

fact the greater part of modern work in probability theory is devoted to the study of stochastic processes (families of non-independent random variables); for a good account of these the reader may turn to another book by Professor Kac appearing almost simultaneously with the present one. For both we are much in his debt.

D. G. KENDALL

MASSEY, H. S. W., AND KESTELMAN, H., *Ancillary Mathematics* (Sir Isaac Pitman & Sons, London, 1959), 990 pp., 75s.

The contents of this book are based on the recently revised syllabus for mathematics as an ancillary subject in the Special Honours Degree courses in Physics and Chemistry in the University of London, and the book is offered as a text-book primarily for first-year students. Its size is due in part to the authors' desire to include as many as possible of the topics proposed for the new syllabus, and in part to their conviction that a more detailed and rigorous training in mathematics is an essential prerequisite to further study in these fields. Some of the more recondite parts have been put in smaller type, an acknowledgment of the fact that they may be difficult for the immature student and may be omitted at a first reading.

The real number system is discussed in the opening chapter, a real number being defined as an infinite decimal, and linked with this is the notion of the limit of a sequence and the general principle of convergence. There is also an account of some inequalities useful in the discussion of convergence. The student beginning a course of mathematics at university will find this chapter formidable, and many with experience of teaching at this level will not be able to share the authors' view that so early an initiation into rigorous mathematics is sound teaching practice however desirable from other viewpoints.

The second chapter is an account of some elementary functions of a single variable introducing the ideas of continuity and convergence. Differentiation and differentials are the topics of the third chapter, the concept of a differential being carefully explained. The chapter on infinite series is in the precise style which characterises the book and is followed by accounts of the exponential function, defined as an infinite series, the logarithmic and hyperbolic functions. Maxima and minima, Taylor's series, and approximate solution of equations are grouped, discussion of them being based on the first mean-value theorem.

The chapter on determinants with applications to electrical problems is orthodox but less so are those on plane analytical geometry, calculus methods being used extensively, and the concept of an invariant being introduced. In addition to the straight line and circle the properties of the conics are investigated from the cartesian, polar and tangent-polar equations, in preparation for a discussion on central orbits in a subsequent chapter, and the general equation of the second degree is discussed.

In the first of three chapters on partial differentiation and related topics (the second and third chapters are later in the book), the elementary notions including Taylor's formula are dealt with competently, and no less satisfying are the subsequent chapters which contain non-linear transformations, differentials, curvilinear coordinates, and maxima and minima of two variables. The chapter on curve-tracing has less appeal and may prove difficult reading for the student.

The chapter on complex numbers does not differ much from the usual treatment found in undergraduate text-books and touches on the theory of functions of a complex variable and transformations. There is a short discussion of the field properties of complex numbers at the end.

There are four chapters devoted to the integral calculus and related topics in the first of which integration as the inverse operation to differentiation is considered and methods of integration are discussed extensively. The properties of the definite integral, defined as an increment in the primitive of the integrand, are discussed in

detail along with some inequalities associated with integrals, the second mean-value theorem, the usual applications to the calculation of geometric magnitudes, and the integral test for convergence. A discussion, much more rigorous than is usual in books aimed at this level, on the integral as the limit of a sum is followed by further consideration of applications from this standpoint. There is a useful commentary on double-limit processes and special functions defined by definite integrals, and the section on integration concludes with an account of numerical methods of integration and interpolation. These chapters are a fine exposition of the subject and are rich in useful results although in parts the undergraduate might feel that there is "much ado about nothing".

In the first of three chapters on the dynamics of a particle, motion in a straight line is systematically treated according to the nature of the accelerating force; no previous knowledge of differential equations is assumed. The physical concepts which emerge on integrating the equations of motion are carefully discussed as are the physical significance of the results. Plane vectors are used in the chapter dealing with the motion of a particle in a plane and prominence is given to motion under a central force. Short historical accounts of some physical problems are a feature of this section. Systems with several degrees of freedom are considered in a chapter on plane vibrational motion. From the equations of transverse motion of a large number of particles attached to a string is deduced the partial differential equation associated with the transverse motion of a string of continuous density. The electrical analogues of dynamical problems are referred to whenever these are of interest. A systematic account of differential and difference equations follows in which an account of the algebra of operators, in particular linear differential and difference operators, is given.

A chapter on three-dimensional dynamics including rigid dynamics is preceded by an account of vector algebra with applications to three-dimensional geometry and spherical trigonometry and includes motion referred to rotating axes.

Boundary-value problems are introduced by considering the vibrating string and it is demonstrated by assuming the string to have non-uniform density that the orthonormal properties of the characteristic functions associated with the problem are not peculiar to circular functions. There is brief mention of Rayleigh's principle, and Laplace's equation and the diffusion equation are also considered but in less detail. The chapter on Fourier series and integrals is very satisfying and ends with a short sketch proof of Fourier's integral theorem.

There are also short chapters on surfaces, curves in space, and multiple integrals, and finally a short account of some variational problems.

This is something more than just another text-book. The distinguished authors have combined their separate outlooks skilfully to produce a treatise on mathematics and its application to the problems of physics and chemistry which should be satisfying to both pure mathematician and applied scientist. On the whole the choice of material and presentation are excellent and the text is adequately supplemented by examples most of which are taken from University of London examination papers. The main criticism would be that perhaps too much is expected too early of the student.

JAMES FULTON

*Mathematics Dictionary*, 2nd ed., edited by GLENN JAMES and ROBERT C. JAMES (D. van Nostrand Co. Inc., Princeton, N.J., 1959), 546 pp., 112s. 6d.

This dictionary, which is an enlargement and revision of a previous volume, is a valuable reference work which provides carefully formulated definitions of the basic terms used in most branches of mathematics taught to undergraduates and scholars. To search the book for some word which is not listed is a game at which any number can play but at which not many will achieve a quick success for only a few important words between *abacus* (pl. *abaci*) and *Zorn's lemma* have been omitted.