CALCULATION OF THE ELECTRICAL CONDUCTIVITY OF PLASMAS WITH FLUCTUATIONS OF THE ELECTROMAGNETIC FIELD

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Abstract. A method to calculate the electrical conductivity of plasmas with electromagnetic waves with small wave number is discussed.

Within the theory of turbulent dynamo the time behaviour of the mean magnetic induction $\langle \mathbf{B}^M(\mathbf{r},t)\rangle$ in a plasma is described by the equations [Krause & Rädler 1980]

$$\frac{1}{\mu_o}\mathrm{rot}\Big(\frac{1}{\sigma}\mathrm{rot}\langle\mathbf{B}\rangle\Big) - \mathrm{rot}\Big(\langle\mathbf{v}\rangle\times\langle\mathbf{B}\rangle+\mathbf{G}\Big) + \frac{\partial\langle\mathbf{B}\rangle}{\partial t} = 0, \quad \mathrm{div}\langle\mathbf{B}\rangle = 0,$$

containing the electrical conductivity σ as a parameter. Here μ_o is the magnetic permeability of the system and $\mathbf{G} = \langle\delta\mathbf{v}\times\delta\mathbf{B}\rangle$ is the electromotive force caused by the fluctuations $\delta\mathbf{v}$ of the convection velocity $\mathbf{v}$, and depending linearly on $\langle\mathbf{B}\rangle$, $\mathbf{G} = \alpha\langle\mathbf{B}\rangle - \beta\mathrm{rot}\langle\mathbf{B}\rangle$. The Ohm's law can be written in the form $\mathbf{j} = \sigma(\langle\mathbf{E}\rangle + \langle\mathbf{v}\rangle\times\langle\mathbf{B}\rangle+\mathbf{G}) = \sigma_T(\langle\mathbf{E}\rangle+\alpha\langle\mathbf{B}\rangle)$ with the turbulent conductivity $\sigma_T = \sigma/(1+\mu_o\sigma\beta)$. The parameters α and β depend on $\mathbf{v}$ and $\delta\mathbf{v}$, and thus are also more or less determined by σ. While, for instance, for very high conductivity the β-parameter was found to be almost independent on the conductivity, $\beta \approx \langle\delta\mathbf{v}^2\rangle\tau/3$ [Rädler 1966], it yields $\beta \approx \mu_o\sigma\langle\delta\mathbf{v}^2\rangle\lambda^2/9$ in rather resistive mediums [Steenbeck 1963] (τ and λ give the time and length scales of the magnetic induction and the plasma convection). For high conductivity it yields $\alpha = -\langle\delta\mathbf{v}\mathrm{rot}\delta\mathbf{v}\rangle/3$, and at low conductivity $\alpha \sim -\mu_o\sigma/3$. Thus the dynamo theory needs a good approximation for the electrical conductivity, which in almost collision-free astrophysical plasmas is often determined by the fluctuations of the electromagnetic fields, $\delta\mathbf{E}$ and $\delta\mathbf{B}$. Thereby the fluctuations generated by ion-acoustic and lower-hybrid-drift instabilities seem to cause plasma resistivity most effectively and to diminish the electrical conductivity by orders in comparison with the values for a plasma without fluctuations. Therefore in this work the special method to calculate the electrical conductivity of plasmas with pure electrostatic fluctuations, $\delta\mathbf{E}||\mathbf{k}$ ($\mathbf{k}$ is the wave vector), developed in [Belyi et al. 1982; Belyi & Meister 1989], is generalized to plasmas with electromagnetic fluctuations. Thus the starting point of the calculations forms a kinetic equation containing an additional collision integral

F. Krause et al. (eds.), The Cosmic Dynamo, 253–254.

$$I_a = -\frac{1}{n_a^o}\frac{\partial}{\partial p_\beta}\int \frac{d\mathbf{k}d\omega}{(2\pi)^4}\mathrm{Re}(\delta N_a \delta F_{a\beta})_{\omega\mathbf{k}xt}$$

$$= -\frac{q_a}{n_a^o}\frac{\partial}{\partial p_\beta}\int \mathrm{Re}\Big\{(\delta N_a \delta E_\beta)_{\omega\mathbf{k}xt}\left(1 - \frac{k_r v_r + k_s v_s}{\omega}\right)$$

$$+(\delta N_a \delta E_s)_{\omega\mathbf{k}xt}\frac{v_s k_\beta}{\omega} + (\delta N_a \delta E_r)_{\omega\mathbf{k}xt}\frac{v_r k_\beta}{\omega}\Big\}\frac{d\mathbf{k}d\omega}{(2\pi)^4}$$

$$(\beta = (x; y; z),\ r \neq \beta \neq s,\ r \neq s),$$

describing the interaction between the charged particles and the waves.

$$(\delta N_a \delta E_\beta)_{\omega\mathbf{k}}$$

$$= \sum_\alpha \left(\frac{i8\pi^2 \omega q_a n_a^o v_\alpha \delta(\omega - \mathbf{k}\mathbf{v}) f_a(\mathbf{r}, \mathbf{p}, t)}{\omega^2 \varepsilon_{\alpha\beta} - c^2 k^2 \delta_{\alpha\beta} + c^2 k_\alpha k_\beta} - \frac{i n_a^o (\delta \mathbf{F}_a \delta E_\beta)_{\omega\mathbf{k}}}{\omega - \mathbf{k}\mathbf{v} + i\Delta}\frac{\partial f_a}{\partial \mathbf{p}}\right),$$

$\delta\mathbf{F}_a = q_a\delta\mathbf{E} + q_a[\mathbf{v} \times \delta\mathbf{B}]$, is the space-time spectral function of the phase-space particle-density fluctuation - electric-field fluctuation correlation, and f_a the particle distribution function. Then the effective collision frequency

$$\nu_{eff} = -\sum_a \frac{q_a}{j_a^2}\int \mathbf{j}_a \mathbf{v} I_a d\mathbf{p}$$

can be found knowing the wave dispersion and the energy spectrum of the waves. In this theory the energy spectrum appears as a parameter, for which experimental results or theoretical estimates can be substituted.

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