Canad. Math. Bull. Vol. **42** (1), 1999 pp. 118–124

Points of Weak*-Norm Continuity in the Unit Ball of the Space $\mathrm{WC}(K, X)^*$

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Abstract. For a compact Hausdorff space with a dense set of isolated points, we give a complete description of points of weak*-norm continuity in the dual unit ball of the space of Banach space valued functions that are continuous when the range has the weak topology. As an application we give a complete description of points of weak-norm continuity of the unit ball of the space of vector measures when the underlying Banach space has the Radon-Nikodym property.

Introduction

Let X be a Banach space. Let $B(X)$ denote the closed unit ball of X. $x^* \in B(X^*)$ is said to be a *point of weak*-norm continuity* (w^*-PC for short) if for any net $\{x^*_\alpha\} \subset B(X^*)$, $x^*_\alpha \to x^*$ in the weak* topology implies $x^*_\alpha \to x^*$ in the norm topology. For a compact Hausdorff space K, if $C(K, X)$, denotes the space of X-valued continuous functions defined on K, equipped with the supremum norm then a complete description of the w^*-PC's of $B\big(C(K, X)^*\big)$ was recently given by Hu and Smith [6] (see [1] for a short proof of their result). In this note we are interested in extending the description given by Hu and Smith to the case of $\mathrm{WC}(K, X)$, the space of X-valued functions on K that are continuous when X has the weak topology, equipped with the supremum norm. Our main result shows that the w^*-PC's of $B\big(\mathrm{WC}(K, X)^*\big)$ and that of $B\big(C(K, X)^*\big)$ have the same description when K is such that its set of isolated points is dense in K. Our result also works for the case of $W^*C(K, X^*)$, the space of X^*-valued functions on K that are continuous when X^* is equipped with the weak* topology.

As an application of these results, we show that for a finite nonatomic measure μ and for any Banach space X, there are no points of weak-norm continuity in $B\big(L^1(\mu, X)\big)$.

Let $\mathcal{K}(X, Y), \mathcal{F}(X, Y), \mathcal{L}(X, Y)$ denote respectively spaces of compact, weakly compact and bounded operators. A result of Ruess and Stegall (Theorem 4 of [12]) says that all the operator spaces mentioned above have the "same" w^*-denting points in the dual unit ball. (Let us recall that an equivalent definition of w^*-denting point is that it is an extreme point and a point of w^*-norm continuity). It is therefore natural to ask if the same is true of w^* PC's. Example 1.6 on page 267 of [4] illustrates that extreme points of $B\big(\mathcal{K}(X, Y)^*\big)$ need not be extreme in $B\big(\mathcal{L}(X, Y)^*\big)$. Taking $Y = C(K)$, it is well-known (see [3, p. 490]) that the spaces of operators mentioned above can be identified as $C(K, X^*)$, $\mathrm{WC}(K, X^*)$ and $W^*C(K, X^*)$ respectively. Thus knowing the complete description of w^*-PC's of $B\big(C(K, X)^*\big)$ we want to know if the same holds for other standard function spaces.

Received by the editors June 1, 1996; revised March 18, 1998.
AMS subject classification: 46B20, 46E40.
Keywords: Points of weak*-norm continuity, space of vector valued weakly continuous functions, M-ideals.

Unlike the case of w^*-denting points, there is no geometric description of w^*-PC's known. Thus the methods of Ruess and Stegall [12] do not seem to work here. Our ideas came from M-structure theory for which we refer the reader to the monograph [4] by Harmand, Werner and Werner.

Main Results

For any discrete set Γ, let $c_0(\Gamma, X)$ and $\ell^\infty(\Gamma, X)$ denote the space of X-valued functions vanishing at infinity and the space of bounded functions, respectively (equipped as usual with the supremum norm).

Let us recall from [4, Chapter I] that a closed subspace $M \subset X$ is said to be a M-*ideal* if there is a projection $P\colon X^* \to X^*$ such that $\|P(x^*)\| + \|x^* - P(x^*)\| = \|x^*\|$ for all $x^* \in X^*$ and $\operatorname{Ker} P = M^\perp$. Any $m^* \in M^*$ has a unique norm preserving extension to X (see [4, p. 11]).

We need two results essentially proved in our earlier work [1], [11].

Theorem 1 *$\Lambda \in B\big(c_0(\Gamma, X)^*\big)$ is a w^*-PC iff $\Lambda = \sum_{i=1}^\infty \alpha_i x_i^*$, where x_i^*'s are w^* PC's of $B(X^*)$ and $\sum_{i=1}^\infty |\alpha_i| = 1$.*

Lemma 1 *Let $M \subset X$ be a closed subspace. If $x^* \in B(X^*)$ is a w^*-PC and is the unique norm preserving extension of x^*/M then x^*/M is a w^*-PC of $B(M^*)$. If M is a M-ideal in X then any w^*-PC of $B(M^*)$ is a w^*-PC of $B(X^*)$.*

For any $k \in K$ and $x \in X$, by $\delta(k) \otimes x$ we denote the functional whose value at any X-valued function f on K is defined by $\big(\delta(k) \otimes x\big)(f) = f(k)(x)$. This is a functional of norm one on all the spaces we will be considering.

Theorem 2 *Let X be a Banach space and K a compact Hausdorff space. Let K^1 denote the set of isolated points of K. If Λ is a w^*-PC of $B\big(\mathrm{WC}(K, X)^*\big)$ then $\Lambda = \sum_{i=1}^\infty \alpha_i \delta(k_i) \otimes x_i^*$ where $\{k_i\}_{i \ge 1} \subset K^1$, $\{x_i^*\}$'s are all w^*-PC's of $B(X^*)$ and $\sum_{i=1}^\infty |\alpha_i| = 1$. If K^1 is dense in K then any Λ of the above form is a w^*-PC of $B\big(\mathrm{WC}(K, X)^*\big)$.*

Proof Using the identification of $C(K, X)^*$ as the space of X^*-valued regular Borel measures, the authors of [2], describe a projection

$$P\colon\ \mathrm{WC}(K, X)^* \to \mathrm{WC}(K, X)^*$$

as follows.

For any $\mu \in \mathrm{WC}(K, X)^*$, consider the measure $\nu = \mu / C(K, X)$. Define

$$P(\mu)(f) = \int f\, d\nu \quad \text{for } f \in \mathrm{WC}(K, X).$$

Then P is of norm one and $\operatorname{Ker} P = C(K, X)^\perp$. Note that Range P is thus isometric to $C(K, X)^*$ and also

$$P\big(\delta(k) \otimes x^*\big)(f) = x^*\big(f(k)\big) = \big(\delta(k) \otimes x^*\big)(f) \quad \text{for any } k \in K,\ x^* \in X^*.$$

https://doi.org/10.4153/CMB-1999-013-8 Published online by Cambridge University Press

Since functionals of the form $\{\delta(k) \otimes x^* : k \in K, \|x^*\| = 1\}$ separate points of $\mathrm{WC}(K, X)$, by an application of the Hahn-Banach separation theorem we have that $\overline{\mathrm{CO}}^{w^*}\{\delta(k) \otimes x^* : k \in K, \|x^*\| = 1\} = B\big(\mathrm{WC}(K, X)^*\big)$ (weak* closed convex hull).

Now since Λ is a w^*-PC, we have that

$$\Lambda \in \overline{\mathrm{CO}}\{\delta(k) \otimes x^* : k \in K, \|x^*\| = 1\} \quad \text{(norm closure)}.$$

Since $\Lambda / C(K, X)$ is now a countably supported measure and since $P(\Lambda) = \Lambda$, we have that

$$\Lambda = \sum_{i=1}^{\infty} \alpha_i \delta(k_i) \otimes x_i^*.$$

If $\alpha_{i_0} \neq 0$ then k_{i_0} is an isolated point and $x_{i_0}^*$ is a w^*-PC of $B(X^*)$.

To see this, note that if $\{k_\alpha\}$ is a net in K such that $k_\alpha \to k_{i_0}$ then for any $f \in \mathrm{WC}(K, X)$

$$\big(\delta(k_\alpha) \otimes x_{i_0}^*\big)(f) = x_{i_0}^*\big(f(k_\alpha)\big) \to x_{i_0}^*\big(f(k)\big) = \big(\delta(k) \otimes x_{i_0}^*\big)(f)$$

since $f(k_\alpha) \to f(k_{i_0})$ in the weak topology. Therefore $\delta(k_\alpha) \otimes x_{i_0}^* \to \delta(k_{i_0}) \otimes x_{i_0}^*$ in the weak* topology of $\mathrm{WC}(K, X)^*$.

Now

$$\sum_{\substack{i=1 \\ i \neq i_0}}^{\infty} \alpha_i \delta(k_i) \otimes x_i^* + \delta(k_\alpha) \otimes x_{i_0}^* \to \sum_{\substack{i=1 \\ i \neq i_0}}^{\infty} \alpha_i \delta(k_i) \otimes x_i^* + \delta(k) \otimes x_{i_0}^* = \Lambda$$

in the weak* topology of $\mathrm{WC}(K, X)^*$ and thus in the norm topology. Therefore $\{k_\alpha\}$ is eventually constant. Hence k_{i_0} is an isolated point of K. A similar argument shows that $x_{i_0}^*$ is a w^*-PC of $B(X^*)$.

Now suppose that K^1 is dense in K. Let $\Lambda = \sum_{i=1}^{\infty} \alpha_i \delta(k_i) \otimes x_i^*$ where $k_i \in K^1$ and x_i^*'s are w^*-PC's of $B(X^*)$ for all i.

From the theorem quoted above, it follows that (as K^1 has the discrete topology) Λ is a w^*-PC of $B\big(c_0(K^1, X)^*\big)$. We shall show that $c_0(K^1, X)$ is a M-ideal in $\mathrm{WC}(K, X)$. Then the conclusion follows as an application of the lemma mentioned above.

Let $W_1 \subset W_2$ be two closed subspaces of a Banach space W. It is easy to see that if W_1 is a M-ideal in W, then it is a M-ideal in W_2 [4, Proposition 1.1.7]. We shall make use of this observation in what follows.

Since K^1 is dense in K, we have

$$c_0(K^1, X) \subset C(K, X) \subset \mathrm{WC}(K, X) \subset \ell^\infty(K^1, X)$$

It is known that $c_0(K^1, X)$ is a M-ideal in $\ell^\infty(K^1, X)$ (see [4, Chapter I]), therefore from the observation above we have that $c_0(K^1, X)$ is a M-ideal in $\mathrm{WC}(K, X)$. Hence Λ is a w^*-PC of $B\big(\mathrm{WC}(K, X)^*\big)$.

Remark 1 We do not know if the second part of the above theorem is true without the hypothesis "K^1 is dense in K". However finite convex combinations of points of the form

$\delta(k) \otimes x^*$ for $k \in K^1$ and a w^*-PC x^* of $B(X^*)$ are w^*-PC's of $B\big(\mathrm{WC}(K,X)^*\big)$. It is easy to deduce now that $B\big(\mathrm{WC}(K,X)^*\big)$ and $B\big(C(K,X)^*\big)$ have the same denting points, thus obtaining a different proof of the Ruess and Stegall theorem when the range is a $C(K)$ space (see also Remark 2).

Our first corollary is a special situation.

Corollary 1 *Let K be a compact extremally disconnected space. Then Λ of the form described above is a w^*-PC of $B\big(\mathrm{WC}(K,X)^*\big)$.*

Proof Let K^1 be the set of isolated points of K. Then $E = \overline{K^1}$ is a clopen subset of K. Now it is easy to see that $f \to \chi_E f$ is a M-projection (see [4, Chapter I]) in $\mathrm{WC}(K,X)\big(C(K,X)\big)$. It hence follows from the above theorem and the lemma quoted before, that Λ is a w^*-PC of $B\big(\mathrm{WC}(K,X)^*\big)$.

The next one is a corollary to the proof of the above theorem and makes use of the fact that there is a norm one projection $P\colon \mathcal{L}\big(X, C(K)\big)^* \to \mathcal{L}\big(X, C(K)\big)^*$ with $\operatorname{Ker} P = \mathcal{K}\big(X, C(K)\big)^\perp$ and such that $P\big(\delta(k) \otimes x^{**}\big) = \delta(k) \otimes x^{**}$. (Note that $\big(\delta(k) \otimes x^{**}\big)(T) = x^{**}\Big(T^*\big(\delta(k)\big)\Big)$ for any $k \in K$ and $x^{**} \in X^{**}$). This follows from Lemma 1 of [7].

Corollary 2 *If Λ is a w^*-PC of $B\Big(\mathcal{L}\big(X, C(K)\big)^*\Big)$ then $\Lambda = \sum_{i=1}^{\infty} \alpha_i \delta(k_i) \otimes x_i$ where $\{k_i\}_{i\geq 1} \subset K^1$, $\{x_i\}$'s are all points of weak-norm continuity of $B(X)$ and $\sum_{i=1}^{\infty} |\alpha_i| = 1$. If further K^1 is dense in K then any Λ of the above form is a w^*-PC of $B\Big(\mathcal{L}\big(X, C(K)\big)^*\Big)$.*

Remark 2 Identifying the space $\mathcal{K}(\ell^1, X)$ as $C\big(\beta(N), X\big)$ and the space $\mathcal{L}(\ell^1, X)$ as $\oplus_\infty X$, the above arguments show that both the spaces have the same w^*-PC's in the dual unit ball, for any X. In the case of non-discrete finite measure space $(\Omega, \mathcal{A}, \mu)$ we note that $\mathcal{K}\big(L^1(\mu), X\big)$ can be identified with $C(K,X)$ where K is the Stone space (and thus an extremally disconnected space) of $L^\infty(\mu)$. This author has observed in [10] that this identification extends to $\mathcal{F}\big(L^1(\mu), X\big)$ onto $\mathrm{WC}(K,X)$. It is also easy to see that $\mathcal{L}\big(L^1(\mu), X\big)$ gets mapped into $W^*C(K, X^{**})$. Therefore all the operator spaces considered have the same w^*-PC's in the dual unit ball.

The next result concerns the w^*-PC's of another well studied class of operators (see [4, Chapter VI]).

Proposition 1 *If $\mathcal{K}(X,Y)$ is a M-ideal in $\mathcal{L}(X,Y)$ then their dual unit balls have the same w^*-PC's.*

Proof Since $\mathcal{K}(X,Y)$ is a M-ideal in $\mathcal{L}(X,Y)$ by the uniqueness of norm preserving extensions (see Proposition 1.12 of [4]), we have that the functional $(y^* \otimes x^{**})(T) = x^{**}\big(T^*(y^*)\big)$ for any unit vectors $y^* \in Y^*$ and $x^{**} \in X^{**}$), is the norm preserving extension of its restriction to $\mathcal{K}(X,Y)$. Since these functionals determine the norm of $\mathcal{L}(X,Y)$, as in the proof of Theorem 1 we see that if F is a w^*-PC of $B\big(\mathcal{L}(X,Y)\big)$ then it is of norm one on $\mathcal{K}(X,Y)$. Hence the conclusion follows from our lemma quoted before. ■

Next set of results concern Proposition 11 and the concluding remarks of [6]. In what follows by a "point of continuity" we mean a point of weak-norm continuity of the identity mapping on $B(X)$. These ideas yield a simple proof of Proposition 11 in [6] and a corresponding result in the injective tensor product case.

We first state a lemma the first part of which follows from Lemma 2.1 in [5] and the converse can be proved by imitating the arguments given during the proof of Lemma 3 in [11].

Lemma 2 *Let P be a L-projection. If x_0 is a point of continuity of $B(X)$, then either $x_0 \in B\big(P(X)\big)$ or $B\big((I-P)(X)\big)$ and is a point of continuity of that space or $P(x)/\|P(x)\|$ and $(I-P)(x)/\|(I-P)(x)\|$ are points of continuity in the unit ball of the range and the kernel of P respectively. Conversely any point of continuity in $B\big(P(X)\big)$ or $B\big((I-P)(X)\big)$ is a point of continuity of $B(X)$.*

Remark 3 For any L-projection P on X, $I \otimes P$ is a L-projection on the projective tensor product space $Y \otimes_\pi X$ (see [4, Chapter VI]). Let $\{y_i\}$ be any sequence of pairwise independent unit vectors in Y such that $line\{y_i\}$ is the range of a L-projection for each i. It is easy to see that for any sequence $\{x_i\}$ of points of continuity of $B(X)$, $F = \sum_{i=1}^{\infty} \alpha_i(y_i \otimes x_i)$ is a point of continuity of $B(Y \otimes_\pi X)$ for all α_i with $\sum_{i=1}^{\infty} |\alpha_i| = 1$.

In particular if $(\Omega, \mathcal{A}, \mu)$ is any measure space then for any sequence $\{\chi_{A_i}\}$ of normalized μ atoms, $F = \sum_{i=1}^{\infty} \alpha_i(\chi_{A_i} \otimes x_i)$ is a point of continuity of $B\big(L^1(\mu, X)\big)$. When μ is purely atomic, it is not difficult to see that these are the only points of continuity. These arguments also cover the case of points of continuity in $B\big(C(K,X)^*\big)$ for a dispersed K.

For a nonatomic measure μ since there are no extreme points in $B\big(L^1(\mu, X)\big)$, the next result complements the results of Hu-Lin [5] when $p = 1$.

Theorem 3 *For any finite nonatomic measure μ and for any Banach space X, there are no points of continuity in $B\big(L^1(\mu, X)\big)$.*

Proof Let F be a point of continuity of $B\big(L^1(\mu, X)\big)$. It follows from Corollary 2.10 of [5] that F is a w^*-PC of $B\big(L^1(\mu, X)^{**}\big)$. Note that $L^1(\mu, X)^*$ can be identified with $\mathcal{L}\big(X, C(K)\big)$ where K is the Stone space of $L^\infty(\mu)$. Since μ is nonatomic clearly K has no isolated points. It now follows from Corollary 2 that there are no w^*-PC's in $B\Big(\mathcal{L}\big(X, C(K)\big)^*\Big)$. This contradiction shows that there are no points of continuity in $B\big(L^1(\mu, X)\big)$.

Corollary 3 *For any compact set K containing a perfect set and for any Banach space X such that X^* has the* R. N. P.*, any point of continuity of the unit ball of $C(K,X)^*$ has the form $F = \sum_{i=1}^{\infty} \alpha_i \delta(k_i) \otimes x_i^*$ where k_i's are in K, x_i^*'s are points of continuity of $B(X^*)$ and $\sum_{i=1}^{\infty} |\alpha_i| = 1$.*

Proof Let $F \in B\big(C(K,X)^*\big)$. Since X^* has the R. N. P., $F \in B\big(L^1(|F|, X^*)\big)$ and is clearly a point of continuity. From Theorem 2 and the remarks made before it we get $F = \sum_{i=1}^{\infty} \alpha_i(\chi_{A_i} \otimes x_i^*)$, where A_i's are $|F|$ atoms. It follows from Theorem 8 on page 51 of [L] that there exist $k_i \in K$ such that $|F|(A_i) = |F|(k_i)$. Therefore F has the required form.

https://doi.org/10.4153/CMB-1999-013-8 Published online by Cambridge University Press

It is easy to see that a point of continuity of the unit ball of a subspace need not be the point of continuity of the unit ball of the entire space (take for example a finite dimensional subspace). In the next proposition we exhibit another class of subspaces where the points of continuity get preserved. Examples that satisfy the hypothesis of this proposition include the spaces considered in Theorem 2 and Proposition 1.

Proposition 2 *Let $M \subset X$ be a closed subspace such that $P\colon X^* \to X^*$ is a norm one projection with $\mathrm{Ker}(P) = M^\perp$ and $B\big(P(X^*)\big)$ is w^*-dense in $B(X^*)$. Any point of continuity of $B(M)$ is a point of continuity of $B(X)$.*

Proof In what follows we identify M^* canonically with $P(X^*)$. Let $J\colon X \to M^{**}$ be defined by $J(x)\big(P(x^*)\big) = x^*(x)$ for $x \in X$ and $x^* \in X^*$. Because of the denseness assumption, clearly J is an isometry whose restriction to M is its canonical embedding in M^{**}. Now suppose m is a point of continuity of $B(M)$. By Corollary 2.10 of [5] again, m is a w^*-point of continuity of $B(M^{**})$. Suppose $\{x_\alpha\}$ is a net in $B(X)$ converging weakly to m. Since $J(\{x_\alpha\}) \to m$ weakly, we have $\|J(\{x_\alpha\}) - m\| = \|x_\alpha - m\| \to 0$. Hence m is a point of continuity of $B(X)$.

Remark 4 Unlike the situation described in Lemma 2, if $X = M \oplus_\infty N$ (the ℓ^∞ direct sum) then $m \in B(M)$ is a point of continuity of $B(X)$ iff N is finite dimensional.

Since comparison of the extremal structures of $B\big(\mathcal{K}(X,Y)^*\big)$ and $B\big(\mathcal{L}(X,Y)^*\big)$ is one of our motivations for this work, we conclude the paper with the following remark.

Remark 5 Hu and Smith have also succeeded in describing the strongly extreme points (see [6], [11] for the definition and this result) of $B\big(C(K,X)^*\big)$ as points of the form $\delta(k) \otimes x^*$, $k \in K$ and $x^* \in B(X^*)$ is a strongly extreme point. Any extreme point of $B(\ell^\infty)$ is a strongly extreme point. Thus the example $C(K) = \ell^\infty$, $X = c_0$ (from [4, p. 267]) alluded to in the introduction also illustrates that strongly extreme points of $B\Big(\mathcal{K}\big(X, C(K)\big)^*\Big)$ need not even be extreme points of $B\Big(\mathcal{L}\big(X, C(K)\big)^*\Big)$. If k is an isolated point then one can show that $\delta(k) \otimes x^*$ is indeed a strongly extreme point of $\mathcal{L}\big(X, C(K)\big)$ whenever x^* is a strongly extreme point of $B(X^*)$ (see Lemma 3 of [11]). We do not know if there are any other strongly extreme points in $B\big(\mathcal{L}(c_0, \ell^\infty)^*\big)$. In the case $\mathcal{F}\big(X, C(K)\big)$, note that since $X^* = \ell^1$ has the Schur property, for the above example $\mathcal{F}\big(X, C(K)\big) = \mathcal{K}\big(X, C(K)\big)$. Similarly if one considers $\mathrm{WC}(K,X)$ where $X = c_0$ and K is any compact set, then since $C(K,X)$ is a M-ideal in $\mathrm{WC}(K,X)$ (see Theorem 7 in [2]) any strongly extreme point of $B\big(C(K,X)^*\big)$ is a strongly extreme point of $B\big(\mathrm{WC}(K,X)^*\big)$ (again from Lemma 3 of [11]). Finally in the infinite dimensional set up extreme points of $B\big(C(K,X)^*\big)$ and $B\big(\mathrm{WC}(K,X)^*\big)$ are the same only when X has the Schur property (see [9]).

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