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ADDENDUM: Π-PRINCIPAL HEREDITARY ORDERS

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Let R denote a complete discrete rank one valuation ring of unequal characteristic, and let p denote the characteristic of the residue class field $\bar{R}$ of R. Consider the integral closure S of R in a finite Galois extension K of the quotient field k of R. Recall (see Prop. 1.1 of [3]) that the inertia group G_0 of K over k is a semi-direct product $G_0 = J \times G_p$, where J is a cyclic group of order relatively prime to p and G_p is a normal p-subgroup of G.

The author has proved in [3] that if $\Delta(f, S, G)$ is a Π-principal hereditary order, then G_p is Abelian; the purpose of the present note is to extend this result by showing that the inertia group G_0 must also be Abelian. The reader should refer to [3] for definitions and notation.

PROPOSITION. *If the crossed product $\Delta(f, S, G)$ is Π-principal, then*
i) *the inertia group G_0 is Abelian,*
ii) *J is a normal subgroup of G.*

Proof. To prove that G_0 is Abelian, it suffices to show that $\Delta(\bar{f}, \bar{S}, G_0)$ is a commutative ring. According to Prop. 1.6 of [3] we may consider a splitting field L of $\Delta(\bar{f}, \bar{S}, G_0)$, so that $\Delta(\bar{f}, L, G_0)$ is isomorphic to the trivial crossed product $\Delta(1, L, G_0)$. Now $\operatorname{rad} \Delta(1, L, G_0)$ is generated by $\operatorname{rad} \Delta(1, L, G_p)$, (see the exercise on p. 435 of [1]), from which it follows that $\Delta(1, L, G_0)/\operatorname{rad} \Delta(1, L, G_0) = \Delta(1, L, J)$. Therefore the factor ring $\Delta(\bar{f}, L, G_0)/\operatorname{rad}(\bar{f}, L, G_0)$ is isomorphic to $\Delta(1, L, J)$.

We proceed to show that $\Delta(\bar{f}, \bar{S}, G_0)$ is isomorphic to a subring of the commutative ring $\Delta(1, L, J)$. The inclusion $(\operatorname{rad} \Delta(\bar{f}, L, G_0)) \cap \Delta(\bar{f}, \bar{S}, G_0) \subset \operatorname{rad} \Delta(\bar{f}, \bar{S}, G_0)$ follows from Lem. 2.4 of [2], and $\operatorname{rad} \Delta(\bar{f}, \bar{S}, G_0) = (0)$ because $\Delta(f, S, G)$ is Π-principal; these facts combine to show that the natural map $\Delta(\bar{f}, \bar{S}, G_0) \to \Delta(\bar{f}, L, G_0)/\operatorname{rad} \Delta(\bar{f}, L, G_0)$ is a monomorphism,

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and this together with the observations in the preceding paragraph completes the proof of part i).

Assertion ii) follows from i) and Prop. 3.2 of [3].

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