1976). In the most recent study (Müller and Hillebrandt 1980), the rebound of the core at nuclear densities generates a shock wave which delays infall of the mantle, so giving the rotational shear time to generate a toroidal magnetic pressure near the core that is close to the thermal pressure. The resulting second hydromagnetic shock can lead to mass ejection with high energy, leaving a central neutron star.

Even a weak magnetic field radically alters the problem of constructing a self-consistent model of a rapidly rotating stellar radiative envelope. Over the bulk of the zone, the advection of angular momentum by the slow, centrifugally-driven Eddington-Sweet circulation is easily off-set by the field, since the speed v_p is much less than any likely Alfvén speed (Mestel 1961). The approximation of ignoring the magnetic as compared with the centrifugal disturbance to hydrostatic and thermal equilibrium is consistent over the bulk of the zone, since

$$\rho v_p^2 << B_p^2/4\pi << \rho\Omega^2 r^2. \tag{2}$$

But in the low-density surface regions the magnetic forces need not be negligible; also, a general perturbing force yields circulation speeds that become large like $\bar{\rho}/\rho$ (Baker and Kippenhahn 1959), so the whole order of approximation could break down.

A limited number of steady self-consistent models have been constructed. The simplest class suppose that the star has achieved radiative equilibrium (with no circulation) - the magnetic forces are significant not only in the surface regions but over the whole radiative envelope. In models with the rotation and magnetic axes parallel, there is as a consequence a general tendency for the fraction of the prescribed total flux that appears above the surface to decrease with increasing rotation rate (Davies 1968; Wright 1969; Moss 1973, 1975). In the appropriate parameter range, the ratio $\bar{B}/B_s$ is $\simeq$ 1500 - much higher than in a Cowling mode, implying $\bar{B} \simeq 10^6$ or more even when B_s is no more than 10^3. Mestel and Moss (1977) have constructed approximate models with a non-vanishing circulation field that reduces deep down to the Eddington-Sweet flow. It is found that the magnetic forces in the surface regions adjust themselves so as to kill off the $\bar{\rho}/\rho$ terms, leaving both $\underset{\sim}{v}_p$ and $\underset{\sim}{B}$ small in the surface regions, and again with the flux strongly concentrated into the deep, high density regions.

It should be emphasized that in much of this work there are severe mathematical difficulties, which force the premature truncation of expansions in orthogonal polynomials. One suspects that this is largely responsible for failure to cover all the physically allowed parameter range. The attempts on analogous non-axisymmetric models have had markedly less success, probably for similar reasons (Monaghan 1973; Moss 1977b). However, the work may give some qualitative understanding of the respective properties of magnetic and non-magnetic early-type stars. It is known that there is a gross anti-correlation between rotation rate and the appearance of magnetic flux above the stellar surface, or of the abundance peculiarities that are often a tracer of such external flux. There is also some tentative evidence that within the class of magnetic Ap stars, the observed fields are systematically weaker in the more rapid rotators (Borra and Landstreet 1980). More effective magnetic braking by a stronger external field may account for this in part, but the tendency for rotationally-driven circulation to concentrate the flux deep in the star may also be playing a role.

All this work has assumed no μ-gradients in the radiative zone. In particular, the model with the Eddington circulation deep down assumes there is a "μ-barrier" at the surface of the convective zone, where the vertical velocity is zero, and where the simple theory predicts large horizontal velocities by continuity. Thus the μ-barrier region is again singular; if the field is at least approximately frozen into the fluid, the associated horizontal component becomes large enough again to contribute to hydrostatic and thermal balance. It is reasonable to look for a self-consistent steady state in which the combined effects of centrifugal and magnetic forces reduce the thermally-driven vertical speed to zero at the barrier. Rough estimates predict that the magnetic contribution should be significant in a layer of thickness $D \simeq r(\bar{B}^2/8\pi\bar{\rho}\Omega^2 r^2)^{1/3}$. However, the attempted construction of such a model has run into even more severe truncation difficulties than in the surface regions. More seriously, it is not clear that one can satisfy the conditions of zero slip and zero stress at the barrier that are strictly required, even though the viscosity of stellar material is so small. It may be that the picture of a μ-barrier separating the domain of a steady increase in μ from an envelope with zero μ-gradient should be questioned (cf. the non-magnetic study of Huppert and Spiegel 1977). Perhaps instead a μ-gradient extends itself steadily through the radiative zone, so that the Eddington-Sweet circulation suffers from "creeping paralysis" (cf. Mestel 1953, 1965b). If so, then the tendency of the magnetic field to be concentrated into high-density regions will be at most temporary; as the circulation dies out, the field over the bulk of the radiative envelope would diffuse back into a Cowling mode. However, residual motions near the surface could still be important for the local field structure, and in particular for the amount of observable flux.

The strength of sub-surface magnetic fields is of particular interest for the theory of Cepheid variables. The observations of magnetic fields in some Cepheids have prompted Stothers (1979) to see what effect a magnetic pressure would have on the pulsational properties of Cepheids. He postulates a small-scale field, with the ratio ν of magnetic to thermal pressure an adjustable parameter. He is able to remove the discrepancy between the pulsational mass and the evolutionary mass if ν has a uniform value of 0.8 in a surface layer comprising 10^{-3} of the stellar mass, implying a field strength of $\simeq 10^4$ gauss at the base of the layer. This model is certainly somewhat ad hoc, but it is worth noting that a frozen-in poloidal magnetic field B_p, consistent with a circulation field v_p, satisfies $B_p \propto \rho v_p$; hence if the circulation speeds were to retain something like the $\bar{\rho}/\rho$ dependence, the distorted magnetic field could have a structure with B only weakly dependent on ρ. A value of 10^4 gauss just below the surface would then not be hydromagnetically inconsistent with a similar value deep down. Certainly one feels that the whole problem of self-consistent, thermally-driven, hydromagnetic flow in the outer layers of a star is still ill understood. Further work should probably pay more attention to the detailed physical properties of the surface regions, especially to the local convective zones.

4. THE OBLIQUE ROTATOR

The case for this simplest model for the magnetic Ap stars (Deutsch 1958; Preston 1971) gains support from the plot of the spectroscopically measured Vsini against the period P of magnetic, spectral and luminosity variations, which shows the points lying below a well-defined hyperbolic envelope PV = constant, to be expected if these stars do not deviate too much from the main-sequence A-star region. Even if a magnetic field is not measured directly, one is always tempted to interpret periodicity in some other feature as the consequence of azimuth-dependent structure with a magnetic field as a possible cause. Equally, any global consequences of non-axisymmetry may be only weakly dependent on the amount of flux that penetrates the photosphere, and so may affect the evolution of observably "non-magnetic" stars.

For definiteness, we suppose the magnetic field symmetric about an axis $\underset{\sim}{p}$ inclined at an angle χ to the rotation axis $\underset{\sim}{k}$. An oblique rotator has some properties in common with the classical problem of a body with three unequal axes of inertia (Spitzer 1958). In slowly-rotating, weakly magnetic stars, the departures from spherical symmetry of the density-pressure field are the superposition of a part symmetric about $\underset{\sim}{k}$ and a part symmetric about $\underset{\sim}{p}$. The motions within such a body can be most simply analyzed into (1) the basic rotation $\Omega\underset{\sim}{k}$; (2) the Eulerian nutation with a frequency ω about $\underset{\sim}{p}$; and (3) a field of "$\underset{\sim}{\xi}$-motions" with frequency ω that ensure that the star remains in hydrostatic equilibrium (Mestel and Takhar 1972; Mestel et al 1980; Nittmann and Wood 1980). To order of magnitude

$$\omega \sim \Omega\varepsilon \ , \qquad \xi \sim \ell \ (\Omega^2 r^3/Gm(r)) \tag{3}$$

where ℓ is the local scale-height. Thus the nutation period $2\pi/\omega >>$ the rotation period $2\pi/\Omega$, but can easily be much less than the stellar lifetime or the Kelvin-Helmholtz time. In slow rotators ξ/ℓ is small, but the appropriate generalization to rapid rotators suggests that the motions would not then be trivial: they could yield mixing of matter

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between part of a stellar envelope and a convective core, so modifying tracks in the H-R diagram; and in fact Nittmann and Wood (1980) have suggested $\underset{\sim}{\xi}$-motion mixing in rapid rotators as an explanation of the blue straggler phenomenon. Also, as already noted, a μ-gradient in a radiative zone can effect indirectly the distribution of magnetic flux through the star.

However, as pointed out by Spitzer, the $\underset{\sim}{\xi}$-motions will be subject to dissipation, which acts as a drain on the rotational kinetic energy of the star. This has the form $h^2/2I$, where h is the angular momentum and I the moment of inertia about the instantaneous axis of rotation. Thus if dissipation is fairly efficient, and nothing else intervenes to affect the flux-distribution, the magnetic axis should rotate in space until the star is rotating about its maximum moment of inertia, and the $\underset{\sim}{\xi}$-motions cease. Following the report (Preston 1971) that magnetic obliquities seemed to be concentrated either at χ large or χ small, the theory was tentatively linked with the requirement that stable magnetic fields must have linked poloidal and toroidal flux of comparable magnitudes. The suggestion was that cases with dominant poloidal flux would be dynamically oblate about the field axis, and so would tend to approach $\chi = 0$, while those with dominant toroidal flux may be prolate and so approach $\chi = \pi/2$. The observational evidence is now more obscure: Hensberge <u>et al</u> (1979) argue that there is no non-randomness in χ, while according to Borra and Landstreet (1980) there is marginal evidence for non-randomness. The immediate applicability of the argument thus depends on the time-scale for dissipation. In Mestel <u>et al</u> (1980) an upper limit is found by constructing the distortions $\underset{\sim}{B}'$ to the magnetic field due to the $\underset{\sim}{\xi}$-motions and then computing the volume integral of the Ohmic dissipation $(c\nabla \times \underset{\sim}{B}'/4\pi)^2/\sigma$. Not surprisingly, the dissipation is strongly peaked at the cooler surface regions. The total dissipation rate is sensitive to the nutation frequency ω and so to ε; in particular, if the field is <u>not</u> as centrally condensed as in the models of Wright, Moss, Mestel and Moss etc., but is closer to Cowling's slowest decaying mode, then the time for Ohmic dissipation of the $\underset{\sim}{\xi}$-motions is too long. One could then imagine an initial gross obliquity and so also the associated $\underset{\sim}{\xi}$-motions lasting through the star's lifetime and so affecting stellar evolution.

It is well, however, to remember that there are other processes that can affect the apparent obliquity, e.g. the precessional torque associated with magnetic braking (Mestel and Selley 1970), or just the slow kinematic effect of horizontal surface motions (Moss 1977a). There may be more powerful dissipative processes affecting the $\underset{\sim}{\xi}$-motions. With more complicated field structures than those studied, there may not be an unambiguous relation between the state with rotation about the maximum moment of inertia and the superficial flux distribution. And if there is some interaction between the convective core and the field in the envelope, e.g. by dynamo generation of new flux, then the theoretical uncertainties are multiplied.

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Any non-axisymmetric feature on a stellar surface is a potential probe of a rotation. Thus Dicke (1979) interprets his most recent solar oblateness measurements, which pick up a 12.2 day period, in terms of the rotation of a perpendicular magnetic rotator in the radiative core. His particular model is questionable on stability grounds, as he postulates a toroidal field with an energy far greater than the poloidal. But if the Ap star fields are primeval, then it is tempting to argue that late-type stars also contain such flux, which is however largely prevented from appearing at the surface by the powerful outer convective zone; in which case, all the questions of stability, obliquity, coupling with the convective zone etc. remain relevant for the central regions.

The problems are frustrating, because of the difficulty in making an unambiguous link with observation. The discovery that some Cepheids have magnetic fields may offer more fruitful scope. For many years it has been known that RR Lyrae shows a 41-day cycle in its light and velocity curves (e.g. Detre and Szeidl 1973). The earlier report that the star has an observable magnetic field has not been confirmed; however, this need not mean that magnetic forces are unimportant in the observed low-density regions. It is well worth trying to construct the back-reaction on the density-pressure field of the forces exerted by a magnetic field that is periodically distorted by an essentially radial pulsation. If significant effects are found, and the basic magnetic field is oblique, then the theory should predict variations with the rotation period.

5. STELLAR CONVECTION ZONES: DYNAMO ACTION

Since the turbulent velocities deep in a convective zone are highly subsonic, one might expect significant magnetic interference with convection for magnetic energy densities much below the thermal. For example, with $\rho \sim 1$ and $v_t \sim 10^4$ cm/sec, $B^2/8\pi \sim \rho\, v_t^2$ requires $B \sim 5 \times 10^4$ gauss. Nearer the surface the turbulent velocities are higher but the densities much lower, so that $B \sim 10^2$ gauss is sufficient. However, for a magnetic field to interfere globally with convective heat transport and so affect seriously the structure and evolution of the star, the field would need to be much stronger: one must compare the field energy with the energy that the turbulence would develop if the temperature gradient retained the strongly superadiabatic value it has if radiative equilibrium were maintained, rather than relaxing to just a fraction 10^{-6} above the adiabatic value, as in the classical Biermann-Cowling estimate. Detailed stability studies by Gough, Moss and Tayler (1966, 1969) show that complete suppression of convection requires a field energy comparable with the thermal energy. Quite apart from conflict with observation, such a strong field could itself be spontaneously unstable. However, a field anchored in a deeper radiative zone may very well stabilize the weak convective regions in the surface regions of early-type stars, so enabling element diffusion to occur and yield abundance anomalies.

A series of laminar flow studies within the Boussinesq approximation (e.g. Galloway, Proctor and Weiss 1977, 1978) have demonstrated how in the presence of a magnetic field an unstable zone maximizes the efficiency of heat transport by assembling the imposed magnetic flux into isolated tubes, so that unimpeded convection can occur in the rest of the zone. A similar picture is suggested (Galloway and Weiss 1979) for a turbulent convective zone. A significant conclusion is that local field-strengths are well above the value given by equipartition with the convective energy, as is indeed observed in sunspots. The upper limit is clearly given by $B \sim \sqrt{8\pi p}$, implying zero thermal or turbulent energy within a flux-tube. One can argue that an external magnetic field may not be totally expelled from a neighbouring strongly convective zone, but rather that some flux penetrates and is concentrated by the turbulence into ropes. The field should thus ensure some dynamical coupling between contiguous stable and turbulent zones.

Accepting that the observed solar fields are generated and destroyed as part of a periodic dynamo, with the solar rotation as a basic parameter, one is led to ask whether magnetic activity could ever rise to a level when it makes a substantial difference to stellar luminosity. We recall the old problem of the "missing heat flux" from a sunspot: a bright ring was expected around a spot, to compensate for the reduced flux into the spot (according to the Biermann picture - though this has been challenged by Parker (1979)). This difficulty was removed by Spruit (1977), who showed that the area of this ring should be too large to yield an observable temperature excess. More recently, Foukal and Vernazza (1979) have found a weak dependence of solar luminosity on magnetic activity, at the level of 7×10^{-4} of the continuum, with periods of 28 days, and consistent with changes in the area of magnetic faculae and sunspots. They argue that this is merely a redistribution of heat flow and not a sign of magnetic influence on the steady heat flow to the photosphere. But would such an effect be completely negligible in a rapidly rotating late-type star - e.g. in the synchronized members of RSCVn close binaries, which as noted appear to show greatly exaggerated solar activity?

Whatever the answer, one can predict that these stars will continue to offer scope for the application of hydromagnetic ideas. Thus Shore and Hall (in Plavec et al 1980) have applied a model analogous to Babcock's solar dynamo, with the two stars in near synchronous rotation, but with the differential rotation less than in the sun. There is also the possibility that the binary structure introduces new effects. For example, Dolginov and Urpin (1979) have discussed the possibility of Herzenberg-type dynamo action in binary systems that are not yet synchronized, and with rotation axes inclined towards the orbital plane.

The rapid development in dynamo theory and the corroboration from observation of a strong dependence of magnetic activity on stellar rotation are very impressive, but one should note that there remain theoretical difficulties. Layzer et al (1979) have criticized the mathematical and physical basis of the "mean field" dynamo equations. They argue for a return to the torsional oscillation model for the solar cycle, which they suggest is not a genuine self-maintained dynamo, but requires a continuous supply of flux from a non-regenerated, irregular field, largely confined to the radiative core. The occurrence of solar-type activity in late-type stars that are probably fully convective is perhaps a difficulty for attempts to replace stellar dynamos by "amplifiers" dependent on an externally anchored source of flux. Dynamo models based more on the individual flux-rope picture face the problem that magnetic buoyancy in a superadiabatic zone can be embarrassingly efficient. A toroidal tube of the strength required to account for the observed solar surface flux and in temperature equilibrium with the non-magnetic surroundings will rise in about a month, as compared with the solar time-scale of years. Several proposals exist for increasing the rise-time. Some workers have argued that the tube will be similar to a tube in a radiative zone - at the same density as its surroundings, but with a lower temperature, so that it rises at the rate fixed by the inward leakage of heat. However, Spruit (1980) has noted that this would not resolve the difficulty, as the tube would be unstable against buckling in vertical planes. Perhaps more plausibly, Zwaan (1978) has proposed that the difference in turbulent pressures maintains the balance between a flux tube with the same density as its surroundings.

We have seen that there is strong observational evidence that magnetic activity in late-type stars increases with rotation. However, this is by no means an obvious consequence of all dynamo theories. For example, Moffat (1970, 1972) has produced a model in which the increasing rotation makes the turbulence more nearly two-dimensional, so reducing the "α-effect". It cannot be ruled out that the fields of early-type magnetic stars are not fossils but are generated in their convective cores by steady dynamos of this type. Moss (1980) has suggested alternatively that rapid rotators produce oscillatory dynamos within their cores, which fail to yield observable fields at the surface, since the fields propagate to the surface as strongly damped waves.

The expected rotation law within convective zones remains ill-understood. Deep within a nearly adiabatic convective envelope, the rotation would be expected to approximate to constancy on cylindrical surfaces, with only minor departures due to locally strong magnetic forces, or to circulation driven e.g. by the Biermann-Kippenhahn anisotropic viscosity (Kippenhahn 1963). It appears that only in a comparatively thin surface layer will the anisotropy and the small scale-height enforce something closer to an $\Omega(r)$ law (Galloway and Mestel, in preparation), and this layer gets thinner in more rapid rotators. How this will affect dynamo action is as yet unclear.

6. MAGNETIC BRAKING OF STELLAR ROTATION

The braking process most studied involves a magnetically-controlled stellar wind (due either to the pressure of a hot corona or to the input of momentum by line absorption from early-type stars). The simple theory yields a flow of angular momentum equivalent to effective co-rotation out to the Alfvénic surface S_A where the wind speed $v = v_A \equiv B/(4\pi\rho)^{1/2}$:

$$- \frac{d}{dt}(k^2 R^2 M\Omega) = -(\frac{dM}{dt})(\Omega R_A^2) \tag{4}$$

where kR is the radius of gyration of the star and R_A a measure of the distance to the Alfvénic surface (Mestel 1966; Weber and Davis 1967). There is in fact something a little paradoxical about what is being demanded. We want the magnetic stresses to be able to control the rotation, and this is satisfied within S_A, where $v^2 < B^2/4\pi\rho$. However, one expects the wind energy density to be comparable with the energy density driving it - e.g. a thermally-driven wind to be supersonic but not hypersonic; in which case, a magnetic energy density that dominates over the thermal should seriously interfere with the outflow of gas. The simplest argument for the solar wind is that a corona heated to $10^{6\circ}$ cannot be held in by the pressure of the interstellar medium; but magnetic field-lines anchored in the solar convection zone can act as a "lid", preventing the expansion of gas with too little thermal energy. In fact, evidence has accumulated that it is the weaker field regions on the sun which are pulled out by the solar wind to become the interplanetary field. The point to note is that (4) shows how the field can increase the efficiency of angular momentum loss for a given mass-loss rate $-\dot{M}$; but a crucial question is how $-\dot{M}$ itself varies with changes in the external magnetic flux (e.g. due to the rotation-dependent dynamo). The maximum braking occurs if the field is supposed pulled out to be nearly radial, so that all the field-lines partake in the braking process (Weber and Davis 1967). A plausible minimum results by supposing the field to be virtually curl-free out to S_A, with extensive dead zones; this yields a braking rate which is only weakly dependent on the strength of the surface field (Mestel 1968). The real value is between these limits, but is difficult to calculate (cf. Okamoto 1974).

The problem is important for understanding the rotational history of the sun and other late-type stars. Anticipating that the external flux and so also R_A increase with Ω, we still need to know to what extent $-\dot{M}$ is reduced as the flux increases. Fortunately, we have observational evidence of the variation of rotation period with age (Kraft 1967) and of the decline in Ca activity with age (Wilson and Woolley 1970). It appears that the specific rate of braking does increase with Ω, implying that the increase of R_A^2 is more important than the interference of the increased magnetic field with $-\dot{M}$. One can plausibly parametrize the net effect by writing $-\dot{M}R_A^2$ in (4) as a simple increasing function of Ω; there results an algebraic rather than an uncomfortable exponential law of variation of Ω with time (Spiegel 1968; Skumanich 1972). On extrapolating back to the zero-age main sequence via the inferred rotations of late-type stars in the young Hyades and Pleiades clusters, it is estimated that the sun began there with only 10 - 20 times its present very slow rotation (Ostriker 1972). This puts a constraint on the later stages of the formation of the sun and similar stars. Even the most efficient processes for angular momentum removal during the pre-opaque phases of star formation are very unlikely to produce proto-stars that can contract all the way to the main sequence without running into centrifugal trouble. It is more plausible that proto-stars reach the pre-main sequence phase as rapid rotators, and subsequently redistribute their remaining angular momentum during the final contraction. The evidence cited implies that late-type stars become slow rotators during this phase. Magnetic effects may again be crucial, e.g. via coupling to a strongly enhanced stellar wind during the Hayashi phase, or between a central condensation and a nascent planetary system.

For stars with cool coronas (of temperatures below $10^{5\,\circ}$) and so without thermally-driven winds, magnetic braking can occur via an accretion process. Gravitationally inflowing gas will compress the field until the magnetic stresses halt the inflow at the Alfvénic surface defined by $B^2/8\pi \sim \rho GM/r_A$. This state is Rayleigh-Taylor unstable: gas can slide into troughs between magnetic planes, and so pick up angular momentum from the star via magnetic pressure gradients. If $\Omega^2 r_A > GM/r_A^2$, the gas is then driven outwards under the centrifugal slingshot, again carrying off the angular momentum of approximate co-rotation at r_A. The process clearly stops when $\Omega^2 r_A^3 = GM$. Rough estimates suggest that the process depends only weakly on the accretion rate; it has a typical time-scale of about 10^7 years (Mestel 1975), and yields a period of a few days at the cut-off. This is certainly of the right order for the majority of the strongly magnetic stars, for which one does not expect strong winds, since they lack outer convection zones, and are not hot enough for radiation-driven winds.

Magnetic coupling between members of a close binary system can interchange spin and orbital angular momentum, and so can be significant in the later stages of star formation. The classical synchronization process - tidal friction - can be efficient for late-type stars with a strong turbulent friction, but it decreases strongly with decreasing ratio of radius to mutual separation. As in the wind braking process, the effectiveness of magnetic synchronization is likely to depend rather critically on the field structure, in particular on the amount of flux coupling the two stars.

A combination of synchronization and efficient loss of angular momentum could be crucial for the evolution of RSCVn and other close binary systems. Break-up into a binary system is one way that a rapidly rotating proto-star can resolve its angular momentum problem; it would be amusing if subsequent magnetic braking led the two stars ultimately to coalesce.

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DISCUSSION

Schatzman: I am a theoretician but I like a theory better when it is in agreement with observations. A simple-minded view suggests that microscopic diffusion (under gravity or under radiation pressure) can take place where the magnetic field stabilizes the convection. However in Ap stars, the regions of strong magnetic field do not coincide with the regions where the elements separated by the Michaud mechanism show up.

Similarly, diffusion explains the general spectroscopic features of Am stars, however it fails to explain in detail any specific case.

Mestel: My comment on the relation between theory and observation was just that I feel defeated if one has to appeal to observation to resolve difficulties within a well-defined theoretical problem, instead of just

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solving the problem completely and then compare its predictions with observation.

I remarked that a weak convective zone in the outermost regions of an early-type star can be suppressed by a magnetic field of the strength observed, and that this is presumably a necessary condition for Michaud diffusion to occur. I did not claim that any particular theory of abundances is in good agreement with observation.

Roxburgh: Would we not expect all stars to have at least a weak magnetic field? Calculations of rotating or close binary stars which reglect the effect of magnetism may not have much relevance to the real world.

Mestel: That tends to be my view, at least for processes that have long timescales. However, while we are still uncertain as to what happens to primeval field in the Hayashi phase, and dynamo theory is still far from complete, studies of the strictly non-magnetic problems retain their interest and possible relevance.

Vilhu: What ideas do you have about how the braking depends on Ω, e.g. if the sun were to rotate more and more rapidly, up to say ~200 km/s?

Mestel: I recall that the Ω^3 law suggested by Skumanich and Spiegel seems to be consistent with the limited number of observations we have.

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