CSIRO PUBLISHING

www.publish.csiro.au/journals/pasa *Publications of the Astronomical Society of Australia*, 2011, **28**, 261–265

Modelling Laboratory Data of Bidirectional Reflectance of a Regolith Surface Containing Alumina

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Received 2010 July 8, accepted 2011 June 8

Abstract: Bidirectional reflectance of a surface is defined as the ratio of the scattered radiation at the detector to the incident irradiance as a function of geometry. Accurate knowledge of the bidirectional reflection function for layers composed of discrete, randomly positioned scattering particles is essential for many remote sensing, engineering, and biophysical applications, as well as for different areas of astrophysics. Computations of bidirectional reflection functions for plane parallel particulate layers are usually reduced to solving the radiative transfer equation by the existing techniques. In this work we present our laboratory data on bidirectional reflectance versus phase angle for two sample sizes of alumina, 0.3 and 1 μm, for the He–Ne laser at wavelengths of 632.8 nm (red) and 543.5 nm (green). The nature of the phase curves of the asteroids depends on the parameters like particle size, composition, porosity, roughness, etc. In the present study we analyze data which are being generated using a single scattering phase function, that is, Mie theory of treating particles as a compact sphere. The well-known Hapke formula, along with different particle phase functions such as Mie and Henyey–Greenstein, will be used to model the laboratory data obtained at the asteroid laboratory of Assam University.

Keywords: comets: general — dust: extinction — scattering — polarization

1 Introduction

The study of the light scattering properties of powdered materials is known to be an important tool for characterizing the physical and compositional properties of asteroids. It is well known that asteroids are covered with fine-grained materials known as regolith layers (Hapke 2005). Hence it is essential that laboratory-based experiments on asteroid analogues can be compared with in situ data and that the theoretical models can be tested. As the phase angle approaches zero, the brightness of asteroids increases very rapidly. This phenomenon is called the opposition effect. Various physical parameters — particle size, porosity, surface roughness, thickness of the layer, etc. — are important to the study of the opposition effect and are being widely investigated in the laboratory (Kamei et al. 1999; Nelson et al. 2000; Kaasalainen et al. 2003). A large amount of literature is available on the physical interpretation of the opposition effect based on shadowing and coherent backscattering (Hapke 2002; Shkuratov, Ovcharenko, & Zubco 2002); however, it is difficult to explain how the opposition effect depends on physical parameters with theoretical models alone.

At large phase angles, not all physical parameters can be studied effectively, but certain very important properties such as composition, grain size, and grain shape can be studied. The most widely used formulae for describing the scattering of light from a particulate surface is the Hapke formula (Hapke 2005) and the Lumme and Bowell formula (Lumme & Bowell 1981).

The Hapke formula requires at least three unknown parameters, two of which become irrelevant for large phase angles. Recently, Hapke et al. (2009) compared the ability of several radiative transfer models to describe the scattering behaviour measured over a wide range of phase angles. Shepard & Helfenstein (2007) studied bidirectional reflectance function for 14 different samples including four Al_2O_3 samples over a phase angle range of from 3° to 130°. Piatek et al. (2004) measured the variation of reflectance as the phase angle varied from 0.05° to 140° for particle sizes ranging from less than to greater than the wavelengths.

In a preliminary work with alumina samples (Deb et al. 2011), it was found that for zero tilt and an observation wavelength of 632.8 nm the phase curve was satisfactorily fitted using the Hapke formula and Mie theory by varying

 10.1071/AS10025 1323-3580/11/03261

the absorption coefficient k. In the present study, we have included more experimental data at two different particle sizes (0.3 µm and 1 µm) and observation wavelengths (632.08 nm and 543.2 nm) for different tilt angles to study the theoretical behavior in more detail. The photometric data at large phase angles for the plane surface of powdered alumina (Al_2O_3) with an average particle diameter of 0.3 µm and 1 µm at the wavelengths of 632.8 nm and 543.2 nm have been generated. In the present analysis, we have used Mie theory — that is, treating the particles as compact and spherical in shape — and the Hapke formula in a Henyey–Greenstein phase function to theoretically calculate bidirectional reflectance and model it with the laboratory data thus obtained.

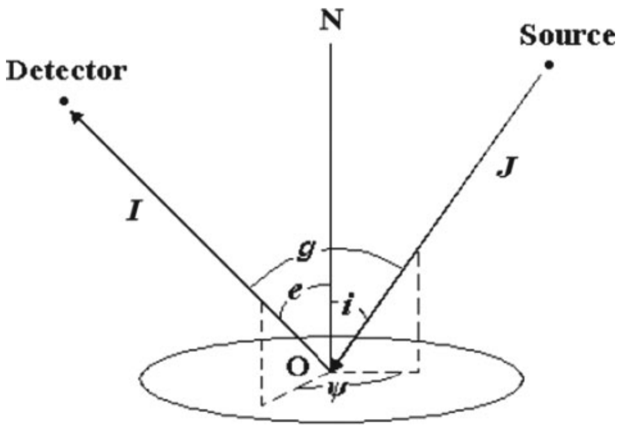

Figure 1 Schematic diagram of bidirectional reflectance.

2 Instrumentation and Sample

The experiment was carried out with the help of a goniometric device at the Department of Physics, Assam University, Silchar, India. The device consists of two metal arms with a common horizontal axis of rotation. The sample surface is placed at the arms' axis of rotation with the help of three translation stages. A miniature goniometer acts as a tilting device for the sample. The two arms can be rotated by ±90° from the zenith and the sample can be tilted by ±20° from the horizontal position perpendicular to the plane of scattering. We used an He–Ne laser of red and green wavelengths as the source of light and a charge-coupled device (CCD) camera as the detector. The sample was placed at the common intersection of the axis of rotation and axes of the source and detector. A diffuser was placed in front of the CCD to reduce the laser speckles produced by the coherent laser beam scattering from the rough surface. The diffuser introduces some uncertainty into the emergent angle, hence to address this we calculated the solid angle and the uncertainty in the emergent angle as 0.028 sr and ±0.32°, respectively. The intensity at any point on an illuminated area, along a given direction, is defined as the power radiated per unit projected area of illumination, in the direction under consideration, per unit solid angle. In this case, the solid angle feature can be neglected as it is constant for a particular instrument and is eventually canceled out when calculating the ratio.

The sample used in the present study is powdered alumina (Al_2O_3). We used two samples with different average values of particle diameter, 0.3 µm and 1 µm. Hereafter, we shall refer to the sample with particles of average diameter 0.3 µm as sample-I and that with particles of average diameter 1 µm as sample-II. Initially, the surface roughness is quite high. To prepare a smooth surface, the sample surface was pressed by a smooth metal plate.

3 Data Collection and Reduction

The tilt angle of the sample was first set at 0°, then varied from ±2° to ±20° at every 2° interval. To simplify the theoretical models, we took tilt angles of 0°, 10°, and 20°. The detector angles (e) were kept fixed at 45° and 63° (the positive sign accounts for forward scattering; see Figure 1) from the zenith. The angle of incidence (i) was varied from 0° to 63° in steps of 9°, hence the phase angles also varied from 45° to 126°. The detector readings were collected at every new angle of incidence, and the images of the sample surface were recorded in the form of flexible image transport system (FITS) image. As the field of view of the detector was larger than the laser spot, geometrical correction (cos i/cos e) was necessary to calculate the intensity values from the detector counts. Corrections for the background were also done for each observation. The reflectance values were calibrated by using $BaSO_4$ (a standard Lambert surface) at incidence angle 0° and detector angle 45°.

4 Theory

Bidirectional reflectance r (i, e, g), defined as the ratio of the reflected intensity (I) to the incidence irradiance (J) measured for the alumina sample, is shown in Figure 1 (which shows the experimental set up).

When a beam of collimated light is incident on a rough surface, the Fresnel laws of reflection are not obeyed by the reflected light as the light gets scattered along all directions in the upper hemisphere. The condition $g = i + e$ holds if the planes of emergence and incidence coincides ($\psi = 0°$ or 180°) and the tilt angle becomes 0°. In the present study, for other tilt angles (i.e. 10° or 20°) the phase angle $g \neq i + e$. The intensity of the scattered beam depends on these three angular parameters.

The bidirectional reflectance r as a function of i, e and g is given by:

$$r(i, e, g) = I(i, e, g)/J. \quad (1)$$

The interrelation among the angle of incidence i, detector angle e, the phase angle g and the tilt angle ϕ is given by:

$$\cos g = \cos i \times \cos e + \sin i \times \sin e \times \cos \phi. \quad (2)$$

https://doi.org/10.1071/AS10025 Published online by Cambridge University Press

4.1 Mie Theory

Mie theory is a single particle light scattering theory, which was derived for the solution of light scattered from a smooth and homogeneous sphere of any size (van de Hulst 1957). It depends on the complex refractive index (n, k) and the size parameter $X = 2\pi a/\lambda$, where a and λ are the radius and wavelength of the light, respectively. Mie theory is strictly only applicable for a single and isolated spherical particle and not directly applicable when there are a number of particles in contact with each other. In the latter case, multiple scattering between one particle and another makes the scattering behavior complicated. However, the approach used in the present study requires a 'single particle phase function' for input into the Hapke formula, then the calculation of multiple scattering is taken into account by the Hapke formula itself. In addition, the 'single particle phase function' of an isolated particle and a particle in regolith differes only by a small amount (e.g. figure 1 of Hapke et al. 2009). This small about has been considered negligible in the present study. To model the laboratory data of bidirectional reflectance, we use Mie theory to calculate the single particle albedo ω and the asymmetry parameter ξ. It is important to note that although particles of alumina can hardly be accepted as smooth and homogeneous spheres, Pollack & Cuzzi (1980) suggest that Mie theory may also be used to calculate the scattering properties of equant irregular particles if the size parameter $X \leq 5$. In the present study, the size parameters for sample-I are 1.49 and 1.73 with red and green wavelengths, hence fulfilling the above criteria.

4.2 Hapke Model

This model describes the scattering of light from a particulate surface, which has been derived from the theory of radiative transfer. The Hapke formula has three main parameters: single particle scattering albedo ω, single particle phase function $p(g)$, opposition surge amplitude B_0, and opposition surge width h. For larger phase angles (i.e. >45°), the effect of $B(g)$ can be neglected. The Hapke formula is given by (Hapke 2002, 2005):

$$r(i, e, g) = (\omega/4\pi)\frac{\mu_0}{\mu_0 + \mu}[\{1 + B(g)\}p(g) + H(\mu_0)H(\mu) - 1] \quad (3)$$

where $\mu_0 = \cos i$, $\mu = \cos e$, $p(g) = (1 - \xi^2)/(1 + 2\xi\cos g + \xi^2)^{3/2}$, $H(\mu_0) = (1 + 2\mu_0)/(1 + 2\gamma x)$, and $H(\mu) = (1 + 2\mu)/(1 + 2\gamma x)$ and $\gamma = (1 - \omega)^{1/2}$.

It is evident that for a brighter surface, the average photon is scattered more times before emerging from the surface, causing the directional effects to be averaged out and the multiple scattered intensity distribution to closely approach the isotropic case. The exact numerical solution for a high albedo surface was obtained by Chandrasekhar (1960). The comparison of exact and approximate solutions for isotropic scattering has been shown by Hapke (1981) (see Figures 3, 4 and 5). In the same paper, Hapke

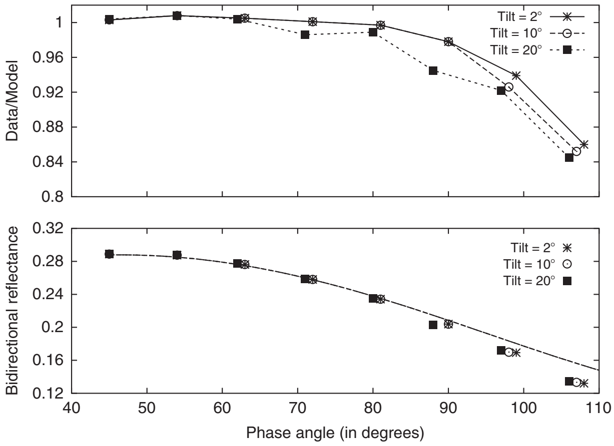

Figure 2 The upper panel shows the matching of data: model values ratio to 1. The lower panel gives the bidirectional reflectance versus phase angle for different tilt angles for sample-I at wavelength $\lambda = 632.8$ nm ($e = 45°$). The solid line in the lower panel represents the model.

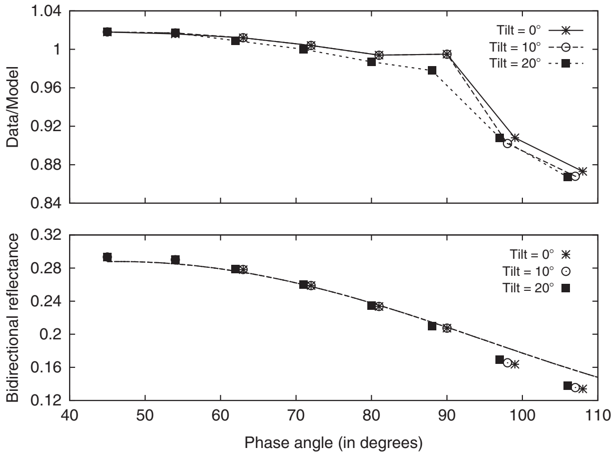

Figure 3 The upper panel shows the matching of data: model values ratio to 1. The lower panel gives the bidirectional reflectance versus phase angle for different tilt angles for sample-II at wavelength $\lambda = 632.8$ nm ($e = 45°$). The solid line in the lower panel represents the model.

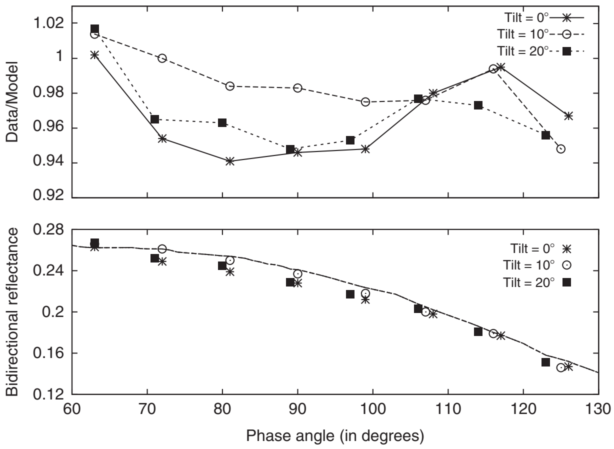

Figure 4 The upper panel shows the matching of data: model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-I at wavelength $\lambda = 543.5$ nm ($e = 63°$). The solid line in the lower panel represents the model.

https://doi.org/10.1071/AS10025 Published online by Cambridge University Press

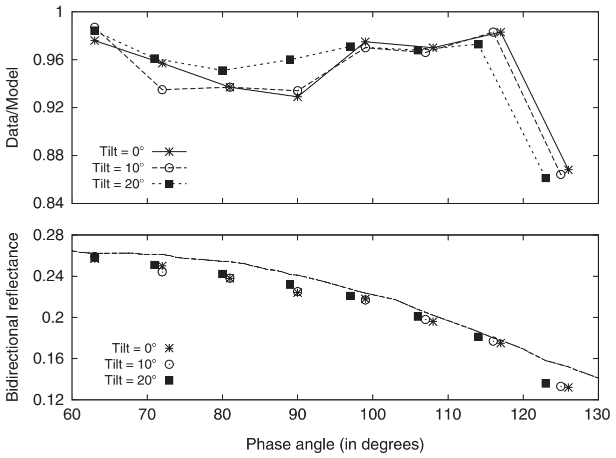

Figure 5 The upper panel shows the matching of data: model values ratio to 1. The lower panel gives the bidirectional reflectance vs phase angle for different tilt angles for sample-II at wavelength $\lambda = 543.5$ nm ($e = 63°$). The solid line in the lower panel represents the model.

compared the H-functions versus μ for several values of single scattering albedo for Chandrasekhar's exact solution with his approximation (see Figure 2) and found that the two solutions agree to better than 3% everywhere. In actual practice, it is observed that single scattering albedo $\omega = 1$ has never been achieved and a slight decrease in the ω value significantly increases the agreement between the exact and approximate solutions. Hapke (1981) reported that when $\omega = 0.975$, the error is only 0.7%. Therefore, the use of Hapke formula is justified for the present analysis. As the theory demands an arbitrary single particle phase function $P(g)$, we use an empirical phase function, that is, the Henyey–Greenstein phase function with one term. This introduces a new unknown parameter ξ, known as asymmetry parameter, which takes a value between -1 and $+1$. The asymmetry factor and single particle scattering albedo are calculated by running Fortran code on Mie theory, as published in Mishchenko et al. (1999, available online at `http://www.giss.nasa.gov/crmim`).

Therefore, using Mie theory in the Hapke formula with the Henyey–Greenstein phase function (Henyey & Greenstein 1941), we calculate approximate theoretical bidirectional reflectance of the powdered alumina sample. In the next section, we show the nature of graphs obtained theoretically and compare them with experimentally obtained graphs.

5 Results and Discussion

The refractive index of alumina at $\lambda = 632.8$ nm is $n = 1.766$ (Gervais 1991) and the absorption coefficient k is known to be very small. In the present study, we tried with different values of the free parameter k and found the best fit value to be $k = 0.00001$ for our model. Similarly, for the green laser of wavelength 543.5 nm and $n = 1.771$, the best fit value of k was found to be 0.000001, which is comparable with earlier work using a tilt angle of 0° (Piatek et al. 2004).

Piatek et al. (2004) studied the absolute reflectance versus phase angle for alumina at different phase angles with average particle diameter less than or equal to wavelength, hence their results are comparable to the sample-I results of the present study. In this study, we have clearly showed how the Hapke model can be used to empirically fit the laboratory data not only for a tilt angle of 0° but also for higher tilt angles (e.g. 10°, 20°). However, there is a basic difference in our calculation of bidirectional reflectance to that of Piatek et al. (2004). In their study, they used a fixed angle of incidence and varied angle of emergence, while in the present study we have two fixed emergent angles, 45° and 63°, and angle of incidence varied from 0° to 63°.

For sample-II, the average particle size is greater than the wavelength of the laser source. Under these conditions, we have found our asymmetry parameter ξ to be positive, which suggests that the phase function is forward scattering for the phase angle range 45° to 126°. This result is in accordance with other previous studies (Mishchenko 1994; Mishchenko & Macke 1997) which report that for non-opaque material in a powder, the single scattering phase function is forward scattering.

At present, we are unable to show whether our results are in accordance or in conflict with Shepard & Helfenstein (2007), who tested the significance of the Hapke photometric model, due to non availability of sufficient photometric data on Al_2O_3 at average particle diameter greater than the wavelength of laser source.

It is evident that the fit to laboratory data of bidirectional reflectance using the Hapke model would be better if we could correlate the results to physical properties such as porosity, roughness, etc. In addition, the assumption that we have taken particles to be smooth spheres may incorporate certain uncertainties in modelling as the particles may be non-spherical in shape.

Acknowledgments

We would like to thank the anonymous referee for his/her valuable comments. We acknowledge ISRO through its RESPOND programme for financial assistance to carry out this work.

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