

WHITE DWARF MODELS OF SUPERNOVAE AND CATAclySMIC VARIABLES

K. NOMOTO and M. HASHIMOTO

Physics Department, Brookhaven National Laboratory;
on leave from the Department of Earth Science and
Astronomy, University of Tokyo, Japan

ABSTRACT: If the accreting white dwarf increases its mass to the Chandrasekhar mass, it will either explode as a Type I supernova or collapse to form a neutron star. In fact, there is a good agreement between the exploding white dwarf model for Type I supernovae and observations. We describe various types of evolution of accreting white dwarfs as a function of binary parameters (i.e., composition, mass, and age of the white dwarf, its companion star, and mass accretion rate), and discuss the conditions for the precursors of exploding or collapsing white dwarfs, and their relevance to cataclysmic variables. Particular attention is given to *helium star cataclysmics* which might be the precursors of some Type I supernovae or ultrashort period X-ray binaries. Finally we present new evolutionary calculations using the updated nuclear reaction rates for the formation of O+Ne+Mg white dwarfs, and discuss the composition structure and their relevance to the model for *neon novae*.

1. INTRODUCTION

The final fate of accreting white dwarfs will be either thermonuclear explosion or collapse, *if* the white dwarf mass grows to the Chandrasekhar mass. Though the exact precursor systems are not known yet, very good agreement between the exploding white dwarf models (carbon deflagration model) and the observed features of Type Ia supernovae (Nomoto 1986a; Woosley and Weaver 1986b) suggests that some accreting white dwarfs actually increase their mass to the Chandrasekhar mass. For Type Ib supernovae, though currently most popular models are explosions of Wolf-Rayet stars, the maximum light spectrum might better be explained by off-center explosions of white dwarfs. Furthermore, recent observations of several interesting binary systems, low mass X-ray binaries, QPOs, and binary radio pulsars have suggested that in these systems a neutron star has formed from accretion-induced collapse of a white dwarf (van den Heuvel 1984; Taam and van den Heuvel 1986).

These variations in the final fate of accreting white dwarfs originate from the differences in the parameters of the binary system, namely, composition, mass, and age of the white dwarf, its companion star, and mass accretion rate. It is interesting to look into whether some cataclysmic variables meet the conditions for the precursors

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of exploding or collapsing white dwarfs. The mass accretion rate is a particularly useful parameter for this purpose.

In §2, we discuss how the evolution of accreting white dwarfs depends on the binary parameters. In §3 - §4, theoretical models of explosion or collapse of white dwarfs are described and compared with the observations of Type Ia and Ib supernovae. In §5, we discuss the evolution of *helium star cataclysmics*, where the mass donor is not a hydrogen main-sequence star but a helium main-sequence star. These systems might be precursors of some Type I supernovae or ultrashort period X-ray binaries. Finally, in §6, we present a new evolutionary calculation to show the formation of O+Ne+Mg white dwarfs and their composition, because recently discovered *neon novae* raised new interest in the O+Ne+Mg white dwarfs.

2. THE FATE OF WHITE DWARFS AS A FUNCTION OF MASS AND ACCRETION RATE

2.1 Heating of Accreting White Dwarfs

Isolated white dwarfs are simply cooling stars that eventually end up as *dark matter*. In binary systems they evolve differently because mass accretion from their companion provides gravitational energy that rejuvenates them. The gravitational energy released at the accretion shock near the stellar surface is radiated away and does not heat the white dwarf interior. However, the compression of the interior by the accreted matter releases additional gravitational energy. Some of this energy goes into thermal energy (compressional heating) and the rest is transported to the surface and radiated away (radiative cooling). Therefore, the interior temperature is determined by the balance between heating and cooling and, thus, strongly depends on the mass accretion rate, \dot{M} .

The actual growth rate of the white dwarf mass, \dot{M}_{eff} , may be smaller than the mass accretion rate, \dot{M} , because some fraction of the accreted matter might be lost as a result of hydrogen or helium shell flashes as discussed below. However, compressional heating is still determined by \dot{M} , not by \dot{M}_{eff} , because the compression rate of matter at a Lagrangian shell of M_r is divided into two terms as

$$\lambda_\rho \equiv d \ln \rho / dt = \lambda_\rho^{(M)} + \lambda_\rho^{(q)},$$

$$\lambda_\rho^{(M)} = (\partial \ln \rho / \partial \ln M)_q (\dot{M}_{\text{eff}} / M),$$

$$\lambda_\rho^{(q)} = -(\partial \ln \rho / \partial \ln q)_M (\dot{M} / M),$$

where $q \equiv M_r / M$ is the mass fraction, and heating near the surface is determined by $\lambda_\rho^{(q)} \propto \dot{M}$, not by $\lambda_\rho^{(M)} \propto \dot{M}_{\text{eff}}$, as seen from Fig. 1 which shows the case of $M = 1.28 M_\odot$ and $\dot{M} = 7 \times 10^{-10} M_\odot \text{ yr}^{-1}$ (see Nomoto 1982a for details).

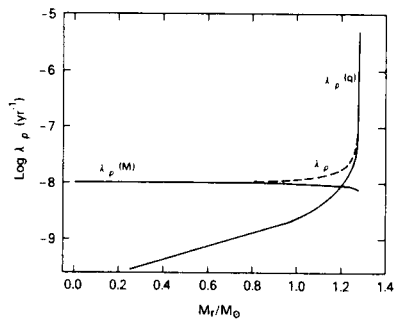


Figure 1

2.2 Accretion of Hydrogen-rich Matter and Hydrogen Flash

First let us discuss the case where the hydrogen-rich matter accretes on the white dwarf. When a certain amount of mass, ΔM_H , is accumulated, hydrogen shell burning is ignited. ΔM_H , which is critical for the subsequent evolution, is determined by \dot{M} and M as seen in Fig. 2 (Nariai and Nomoto 1979; Nomoto 1982a). For $M < 0.4 \dot{M}_{RG}$ where $\dot{M}_{RG} = 8.5 \times 10^{-7} (M/M_\odot - 0.52) M_\odot \text{ yr}^{-1}$, the hydrogen shell burning is unstable to a flash. The flash gives rise to an expansion of the accreted envelope, a part of which is lost from the system. For larger M and ΔM_H (i.e., smaller \dot{M}), the flash is stronger and causes larger mass loss. Therefore the white dwarf mass M can hardly increase for $\dot{M} < 10^{-8} M_\odot \text{ yr}^{-1}$, especially for nova-like explosion (Fujimoto and Taam 1982; MacDonald 1983, 1984).

2.3 Accretion of Helium and Helium Flash

On the other hand, a large portion of ΔM_H can be processed into helium for the accretion as rapid as $\dot{M} \sim 10^{-8} - 10^{-6} M_\odot \text{ yr}^{-1}$. Then a helium layer is gradually built up on the C+O white dwarf. This process is equivalent to the accretion of helium from a helium star (helium dwarf or helium star cataclysmics; see §5). Again a helium flash is ignited when a certain amount of helium, ΔM_{He} , is accumulated. ΔM_{He} is a function of \dot{M} and a mass of underlying C+O core M_{CO} as given in Fig. 3 (Kawai et al. 1986). The flash is stronger for larger M and ΔM_{He} (i.e., lower \dot{M}) because of higher ignition density. It grows even into a *helium detonation* for $\dot{M} < \dot{M}_{det} \sim 4 \times 10^{-8}$ (Nomoto 1982b; Woosley et al. 1986) as indicated by the dashed line in Fig. 3 (see §3.2 - §3.3).

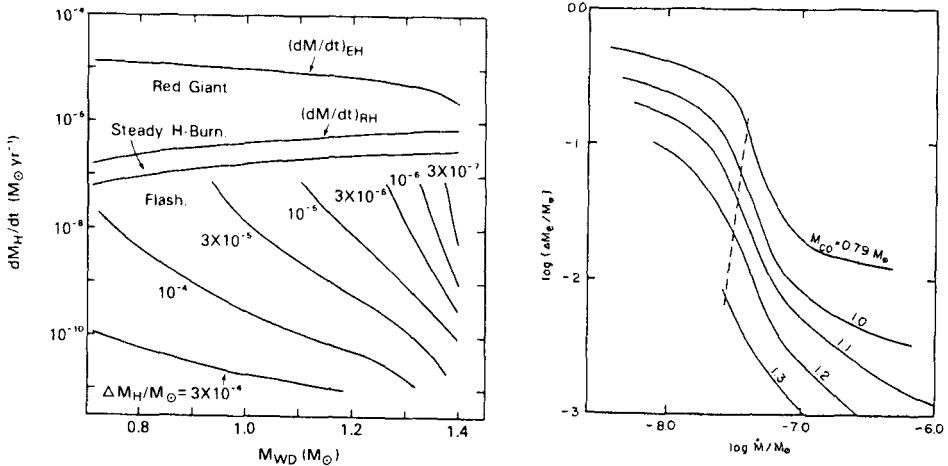


Figure 2 (left): The mass of accreted hydrogen-rich envelope, ΔM_H , at the ignition of a hydrogen flash as a function of M and \dot{M} .

Figure 3 (right): The mass of accreted helium envelope, ΔM_{He} , at the ignition of a helium flash as a function of \dot{M} and the mass of underlying C+O core, M_{CO} .

For higher \dot{M} , the helium flash is mild and will increase the C+O core mass (Taam 1980; Fujimoto and Sugimoto 1982). Eventually carbon ignition occurs at the center and induces a *carbon deflagration* (§3.1). The above scenario holds also for O+Ne+Mg white dwarfs.

2.4 Off-Center Flash in Merging Double White Dwarfs (CO - CO , He - He Pairs)

When two white dwarfs start to merge owing to gravitational wave radiation (Iben and Tutukov 1984; Webbink 1984), rapid mass transfer from the less massive white dwarf is expected. In spherical accretion models for C+O - C+O pair (Nomoto and Iben 1985), accretion faster than $2.7 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ ignites a mild off-center carbon flash at $M_r = M_{r,\text{ig}}$ when the white dwarf mass reaches M_* given in Fig. 4 as a function of \dot{M} (Kawai et al. 1986, where $\Delta M_e = M_* - M_{r,\text{ig}}$). The carbon burning front, then, propagates inward all the way to the center and converts the C+O white dwarf into an O+Ne+Mg white dwarf (Saio and Nomoto 1985; Woosley and Weaver 1986a).

For a He - He dwarf pair, accretion faster than $2 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ ignites a mild off-center helium flash (Nomoto and Sugimoto 1977). The helium burning front also propagates to the center (Fig. 5; Saio and Nomoto 1986). There is a difference from the C+O - C+O case, however. The white dwarf changes into a helium main-sequence star, not directly into a C+O white dwarf, because only 10 percent of helium burns during the propagation. After helium is exhausted, the star changes into a C+O white dwarf.

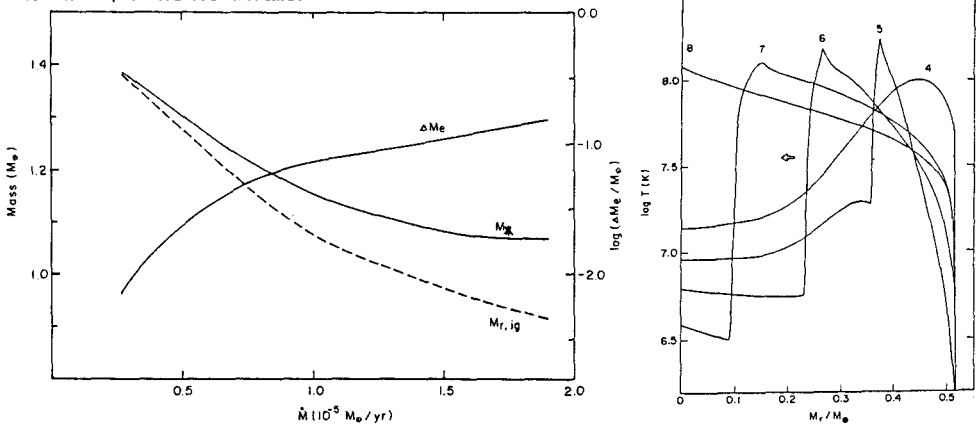


Figure 4 (left): The white dwarf mass (M_*), the location of the carbon burning shell ($M_{r,\text{ig}}$), and $\Delta M_e = M_* - M_{r,\text{ig}}$ at the carbon ignition. These values depend only on \dot{M} . For $\dot{M} > 2.7 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$, central ignition occurs.

Figure 5 (right): Change in the temperature profile during the propagation of the helium burning front (stages 4 - 8) for the accreting helium white dwarf with $\dot{M} = 1 \times 10^{-7} M_{\odot} \text{ yr}^{-1}$. At stage 8, the front reaches the center and the star becomes a helium main-sequence star.

2.5 Long Term Evolution leading to Supenova Explosion or Collapse

When the underlying C+O core grows gradually, the white dwarf evolves in the following way and its final fate depends critically on the initial mass M_{CO} (i.e., the mass at the onset of accretion) as well as \dot{M} . Compression first heats up a layer near the surface because of the small pressure scale height there (see $\lambda_\rho^{(g)}$ in Fig. 1). Later, heat diffuses inward. The diffusion timescale depends on \dot{M} and is small for larger \dot{M} because of the large heat flux and steep temperature gradient generated by rapid accretion. For example, the time it takes the heat wave to reache the central region is about 2×10^5 yr for $\dot{M} \sim 10^{-6} M_\odot \text{ yr}^{-1}$ and 5×10^6 yr for $\dot{M} \sim 4 \times 10^{-8} M_\odot \text{ yr}^{-1}$. Therefore, if the initial mass of the white dwarf, M_{CO} , is smaller than $1.2 M_\odot$ (this value is larger if $\dot{M}_{\text{eff}} < \dot{M}$), the entropy in the center increases substantially due to the heat inflow and carbon ignites at relatively low central density ($\rho_c \simeq 3 \times 10^9 \text{ g cm}^{-3}$). On the other hand, if the white dwarf is sufficiently massive and cold at the onset of accretion, the central region is compressed only adiabatically and thus is cold when carbon burning is ignited in the center. In the latter case, the ignition density is as high as $10^{10} \text{ g cm}^{-3}$ (e.g., Isern et al. 1983).

Accordingly, the ultimate fate of accreting C+O white dwarfs depends on \dot{M} and the inintial mass of the white dwarf M_{CO} , as summarized in Fig. 6. \dot{M} denotes the growth rate of the white dwarf mass irrespective of the composition of the accreting matter. A similar diagram for the O+Ne+Mg white dwarfs is shown in Fig. 7. Details on the hydrodynamical models indicated in Figs. 6 - 7 will be discussed in §3 - §4.

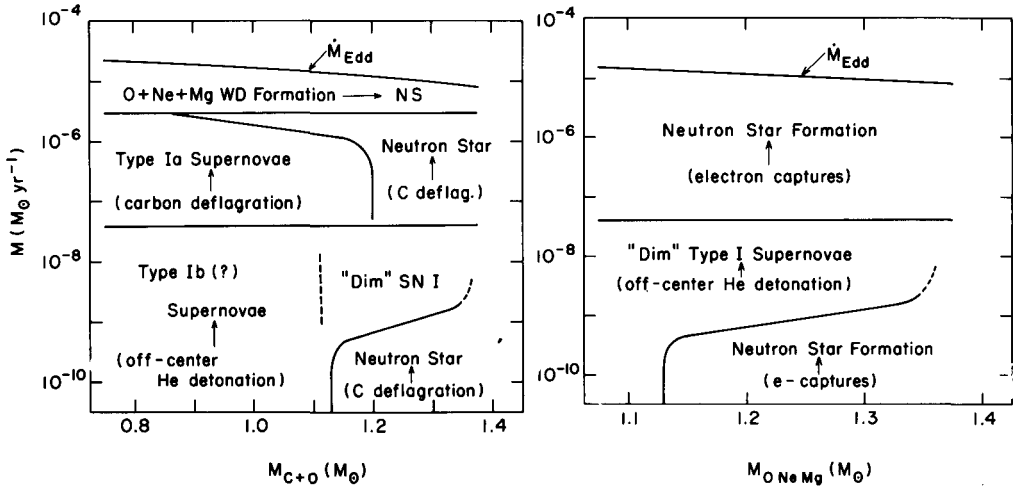


Figure 6 (left): The final fate of accreting C+O white dwarfs expected for their initial mass M_{CO} and accretion rate \dot{M} .

Figure 7 (right): Same as Fig. 6 but for O+Ne+Mg white dwarfs.

2.6 Candidates of the Supernova or Collapse Precursors

As discussed in §2.2, rather high accretion rates ($\dot{M} > \sim 10^{-8} M_{\odot} \text{ yr}^{-1}$) are required in order for the accreting white dwarf to become a Type I supernova or neutron star because nova explosions and helium detonations should be avoided. Possible binary systems to realize such accretion rates include:

- (1) Case A mass transfer on the thermal timescale from a main-sequence star of $\sim 2 M_{\odot}$ ($\dot{M} \sim 10^{-7} M_{\odot} \text{ yr}^{-1}$) (Iben and Tutukov 1984),
- (2) Mass transfer on the nuclear timescale from a subgiant of $\sim 0.8 M_{\odot}$ ($\dot{M} \sim 10^{-7} - 10^{-8} M_{\odot} \text{ yr}^{-1}$) (Webbink et al. 1983).
- (3) Accretion of helium from a helium main-sequence star on the timescale of gravitational wave radiation ($\sim 2 - 9 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$) (see §5; Savonije et al. 1986; Iben et al. 1986). For the above cases 2 - 3, the $^{14}\text{N}(e^{-}, \nu)^{14}\text{C}(\alpha, \gamma)^{18}\text{O}$ reaction would be important to lower \dot{M}_{det} from 4×10^{-8} to $1 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ (Hashimoto et al. 1986).
- (4) Double C+O white dwarfs if $\dot{M} < 2.7 \times 10^{-6} M_{\odot} \text{ yr}^{-1}$ or if non-spherical effects prevent off-center carbon burning from igniting or propagating. If \dot{M} is higher, collapse of an O+Ne+Mg white dwarf would result.

Since typical accretion rate for cataclysmic variables is rather low ($< 10^{-8} M_{\odot} \text{ yr}^{-1}$), it seems unlikely that the white dwarfs in cataclysmic variables evolve into Type I supernovae or collapse unless \dot{M} changes to exceed $\sim 10^{-8} M_{\odot} \text{ yr}^{-1}$ or M is already very close to the Chandrasekhar mass.

3. MODELS FOR SUPERNOVAE OF TYPE Ia AND Ib

3.1 Type Ia Supernovae

As discussed in §2, for relatively high accretion rates ($2.7 \times 10^{-6} M_{\odot} \text{ yr}^{-1} > \dot{M} > 4 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$), a carbon deflagration starts in the white dwarf's center at a relatively low central density ($\rho_c \sim 3 \times 10^9 \text{ g cm}^{-3}$) (e.g., Ivanova et al. 1974; Nomoto et al. 1984 and references therein). The *convective* deflagration wave then propagates outward at a subsonic velocity. The density the wave encounters is decreasing due to the expansion of the white dwarf.

The products of explosive nucleosynthesis depend on the temperature and density at the deflagration front and, thus, vary from layer to layer. In the center, iron peak elements are produced. In particular, about $0.6 M_{\odot}$ of ^{56}Ni is synthesized. In the outer layers, intermediate mass elements such as Ca, Ar, S, Si are produced. The white dwarf is disrupted completely and no neutron star residue remains (Nomoto et al. 1984; Thielemann et al. 1986; Woosley et al. 1984).

The carbon deflagration model can account for the light curves, early time spectra, and late time spectra of Type Ia supernovae as follows (see Nomoto 1986a; Woosley and Weaver 1986b for reviews and references therein):

- (1) The theoretical light curve based on the radioactive decays of ^{56}Ni and ^{56}Co into ^{56}Fe fits the observations well.
- (2) The synthetic spectrum at maximum light is in excellent agreement with the observed spectrum of SN 1981b as seen in Fig. 8 (Branch et al. 1985; Wheeler and Harkness 1986).
- (3) At late times, the outer layers are transparent and the inner Ni-Co-Fe core is exposed. Synthetic spectra of emission lines of [Fe II] and [Co I] agree quite well with the spectra observed at such phase (Woosley et al. 1984).

3.2 Type Ib Supernovae

Recent observations has established the existence of another kind of Type I supernovae, designated Type Ib (SN Ib) (e.g., Wheeler and Harkness 1986 and references therein). The SN Ib spectra are characterized by the lack of the 6125 Å Si feature at maximum light and the appearance of oxygen emission lines at late times (e.g., Gaskell et al. 1986). The currently popular progenitor models for SN Ib are Wolf-Rayet stars (e.g., Wheeler and Levreault 1985; Begelman and Sarazin 1986). However, a large mass of Wolf-Rayet stars may yield a light curve whose decline is too slow to be compatible with SN Ib observations (Wheeler and Levreault 1985).

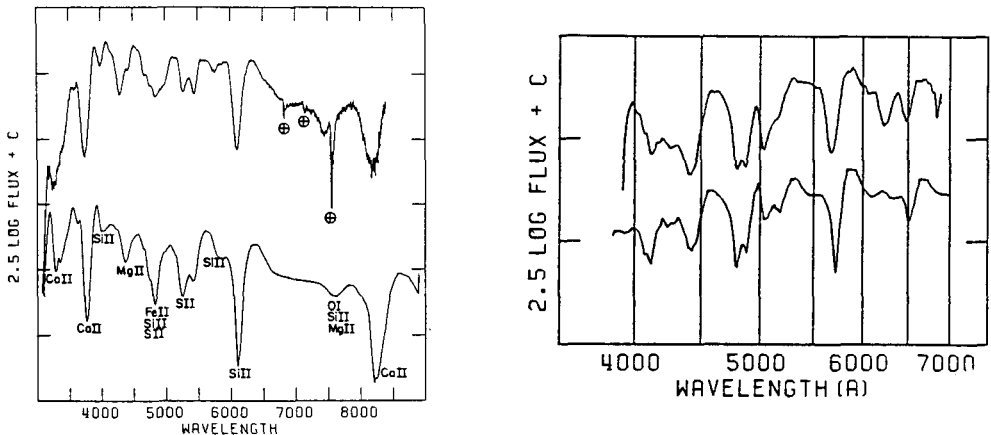


Figure 8 (left): The maximum-light spectrum of Type Ia SN 1981b (top) is compared to a synthetic spectrum for the carbon deflagration model (Nomoto et al. 1984; Branch et al. 1985). In this model outer layer is assumed to be mixed.

Figure 9 (right): The maximum-light spectrum of the Type Ib SN 1984I (upper; Wheeler and Levreault 1985) is compared with a synthetic spectrum (lower; Branch and Nomoto 1986). In the synthetic spectrum the blueshifted absorption component of He I $\lambda 6678$ appears near 6500 Å and He I $\lambda 5876$ appears near 5850 Å. Other features are produced primarily by Fe II lines.

Branch and Nomoto (1986) have suggested that the observed spectra are better explained by an accreting white dwarf model. In Fig. 9, the maximum-light spectrum of SN 1984I is compared with a synthetic spectrum. Two of the absorption lines in the red are identified as He I lines and other features are well explained as Fe II lines. Here the expansion velocity at the photosphere is $8,000 \text{ km s}^{-1}$. In addition, ultraviolet features can fit with a synthetic spectrum of Co II and Fe I lines if the photospheric velocity is $12,000 \text{ km s}^{-1}$ (Branch and Venkatakrisna 1986). The above interpretation suggests that Fe, Co (decaying), and He are in the outer high-velocity layers and oxygen is in the inner layers.

The existence of such high velocity Fe and, especially, Co is difficult to explain with the Wolf-Rayet model. Branch and Nomoto (1986) have speculated that the progenitors of SN Ib are white dwarfs having $\dot{M} < 4 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$. Such a slow accretion induces an off-center helium detonation that will occur at a point rather than all over a spherical shell (Fig. 5). The outer helium layer will burn to mostly ^{56}Ni with a trace He and the inner C+O core will remain unburned. A part or most of the C+O will be ejected following ^{56}Ni layer. Since the ejected mass of ^{56}Ni will be as small as $0.1 - 0.3 M_{\odot}$, the peak luminosity of this model is lower than that for SN Ia by a factor of 2 - 6. This is consistent with the observations of SN Ib. If the white dwarf accretes matter with an efficiency of only 0.03 - 0.1 from a wind ($10^{-6} - 10^{-7} M_{\odot} \text{ yr}^{-1}$) of a relatively massive ($4 - 7 M_{\odot}$) red giant companion (Iben and Tutukov 1984), the model would be consistent with relatively young nature of SN Ib. Further, this scenario is consistent with the radio observations of SN Ib in that they can be explained by the interaction of supernova ejecta with the circumstellar shell (Sramek et al. 1984; Chevalier 1984).

If this speculation is correct, the precursor systems of Type Ib supernovae would be *symbiotic stars* rather than cataclysmic variables.

3.3 Dim Type I Supernovae

If an off-center *single* detonation occurs on a very massive white dwarf ($> \sim 1.1 M_{\odot}$), the resulting supernova will be rather *dim*, because the accumulation of only a small amount of helium ($\sim 0.01 - 0.1 M_{\odot}$) can lead to the helium detonation (Fig. 5). In most cases, an unburned C+O core will be left behind as a white dwarf. Such dim supernovae (Branch and Doggett 1985) are more likely to be associated with O+Ne+Mg white dwarfs since their masses are larger than $\sim 1.2 M_{\odot}$.

4. NEUTRON STAR FORMATION FROM ACCRETION-INDUCED COLLAPSE OF WHITE DWARFS

Possible models for the white dwarf collapse involve solid C+O white dwarfs, in which carbon and oxygen may or may not have chemically separated (Canal et al. 1980; Isern et al. 1983) and O+Ne+Mg white dwarfs (Nomoto et al. 1979).

4.1 C+O White Dwarfs

The C+O white dwarfs could either explode or collapse, depending on the conditions of the white dwarfs and binary systems in which they are formed. Chemical separation in such objects is still hypothetical and its timescale is not accurately known. It takes a carbon fraction of only a few percent to sustain a deflagration. Therefore, it is worth determining the critical condition for which a carbon deflagration induces the collapse of a C+O white dwarf rather than its explosion. Recently Nomoto (1986b,c) has examined such a critical condition. If a carbon deflagration is initiated in the center of the white dwarf when $\rho_c \simeq 10^{10} \text{ g cm}^{-3}$ and if the propagation velocity of the deflagration wave is slower than a certain critical speed, v_{crit} , the outcome is collapse, not explosion, as seen in Figs. 10 - 11 (Nomoto 1986b,c; see also Isern et al. 1984). For $v_{\text{def}} > v_{\text{crit}}$, complete disruption results (and the ejecta contain too much neutron-rich matter). The value of v_{crit} depends on ρ_c at carbon ignition. For $\rho_c \simeq 1 \times 10^{10} \text{ g cm}^{-3}$, $v_{\text{crit}} \sim 0.15 v_s$ where v_s is the sound speed. A lower ρ_c implies a lower v_{crit} . In our case of $\rho_c \simeq 1 \times 10^{10} \text{ g cm}^{-3}$, for both conductive and convective deflagrations $v_{\text{def}} < v_{\text{crit}}$ and, therefore, collapse will result (see Nomoto 1986b,c for more details).

Such a high central density is reached in two regions of the $\dot{M} - M_{\text{CO}}$ plane of Fig. 6. One is defined by $\dot{M} > 4 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$ and $M_{\text{CO}} > 1.1 M_{\odot}$, while the other is defined by $\dot{M} < 10^{-9} M_{\odot} \text{ yr}^{-1}$ and $M_{\text{CO}} > 1.13 M_{\odot}$. The frequency of

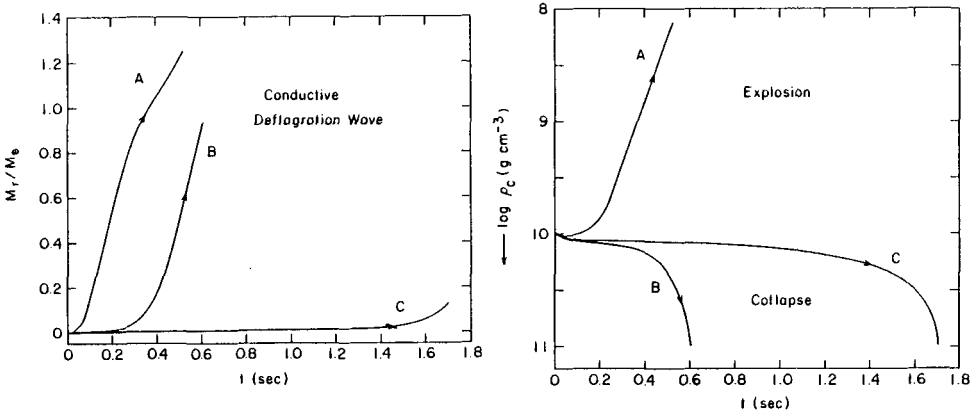


Figure 10 (left): Propagation of the conductive deflagration wave in the C+O white dwarf. The location (M_r) of the deflagration front is shown as a function of time, t , for three cases (A, B, C) of parametrized conductivity.

Figure 11 (right): Change in the central density of the C+O white dwarf associated with the propagation of the conductive deflagration wave. Relatively slow propagation in Cases B and C leads to the increase in ρ_c , i.e., collapse of the white dwarf. On the other hand, faster propagation in Case A induces the explosion of the white dwarf.

such systems may be small. First, for accretion at $\dot{M} < 10^{-9} M_{\odot} \text{ yr}^{-1}$, a nova-like explosion or helium detonation will prevent the white dwarf mass from growing. Secondly, massive C+O white dwarfs ($> 1.2 M_{\odot}$) may be rare. The formation of such white dwarfs might be prevented if the precursor star lost its hydrogen-rich envelope by either a stellar wind or Roche-lobe overflow before its degenerate C+O core could grow substantially.

4.2 O+Ne+Mg White Dwarfs

As seen in Fig. 7, the accretion-induced collapse is the outcome for a wider range of parameter space for O+Ne+Mg white dwarfs. The triggering mechanism of the collapse is electron capture on ^{24}Mg and ^{20}Ne (Nomoto et al. 1979; Miyaji et al. 1980). The initial mass of the white dwarf, M_{ONeMg} , is larger than $\sim 1.1 M_{\odot}$ (Nomoto 1980, 1984a). In many cases, M_{ONeMg} is very close to the Chandrasekhar mass, so that only a small mass increase is enough to trigger collapse. However, an O+Ne+Mg white dwarf forms from an 8 - 10 M_{\odot} star (Nomoto 1984a). The number of such systems may be significantly smaller than the number of systems containing C+O white dwarfs whose precursors are 1 - 8 M_{\odot} stars, perhaps, by four order of magnitude (Iben and Tutukov 1984). Even so, the number of low mass X-ray binaries is much smaller than the number of Type I supernovae and the statistics may be consistent (Webbink et al. 1983).

5. HELIUM STAR CATAclysmics

5.1 Mass Transfer from a Helium Main-Sequence Star due to Gravitational Wave Radiation

In recent studies of possible evolutionary paths of close binary stars, Tornambè and Matteucci (1986) and Iben and Tutukov (1986) have found that, for a relatively large range of initial parameters, systems may evolve into a configuration consisting of a more massive C+O dwarf and a less massive helium star close enough that mass transfer from the helium star to the degenerate dwarf can be driven by gravitational wave radiation (GWR). If GWR is the only source of angular momentum loss, accretion rates \dot{M}_{He} expected for such *helium star cataclysmics* can be approximated by (Iben et al. 1986)

$$\dot{M}_{\text{He}} (M_{\odot} \text{ yr}^{-1}) = 10^{-7.85} M_{\text{CO}} M_{\text{He}}^{-0.84} M_{\text{tot}}^{-0.76} (1.355 - M_{\text{He}}/M_{\text{CO}})^{-1} \quad (1)$$

where M_{CO} and M_{He} are the masses (in solar units) of the accreting object and of the donor, respectively, and $M_{\text{tot}} = M_{\text{CO}} + M_{\text{He}}$. In Fig. 12 we plot curves of constant \dot{M}_{He} that follow from equation (1). The region bounded by the dash-dot curve defines where initial component masses are predicted to lie (Iben and Tutukov 1986). Several evolutionary tracks are shown in Fig. 12 (lines A,B,C, and D). Here the coordinate M_{CO} is to be interpreted as the sum of the initial mass of the C+O

dwarf and the mass ΔM_{He} of accreted helium.

We note first that, for all permissible initial masses, $\dot{M}_{\text{He}} < 4 \times 10^{-8} M_{\odot} \text{ yr}^{-1}$. Therefore, for initial masses in, say, the upper half of the allowed region, and for systems in which the abundance of ^{14}N is low, a helium detonation will terminate the active life of the system (track A). The resulting explosion would look-like a Type Ib supernova (see §3.2).

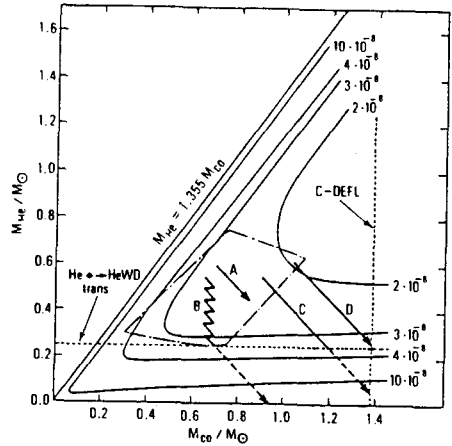


Figure 12

5.2 Mass Transfer after Helium Burning is Quenched

For systems with initial masses in the lower portion of the allowed region, several additional factors must be taken into account. When its mass decreases below some minimum value $M_{\text{min}} \sim 0.28 - 0.3 M_{\odot}$, a helium star can no longer burn helium and it will attempt to evolve into a degenerate dwarf. Evolutionary calculations (Savonije et al. 1986; Nomoto et al. 1986) have shown that, as M_{He} decreases below M_{min} and helium burning goes out, the timescale of cooling and contraction of the helium star becomes much larger than the orbital decay timescale (τ_{GW}). As a result, the radius of the helium star is smaller than the value given by the mass-radius relationship used to obtain equation (1) but significantly larger than that of a fully degenerate configuration. This prevents the donor from detaching from its Roche-lobe. Moreover, the mass transfer rate is enhanced above that given by equation (1) and reaches a maximum of $\sim 10^{-7} M_{\odot} \text{ yr}^{-1}$ because $\tau_{\text{GW}} (\propto M_{\text{He}}^{-1})$ is proportional to the fourth power of the radius. When the star becomes sufficiently degenerate, its radius starts to increase as its mass decreases. Then the mass accretion rate declines monotonically as τ_{GW} increases.

Thus, thermal inertia appears to have the effect of considerably enhancing the mass transfer rate above \dot{M}_{det} for $M_{\text{He}} < M_{\text{min}}$. The resulting enhanced heating of the accreted helium layer will ignite a mild helium flash, not a detonation, unless the helium layer has already been too massive to avoid the detonation. Such a system will end up as a C+O white dwarf because total mass of the system is smaller than $1.38 M_{\odot}$ in most cases.

5.3 The Role of the NCO Reaction and Additional Angular Momentum Loss

The picture is altered if we invoke an additional source of angular momentum loss at a strength comparable to the GWR source and/or a sufficient abundance of ^{14}N in the accreted helium for the efficient $^{14}\text{N}(e^-, \nu)^{14}\text{C}(\alpha, \gamma)^{18}\text{O}$ reaction (Hashimoto et al. 1986). Now $\dot{M}_{\text{He}} > \dot{M}_{\text{det}}$ as far as $M_{\text{He}} > \sim 0.1 M_{\odot}$, so that all models which have initial masses in the region suggested by binary scenarios will experience relatively mild flashes. Several untested possibilities for evolutionary paths arise. If the mass accreted between two successive flashes is converted into carbon and oxygen that remains on the accreting star, then there exists a region in initial parameter space such that the underlying CO core will grow until it reaches the critical value of $1.38 M_{\odot}$, at which point a carbon deflagration will be ignited. In other words, Type Ia supernovae would be the possible outcome of the evolution starting from the upper-right of track C in Fig. 12 where $M_{\text{He}} + M_{\text{CO}} > 1.48 M_{\odot}$.

5.4 Helium Star - O+Ne+Mg White Dwarf Pair and the Origin of Ultrashort Period X-Ray Binaries

If the helium star has an O+Ne+Mg white dwarf companion, the initial mass of the white dwarf is larger than $\sim 1.1 M_{\odot}$, even as large as $\sim 1.36 M_{\odot}$ (§6). Therefore, for a wider range of initial parameters in Fig. 12, the O+Ne+Mg white dwarf evolves to collapse if $\dot{M} > \dot{M}_{\text{det}}$. This is possible for the helium star of initial mass $< 0.3 M_{\odot}$ even if GWR is the only source of angular momentum loss, and for a much wider range of initial mass if the additional effects operate as discussed in §5.3. Then the system would become an ultrashort period X-ray binary system such as X-ray pulsar 1E2259+586 whose orbital period is 38 min (see Savonije et al. 1986 for more details). Accretion-induced collapse in *helium star cataclysmics* would not occur for C+O white dwarfs if the initial mass is bounded as in Fig. 12 (Iben and Tutukov 1986) because $M_{\text{CO}} > \sim 1.2 M_{\odot}$ is required for the collapse to occur (Fig. 6).

5.5 S - Process Nucleosynthesis

If $\dot{M} < \dot{M}_{\text{det}}$ and a large fraction of the matter accreted between pulses is ejected during pulses, then, since s-process isotopes are formed in the convective shell during pulses (Iben 1981; Fujimoto and Sugimoto 1982), helium star cataclysmics may be important sources of s-process isotopes (Iben and Tutukov 1986). If the NCO reaction operates, the neutrons are produced by the $^{14}\text{N}(e^-, \nu)^{14}\text{C}(\alpha, \gamma)^{18}\text{O}(\alpha, \gamma)^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}$ reactions. If a significant fraction of the material processed by helium flashes is not ejected during pulses, a Type Ia supernova will occur and eject s-process elements which remain in the outer unburned layers (Nomoto et al. 1984).

6. NEON NOVAE AND COMPOSITION OF O+Ne+Mg WHITE DWARFS

6.1 Neon Novae

The evolution of O+Ne+Mg white dwarfs has been discussed in the context of white dwarf collapse assuming their formation from certain stars (Finzi and Wolf 1967; Nomoto et al. 1979). The actual stellar evolution through the formation of O+Ne+Mg white dwarfs has been shown in the calculation of helium stars by Nomoto (1980, 1981, 1984a,b). Recently the O+Ne+Mg white dwarf has attracted another attention because some novae show enrichment of Ne, Mg, Na, and Al in the ejecta (e.g., Williams et al. 1985; Gehrz et al. 1985). One of the possible models is that the *neon novae* occur on the O+Ne+Mg white dwarfs (Law and Ritter 1984; Delbourgo-Salvador et al. 1985; Starrfield et al. 1986; Truran and Livio 1986). This model invokes some mixing between the accreted matter and the underlying core material so that the composition of the ejecta must reflect the abundance of the O+Ne+Mg core. To confirm this idea, we have to know the original composition structure of the O+Ne+Mg white dwarf and compare the theoretical prediction of nucleosynthesis with the observed abundance. Regarding the composition, recently inflated $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate changes nucleosynthesis during the course of stellar evolution. We report below our new evolutionary calculation for the formation of the O+Ne+Mg white dwarfs (Nomoto and Hashimoto 1986b).

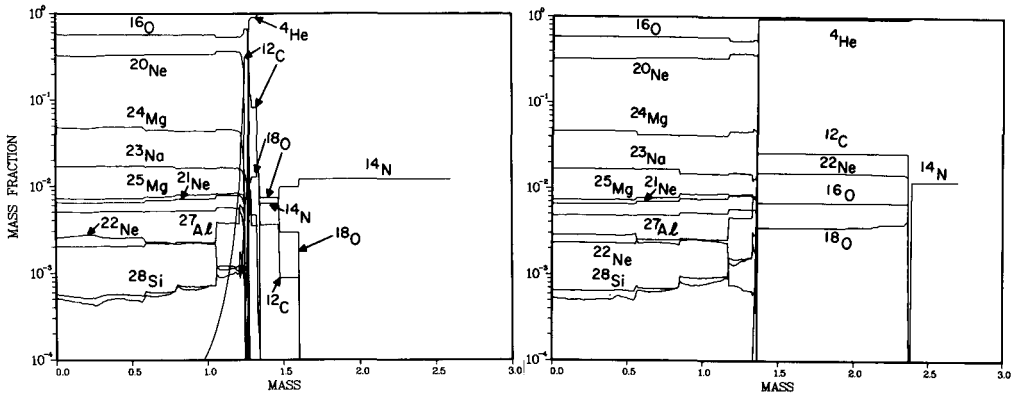
6.2 Evolution of Helium Stars in Close Binaries through Formation of O+Ne+Mg White Dwarfs

If one of the component star of a close binary is a 8 - 13 M_{\odot} star, it becomes a helium star of mass $M_{\alpha} \sim 2 - 3.3 M_{\odot}$ after tidal mass loss. The helium star in this mass range undergoes non-degenerate carbon burning and forms a semi-degenerate O+Ne+Mg core (Nomoto 1984a; Habets 1985, 1986). For $2.8 < M_{\alpha} < 3.2 M_{\odot}$, neon burning is ignited off-center and the neon-oxygen burning front propagates inward all the way to the center. Subsequent evolution forms an iron core (Hashimoto and Nomoto 1986; see Nomoto and Hashimoto 1986a). For $M_{\alpha} < 2.8 M_{\odot}$, the advance of the carbon burning shell (i.e., growth of the O+Ne+Mg core) stops near the helium burning shell at $M_{\text{Heb}} = 1.15, 1.27, \text{ and } 1.36 M_{\odot}$ for $M_{\alpha} = 2.5, 2.6, \text{ and } 2.7 M_{\odot}$, respectively. Hence the O+Ne+Mg core mass does not exceed the critical mass of $1.37 M_{\odot}$ for neon ignition and, thus, neon burning is never ignited. Afterwards, the core becomes strongly degenerate. The radius of the helium star increases as the O+Ne+Mg core grows and reaches as large as $\sim 1000 R_{\odot}$ if it is a single star. In a close binary system, certainly Roche lobe overflow occurs and an O+Ne+Mg white dwarf of mass $M_{\text{WD}} \simeq M_{\text{Heb}}$ is left.

6.3 Composition Structure of O+Ne+Mg White Dwarfs

In Figs. 13 - 14, the composition structures of helium stars of $M_\alpha = 2.6$ and $2.7 M_\odot$ near the onset of Roche lobe overflow are shown. After the helium envelope is lost, the dwarf residue consists of an O+Ne+Mg core and an outer C+O layer. The C+O layer is thinner for larger white dwarf mass because the density at the bottom of the C+O layer is $\sim 1.5 \times 10^5 \text{ g cm}^{-3}$ and a mass of overlying layer is smaller for larger mass white dwarfs which have smaller radius. For example, $\Delta M_{\text{C+O}} = 0.036$ and $0.005 M_\odot$ for $M_{\text{WD}} = 1.27$ and $1.36 M_\odot$, respectively. This implies that a larger C+O layer must be eroded by accretion and outburst until neon-rich layer is exposed (Truran and Livio 1986).

The abundances taken from Figs. 13 - 14 are summarized as follows. For $M_\alpha = 2.7 M_\odot$ ($M_{\text{WD}} = 1.36 M_\odot$), mass fractions are $X(\text{C}) = 0.30$, $X(\text{O}) = 0.66$, $X(\text{Ne}) = 0.019$ in the C+O layer. For the core at $M_r = 1.34 M_\odot$, $X(\text{O}) = 0.54$, $X(\text{Ne}) = 0.37$, $X(\text{Na}) = 0.013$, $X(\text{Mg}) = 0.049$, $X(\text{Al}) = 0.01$, and $X(\text{Si}) = 0.003$. The abundances of the $1.26 M_\odot$ white dwarf are similar. Compared with the old models (Nomoto 1984a), the C/O ratio in the C+O layer and the Ne/O, Mg/O ratios in the core are significantly smaller in new models because of the larger ^{12}C (α, γ) ^{16}O reaction rate. If the O-Ne-rich core material constitutes 20 - 30 percent of the nova ejecta, the abundance of the O+Ne+Mg white dwarf is consistent with the observed abundance of Nova CrA 1981 (Williams et al. 1985).



Figures 13 - 14: Composition of the helium stars of $M_\alpha = 2.6 M_\odot$ (left) and $2.7 M_\odot$ (right). After the Roche lobe overflow, O+Ne+Mg white dwarfs of masses $1.27 M_\odot$ and $1.36 M_\odot$ form from these stars, respectively.

Another characteristic of the O+Ne+Mg white dwarfs compared with C+O white dwarfs is a significantly larger mass ($> \sim 1.1 M_{\odot}$), even close to the Chandrasekhar mass, and, hence, smaller radius. The lower mass limit of O+Ne+Mg white dwarfs is somewhat larger than $1.06 M_{\odot}$ below which no carbon ignition occurs, because the core mass still increases after carbon ignition until the Roche lobe overflow commences. The upper mass limit is $1.37 M_{\odot}$ above which neon is ignited (Nomoto 1984a). Because of the large white dwarf mass, ΔM_{H} at the ignition of hydrogen flash is as small as $10^{-6} - 10^{-5} M_{\odot}$ (Fig. 2) so that the ejecta mass is small (Starrfield et al. 1986).

7. CONCLUDING REMARKS

Though there is a good agreement between the exploding white dwarf models and the observed spectra and light curves of Type Ia supernovae, the exact precursor systems where the white dwarf mass grows to the Chandrasekhar mass are not known yet. Typical cataclysmic variables seem not to be the promising candidates of supernova precursors because the mass accretion rate of the cataclysmic variables is too low to increase the white dwarf mass up to the Chandrasekhar mass. However, related systems such as double white dwarfs, helium star cataclysmics, and symbiotic stars deserve further exploration.

Recent breakthrough in supernova studies has been brought about by the calculations of theoretical spectra at various stages of supernova outburst which are in good agreement with the observed spectra (§3). For this purpose, radiation hydro code has been developed by several groups. If the same approach is possible for nova studies by calculating model atmospheres and synthetic spectra, it would certainly be quite fruitful. Besides radiation hydrodynamics, detailed nucleosynthesis calculation based on realistic O+Ne+Mg white dwarf models are needed for the study of neon novae.

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