CONVECTION IN THE SUN

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ABSTRACT. The present knowledge of the dynamics and structure of the solar convection zone is reviewed with the aim of checking current assumptions and conjectures against laboratory experiments and numerical modeling of thermal convection. Buoyancy is the only forcing considered. Rotation and magnetic fields are explicitly avoided. Nor are departures from planar geometry considered , except as regards large scale structures. Local theories are reviewed in section §2, hydrodynamic models in §3, non-local theories in §4, the global structure of the convection zone is discussed in §5 and the flow patterns in §6.

1. Introduction

The solar convection zone is the site of very rich dynamics, the intricacies of which make it difficult to look through it into the deeper interior. Therefore, solar convection theory has often been approached with the avowed aim of obtaining a recipe for computing gradients and fluxes in places where a radiative gradient would be unstable. We shall take this view in the first part of the present review. Yet the solar convection zone is still the site of a highly structured magnetohydrodynamic flow, where buoyancy is thought to be one of the dominant external forcings. This view will be taken later on and buoyancy induced solar structures will also be reviewed.

The purpose of the present paper is to examine the solar convection zone as a whole, with laboratory and numerical experiments in mind. We shall attempt to stress similarities as much as possible and, as a consequence, we shall avoid discussing any process dominated by forcings other than buoyancy. Rotation and magnetic fields will not be considered at all, and the geometry will be assumed to be planar unless stated otherwise. However, as most of the laboratory experiments and numerical simulations in thermal convection are extremely dependent on details irrelevant to a stellar interior - the shape of the container, the boundary conditions, etc.- any extrapolation of numerical results or laboratory measurements is to be looked at carefully.

2. On local theories of stellar convection.

Local theories are widely used for modeling thermal convection in stars, as they are the simplest available algorithms, though some attempts at modeling stellar convection zones with non-local theories exist and will be reviewed in §3 and §4. Mixing-length scaling is widely accepted and consistency with it is required for any model, no matter whether it is local or non-local. However, local models can be derived without any reference to phenomenology. We shall take this view because

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detailed physics often masks more fundamental similitudes.

In order to summarize the theory, let us begin by writing

$$\partial_r T = -\mathcal{F}(p, m, L, T) \quad (2.1)$$

where $\{p, m, L, T\}$ are the four dependent variables of stellar structure: pressure, mass, luminosity and temperature. In order to produce a convenient frame for a local theory we can define the Nusselt number N as

$$N = \frac{F_T - F_A}{F_R - F_A}$$

where $F_T = -k\ \partial_T$ is the total flux, $F_R = -k\ \partial_r T$ would be the flux if this convection were artificially inhibited, and $F_A = -k\ \partial_A$ is the flux transported radiatively by the adiabatic gradient. We shall also introduce for later use the definitions

$$\partial_T = -k^{-1}\frac{L}{4\pi r^2} \quad (2.2a)$$

$$\partial_A = (1-\gamma^{-1})\ \frac{T}{p}\partial_r p \quad (2.2b)$$

where $k = 4acT^3/3\kappa\rho$. Using the previous definition of N, $\partial_r T$ can be written as

$$\partial_r T = \frac{N-1}{N}\partial_A + \frac{1}{N}\partial_T \quad (2.3)$$

In (2.3) the radiative and convective equilibria are attained, respectively, for $N-1 \ll 1$ and $N-1 \gg 1$. Therefore, we do not expect significant differences from any theory giving $N-1$ very large values inside the convection zone and very small ones, or zero, outside it. Any significant difference between two models concerns only the intermediate values of N.

2.1 DIMENSIONAL ANALYSIS

In a stratified medium the Nusselt number can be written as $N = N(Ra, \sigma, Z)$ - see, for instance, Massaguer & Zahn (1980)- where Ra and σ are, respectively, the Rayleigh and the Prandtl numbers

$$Ra = \frac{g\ d^4(\delta/\ T)}{(k/\rho C_P)\ \nu}[-(\partial_T - \partial_A)]$$

$$\sigma = \frac{\nu}{k/\rho C_P}$$

In these expressions each variable retains its usual meaning with d being the thickness of the convection zone and δ/T the thermal expansion coefficient. The parameter Z introduced above is defined as a measurement of the density contrast across the convective layer, say $Z = d/H_P$ where $H_P = 1/\partial_r(lnp)$ is the pressure scale height. The choice of H_P as a local scale height instead of, say,

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$H_T = 1/\partial_r(lnT)$ is indeed arbitrary, but is consistent with the theory, as H_P can be explicitly written in terms of the independent variables of the problem. This choice still deserves more substantial criticism as hydrodynamic models take Z as a global variable (Spiegel 1965). In the present derivation, Z as well as N have been arbitrarily turned into local variables.

The definitions of Ra and Z imply a knowledge of d, which is not a variable of the problem and cannot be included in a local model. Thus we must expect R and Z entering N as RaZ^{-4} . There is no direct test for the RaZ^{-4} law, but it displays the tendency expected from numerical computations. The only exception is the so-called up-solutions in the modal computations by Massaguer & Zahn (1980), but these solutions seem to be very peculiar - see Hurlburt, Toomre & Massaguer (1984) for a discussion.

Stellar plasmas are almost inviscid and largely conducting fluids, hence their Prandtl number σ is much smaller than one. It has been conjectured, and is widely accepted that under such circumstances the large scale motion is to be independent of the molecular viscosity. From the previous assumptions the Nusselt number is to be a function of the product $\Lambda = Ra\ Z^{-4}\ \sigma$, which can be explicitly written as

$$\Lambda \;=\; \frac{g\ H_P^4\ (\delta/\ T)}{(k/\rho\ C_P)^2}[-\ (\partial_T - \partial_A)] \tag{2.4a}$$

So, we can finally write $N = N(\Lambda)$. A local theory is expected to provide such a relationship. It can be derived from phenomenology, from experiments - either numerical or laboratory - or from turbulence theory, as will be discussed below.

Mixing-length theories take a slightly different point of view. They assume $N = N(\Lambda_*)$, where Λ_* is the local value of Λ defined as

$$\Lambda_* \;=\; \frac{g\ H_P^4\ (\delta/\ T)}{(k/\rho\ C_P)^2}[-\ (\partial_r T - \partial_A)] \tag{2.4b}$$

Then, from (2.3) we obtain the implicit equation $\partial_r T = -\mathcal{F}(p, m, L, T; \partial_r T)$ instead of (2.1). As an example of such an equation we can mention the well known Bohm-Vitense's cubic equation.

In the limit $\Lambda \ll 1$ most models agree with $N - 1 \sim \Lambda^2$ or $N - 1 \sim \Lambda_*^2$. As both limits will be shown to be equivalent, this seems to be a well established asymptotic law, but this is a limit displaying poor interest as it implies $\partial_r T \simeq \partial_T$. For the opposite limit, $\Lambda \gg 1$, mixing-length theories assume $N \sim \Lambda_*^{1/2}$. This law is the result of forcing the large scale dynamics - say the heat flux - to be independent of molecular conductivity. Yet experimental results would agree better with a $\Lambda^{1/3}$ power-law. For a discussion on power-laws, the reader is directed to Spiegel's (1971) review, but it can be anticipated that the $\Lambda_*^{1/2}$ and $\Lambda^{1/3}$ power-laws are asymptotically equivalent. By using the alternative definitions (2.4a,b) the Nusselt number can be written as $N = \Lambda\ /\ \Lambda_*$, and for a given law $N = N(\Lambda)$ we obtain Λ_*=f(Λ).

Let us now assume, for simplicity, $N - 1 = Q\Lambda^n$ with $n > 0$. Then, for $\Lambda \ll 1$ and $\Lambda \gg 1$ the limits $\partial_r T = \partial_R$ and $\partial_r T = \partial_A$ are recovered from (2.3), no matter what the value of n is. And, in addition

$$\Lambda\ /\ \Lambda_* \;=\; 1 \;+\; Q\ \Lambda^n \qquad (n\ >\ 0) \tag{2.5a}$$

An alternative derivation with $N - 1 = Q\Lambda_*^n$ would give

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$$\Lambda \, / \, \Lambda_* \; = \; 1 \; + \; Q \, \Lambda_*^n \qquad (n \; > \; 0) \tag{2.5b}$$

Each one of the equations (2.5) can be solved for Λ_* , and $\partial_r T$ can then be obtained as a function of ∂_T and ∂_A. Therefore, given a power-law in terms of either Λ or Λ_*, the converse can be obtained. In particular, the mixing-length asymptotic power-law $N-1 \propto \; \Lambda_*^{1/2}$ can be written as $N-1 \propto \; \Lambda^{1/3}$, thus making it possible to establish contact between laboratory experiments and stellar convection theory.

As shown by Gough & Weiss (1976) a couple of power-laws, one for small and one for large Λ values are as good as any local model. To be precise, a simple local model can be written as

$$N-1 \; = \; \begin{cases} Q_1 \Lambda_*^2 & \Lambda_* \leq \Lambda_c \\ Q_2 \Lambda_*^n & \Lambda_* \leq \Lambda_c \end{cases} \tag{2.6}$$

where the Q coefficients embody the so-called mixing-length parameter - i.e.: the ratio of the local scale height to H_P -, n takes the value $n = 1/2$ for mixing-length theories, but here it is kept free to allow for departures from them, and Λ_c is imposed so as to make $N-1$ continuous. In practice, Q_1 can be taken to be zero and the sole concern of a local theory is to decide the values of Q_2 and n. These values can be obtained from phenomenology (Bohm-Vitense 1958), calibrated from stellar evolution (Gough & Weiss 1976), inferred from laboratory measurements or even computed from turbulence theory (Canuto, Goldman & Chasnov 1987).

At this point, it is worth mentioning that recent experiments of convection in low-Prandtl-number fluids (Heslot, Castaing and Libchaber 1987) suggest departures from the $\Lambda_*^{1/2}$ asymptotic power-law. Such a departures are still not well understood, so they can hardly be included in a phenomenological model. And they provide a good reason for restating local theories on a well established frame.

3. Hydrodynamic models.

Hydrodynamic models are usually derived from the dynamic equations (continuity, momentum and heat) as low order moment equations, often with *ad hoc* closures. Statistical relationships obtained from numerical experiments, such as those from Chan & Sofia (1989) for compressible convection, can be very useful for setting these closures, though few experiments have been devised with this purpose in mind, and most of the closures have been conjectured from phenomenological arguments.

Recently derived hydrodynamic models can be found in Xiong (1979, Unno, Kondo & Xiong 1985), van Ballegooijen (1982), Eggleton (1983), Kuhfuβ (1986) and Tooth & Gough (1988). All these models are consistent with mixing-length scaling, which in turn can be viewed as a low order closure model. All these models attempt to give a detailed description of the dynamics of the convection zone, with their final goal being to describe more than just the mean stellar structure. Magnetic fields, stellar pulsation, etc. fall within their scope, so they are non-local and time-dependent. Unfortunately, only a few of these models have been checked against real stellar modeling or laboratory experiments and this is the very test for the underlying hypotheses.

More systematic closure techniques, such as those used in meteorology or engineering have not yet become popular among astrophysicists, possibly because of the difficulties in modeling turbulence in stratified fluids but, also, because of the temporal and spatial resolution they require. The work done by Cloutman & Whitaker (1980) and Marcus, Press & Teukolsky (1983) is pioneering in this respect, but more remains to be done.

A recent work that deserves more substantial comment is the attempt by Canuto, Goldman & Chasnov (1987) to extend the small scale turbulence theory of Kolmogorov and Heisenberg in order to model the largest scales of the flow. Their work follows an early attempt by Ledoux, Schwarzchild and Spiegel (1961) - see also Spiegel (1962). The model provides a detailed description of the - steady - kinetic energy spectrum, thus establishing a close connection with the more sophisticated modern turbulence theories, such as the so-called direct interaction approximation (Kraichnan 1964) and the renormalization group techniques (Yakhot & Orszag 1986).

The master equation for this model is the energy balance, written in terms of the turbulence spectrum as

$$\int_{k_0}^{k} F(k')n_s(k')dk' \;=\; \nu_t(k) \int_{k_0}^{k} F(k')k'^2 dk$$

where k_0 is lower cut-off wavenumber, $n_s(k)$ is the growth rate for the mode k, $F(k)$ is the average kinetic energy per wavenumber and ν_t is the eddy viscosity, which can be formally written as

$$\nu_t(k) \;=\; \int_{k}^{\infty} \frac{F(k')}{n_c(k')} dk$$

where $n_c(k)$ is a correlation frequency.

In order to solve the system, $n_s(k)$ is known from the linearized dynamic equations - i.e.: the dispersion relation -, therefore allowing us to include any body force - i.e.: magnetic fields, rotation, etc.- and $n_c(k)$ is to be given as a closure. By assuming the small scale closure of Heisenberg and Kolmogorov to be valid everywhere, the universal law for the inertial range $F(k) \approx \varepsilon^{2/3} k^{-5/3}$ is recovered for the small scales. But this closure has been shown to be inconsistent for large scales, where the energy balance equation itself imposes $\nu_t(k_0) = n_s(k_0)/k_0^2$ (Canuto, Goldman & Hubickyj 1984). If instead of that closure, the following closure is assumed, $\nu_t(k) = \gamma n_c(k)/k^2$, both the large and the small scales can be modeled self-consistently.

From this model, the asymptotic law $N \sim QRa^{1/3}$ can be recovered, and the value of Q has been computed with remarkable accuracy for convection in water, so extending local theory in the most natural way. Therefore, the model provides a promising frame for discussing hydrodynamics in the asymptotic ranges that conform the stellar regimes, possibly including any external forcing. Results concerning astrophysical plasmas are not that good, as the computed mixing-length coefficient Q_2 in (2.6) is off by a factor of two if compared with the calibration done by Gough & Weiss (1976). This failure seems to be of a very fundamental nature, as it is associated with the difficulty of properly modeling shear processes.

4. Non-local theories of convection

Local models are well suited to describe homogeneous or slowly varying media, as invariance properties constitute their conceptual framework. Boundaries or turning points -say the edge of the convection zone $\partial_T = \partial_A$ - are the genuine elements for breaking such an invariance. Local theories could be extended to include fixed boundaries by, say, turning (2.1) into $\partial_r T = -\mathcal{F}(p, m, L, T; z)$, with z being the distance to the boundary, much as in Prandtl's boundary layer theory. However nothing similar can be done if boundaries are moveable, or their position is not known *a priori*. Also, extending local models for dealing with convection beyond turning points requires changing $\mathcal{F}$ into a functional.

One of the simplest ideas for turning a local model into a non-local one was that of Shaviv & Salpeter (1973). Consistent with mixing-length theory they assumed that any parcel of fluid can travel a length $\ell = H_P$ before losing its identity. In their model the dynamics was reduced to a balance between buoyancy driving and kinetic energy production while the parcel was flying a distance ℓ.

Marcus, Press & Teukolsky (1983) suggested that a more realistic model should, in addition, include shear instabilities as a destabilizing mechanism. Convective motion from the unstable region stirs the neighbouring layers, thus eroding the stable layer. In a shearing motion the erosion will progress until it reaches a front edge where no fluid parcel can be overturned against buoyancy by shear stresses. The stability for this edge is given by the Richardson criterion $Ri > 0.25$, with the Richardson number Ri defined as

$$Ri = \frac{\mathcal{N}}{\partial_r U}$$

where U is the shear velocity, r is taken across the edge and $\mathcal{N}$ is the Brunt-Vaissala frequency defined by $\mathcal{N}^2 = gT^{-1}(\partial_r T - \partial_A)$. As discussed by Marcus, Press & Teukolsky, if shear stresses are to be stabilized by buoyancy, $\mathcal{N}^2$ at the edge is to be large and positive. Therefore, the temperature must change abruptly in order to give a large $\partial_r T$ value. This model introduces a new length-scale λ, defined by $\partial_r U \simeq U/\lambda$, with the result that the penetration depth may depend on the eddy size. The smaller the eddy size the larger the penetration depth.

A simple method of modeling convection with penetration by taking into account buoyancy and shear stresses is to assume for the edge of the convective zone a horizontal plane from which motion starts as plumes rising from heated point sources. Plume convection is a phenomenon well known in meteorology (see Turner 1973 for a review) and plume models have been widely tested from experiments in order to model entrainment of ambient material into the plume itself, a process which is a difficult one to model.

In Schmitt, Rossner & Bohn's (1984) work the plume model serves only to estimate the penetration depth below a locally modeled convective zone. However, the numerical simulations by Hurlburt, Toomre & Massaguer (1986) show that plumes are generated at the upper boundary of the convection zone, cross the unstable layer and finally die in the lower stable layer after some penetration. Thus it seems more appropriate to consistently model the whole convection zone, including penetration, with plumes.

In its simplest form a plume is an axisymmetrical mass flow diverging from a fixed point. Buoyancy work and entrainment of external material into the plume conform its dynamics through the balance of mass, momentum and buoyancy flux.

For a stratified fluid we can write

$$\begin{aligned} \partial_r\,(b^2\rho v) &= 2\alpha b\rho v \\ \partial_r\,(b^2\rho v^2) &= b^2 g\,\rho' \\ \partial_r\,(b^2\rho v S') &= -b^2\rho v\partial_r S\; +\; 2b\alpha\rho v S'_{ext} \end{aligned} \tag{4.1}$$

where $b = b(r)$ is the radius of the plume's cross section, α is an entrainment coefficient, S is the mean entropy profile, S' stands for entropy fluctuations in the plume and S'_{ext} is the entropy fluctuation in the ambient fluid. The convective flux can be written as $F_c = \rho v T S' + k\partial_r$ T + F_{ext}, where F_{ext} is the external heat flux. A convenient modeling of S', S'_{ext} and F_{ext} closes the problem. If there is no entrainment of material, $\alpha = 0$, (4.1) can be reduced to Shaviv & Salpeter's model, but now with the fluid parcels being born at the boundaries, from where they fly across the whole layer.

5. Numerical simulations and laboratory experiments

In the previous sections we have reviewed the most relevant attempts at modeling stellar convection. None of these models have been derived from first principles and some knowledge of the phenomenology was required for their closure. Neither laboratory experiments nor numerical simulations can be easily extrapolated to the stellar case but they provide useful information about some particular aspects of the problem such as convection in a compressible or stratified fluid, penetrative convection or convection in low-Prandtl-number fluids. We shall review the most relevant of these results below.

5.1 CONVECTION IN COMPRESSIBLE OR STRATIFIED FLUIDS

Considerable efforts have recently been put into modeling convection in highly stratified fluids. Unfortunately neither laboratory experiments nor measurements in the Earth's atmosphere can provide the information required. In laboratory convection compressible fluids behave as if they were non-stratified -i.e.: Boussinesq fluids- with $Z \ll 1$, and the situation is quite similar for the Earth's atmosphere, where convection is confined to a region of height $Z \approx 1$. Therefore, phenomenology for highly stratified convection relies on numerical simulation.

In a layer spanning a large density contrast between top and bottom, $\rho_{top}/\rho_{down} \ll 1$, the local scale height at the top of the layer is much smaller than the depth of the layer itself, thus requiring large spatial resolution in numerical codes. Requirements of spatial resolution in compressible convection may come from two different sources: diffusion lengths, either conductive or viscous, and local scale heights. The former can be, and must be, conveniently parameterized to fit into today's computers. The latter simply cannot. Therefore, the bottleneck for hydrodynamic modeling of solar type stars is density stratification, with diffusion scales parameterized so as to be keep them smaller than local scale heights.

A recent attempt at modeling convection conveniently in a highly stratified layer is the three-dimensional numerical simulation of Chan & Sofia (1986, 1989). The latter assumed for the diffusion term Smagorinski's (1963) recipe, which is a well known parameterization procedure for incompressible fluids and one that can be used in the present context if the eddy diffusion lengths are taken to be much smaller than any local scale height. A very systematic discussion of subgrid scale turbulence with astrophysics in mind can be found in Marcus (1986). In which he also reports on the most dangerous flaws for unresolved numerical simulations. The main goal of Chan & Sofia's work was to assess or repudiate hypotheses underlying mixing-length theory or other closures used in building hydrodynamic models. Their results support most of the physics in mixing-length theory.

A major consequence of the stratification is the asymmetry between the upper and lower layers. As shown by Hurlburt, Toomre & Massaguer's (1984) two-dimensional numerical simulations, the asymmetry between both layers is not just a matter of scaling. Rescaling the layers by a local scale height, say H_P , would not be of any help. In the cellular flow which they obtained, the centre of the cell goes down while the density contrast is being increased, -i.e.: the separation between the upper streamlines becomes wider at the expense of the lower ones- thus becoming wider in places where H_P is smaller. Therefore, in a highly stratified atmosphere, mixing-length hypotheses are fulfilled only a number of local scale heights apart from the upper boundary.

A different description of these upper layers is proposed by Chan, Sofia & Wolf (1982) on the grounds of two-dimensional numerical simulations. They obtained for these layers some time-dependent eddies with sizes of the order of the local scale height, and recent three-dimensional numerical computations seem to support their results (Chan & Sofia 1989). However, their computations arouse some criticisms of a very general nature. First of all, the aspect ratio chosen for their box might be too small, and secondly, the choice of Smagorinski's recipe to parameterize the smaller scales results in an enlarging of the velocity boundary layers, from which the thickness of the upper boundary layer might become comparable to the pressure scale height, so producing a mismatch of both length-scales.

In Hurlburt, Toomre & Massaguer (1984) it was found that convection in a box of aspect ratio $A < 4$ with periodic lateral boundary conditions produces artificial time-dependence. This might also be the reason for the time-dependence seen in Ginet & Sudan (1987) as they took $A = 1$. In fact small aspect ratios might enhance pressure fluctuations, which is very dangerous when modeling convection in a compressible fluid.

5.1.1 *Anelastic approximation for compressible convection*

Numerical simulation of convection in a compressible fluid has to deal with propagation of sound waves. This is an unwanted effect as these waves impose very small time steps for the integration of the time-dependent equations whereas unless convection is nearly sonic, their contribution to the energy balance is negligible. Anelastic approximation is a second order asymptotic expansion in terms of the Mach number M (Gough 1969). Acoustic frequencies can be filtered out with the system still remaining energetically consistent at the chosen approximation order.

A widespread version of the anelastic approximation reduces the continuity equation to a divergence-free condition for the mass flux $\nabla . \rho \boldsymbol{v} = 0$. Neglecting the eulerian time derivative of the density, $\partial_t \rho$, in the continuity equation is a crucial requirement for avoiding any pressure mode, but this might be inconsistent. If we split the density as $\rho = \bar{\rho} + \rho'$, where $\bar{\rho} = \bar{\rho}(z,t)$ is the horizontal average, the anelastic scaling implies $\rho'/\bar{\rho} = O(M^2)$ and the continuity equation splits as

$$\partial_t \bar{\rho} + \partial_z \overline{\rho v}_z = 0$$

$$\nabla . (\rho \boldsymbol{v})' = 0$$

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with overbars and primes meaning, respectively, horizontal averages and fluctuations.

Ginet & Sudan (1987), by imposing the divergence-free condition for the mass flow realized some leakage of mass. Also Chan & Sofia (1989) measured in their three-dimensional fully compressible computations a non-zero vertical mass flux $\overline{\rho v_z}$, which is inconsistent with the assumption $\partial_t \bar{\rho} \simeq 0$. With stellar pulsation theory in mind this mean vertical mass flux has been called the radial mode by Gough (1969) and has been included by Latour, Spiegel, Toomre & Zahn (1976) in their derivation of the anelastic modal equations . Hence, the purely radial acoustic modes may couple thermal convection with radial pulsations.

5.2 PENETRATIVE CONVECTION AND OVERSHOOTING

Penetration and overshooting are names sometimes used as synonyms and sometimes not, but which attempt to describe different situations. Let us assume that the structure of a star is computed using a local model for convection. We may then ask how deep into the stable zone convection will penetrate if the structure of the star is kept unperturbed. In mixing-length terminology we should ask how far a parcel of fluid will *overshoot* the boundary of the convective zone. *Penetration* would be better used to describe those situations where convection has been computed self-consistently for the stable and unstable layers as a whole.

5.2.1 *Downward penetration*

As mentioned in §4 penetration into the stable layer can be controlled by two different mechanisms: buoyancy and shear. Shear depends critically on eddy sizes and buoyancy depends, through the Brunt-Vaissala frequency $\mathcal{N}^2$, on the relative stratification between the stable and the unstable layers

$$S = -\frac{[\partial_T - \partial_A]_{unstable}}{[\partial_T - \partial_A]_{stable}}$$

Penetration increases by increasing S and decreasing the shear length ℓ. In fact, as shown by Hurlburt, Toomre & Massaguer (1989), S and ℓ are somehow linked. By decreasing the stiffness of the stable layer (i.e.: increasing S), the penetration motion changes from being cellular (almost confined between flat boundaries) to being plume-like, with the velocity field being concentrated in narrow vertical vorticity sheets (i.e.: eddies with small ℓ values).

Cellular motion is possible only if the adjoining layers are very stable. In a Boussinesq fluid, and with the assumption of cellular convection, Zahn, Toomre & Latour (1982) estimated the distance Δ from the edge of the unstable zone to the first zero of the velocity to be proportional to the relative stability between the two layers, $\Delta \propto R/R_s$, where R and R_s are, respectively, the Rayleigh numbers of the unstable and stable layers. Massaguer, Latour, Toomre & Zahn (1984) generalized the $\Delta \propto R/R_s$ law for downwards penetration in a highly stratified fluid, but their results seem to overestimate penetration.

Hurlburt, Toomre & Massaguer (1989), from a two-dimensional numerical simulation in a compressible fluid, propose the more conservative law $\Delta \propto S^{1/2}$ for their plume-like penetration. Their plumes are laminar but they are still remarkably invariant with depth, which indicates that entrainment is contributing significantly to their dynamics, thus explaining why penetration is to be smaller

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in plumes than in cellular motion.

5.2.2 *Upward penetration*

The upward penetration in a highly stratified fluid is substantially different from the downward one. While the latter, roughly speaking, displays a Boussinesq physics, the former is dominated by non-Boussinesq effects. Going upwards the local scales shrink very fast and pressure fluctuations increase in magnitude until they overcome temperature in the buoyancy work. Buoyancy braking slows the motion down, so as to produce a very stable layer on top of the convection zone. This can explain why upward penetration is always cellular instead of being plume-like (Hurlburt, Toomre & Massaguer 1986). In spite of this enhanced stability, upward penetration can extend over a significant fraction of a local scale height, although measured in terms of the total layer thickness the unstable layer is very shallow.

5.2.3 *Downward penetration of small amplitude velocity fields*

So far, we have only considered large amplitude velocity fields, meaning by this a flow strong enough to flatten the entropy gradient. For the downward penetration this region includes, roughly speaking, the first countercell -i.e.: the region where the convective flux reverses its sign. Below this point the convective flux is so small that the temperature gradient cannot be distinguished from the radiative one. Below the first countercell the velocity field decreases its amplitude significantly, as do the transport coefficients. Time scales for turbulent diffusion and mixing become much larger than in the bulk of the convection zone, but diffusion and mixing can still be effective in these regions (Hurlburt, Toomre & Massaguer 1989).

In Hurlburt, Toomre & Massaguer (1986), much as in the ice-water experiments (Adrian 1975), the plume-like motion is time-dependent, thus forcing gravity waves in the stable layer. The Fourier spectra for these waves may be strongly dependent on the resonance properties of the whole cavity below the convection zone. Absorbing or reflecting walls can drastically change the amplitudes of the Fourier spectrum, but waves can also be absorbed or reflected selectively by the plumes themselves. As a result the feed-back between the gravity waves and the external forcing may be channeling energy towards some preferred frequencies. This might explain why the computed spectra for gravity waves is substantially different from that measured in the ocean (Phillips 1966).

6. Detailed structure of the solar convection zone

Current estimates give a depth for the solar convection zone of between twenty and thirty percent of the total radius of the star - i.e.: 150.000 km to 200.000 km. Although recent work by Christensen-Dalsgaard, Gough & Thomson (1989), based on the measurement of the sound speed in the solar interior by using helioseismological data, shows and adiabatic region delimitated by a sharp edge at a depth of twenty-three percent of the solar radius.

The Rayleigh number value based on the largest local pressure scale height, $H_P \approx 50.000$ km, and on a turbulent viscosity takes the value $R \approx 10^{12}$. And the Prandtl number based on the radiative diffusion coefficient is $\sigma \approx 10^{-9}$, thus giving $\Lambda = R\,\sigma > 10^3$ everywhere except, perhaps, near the photosphere - the reader must be aware that the Λ-value is independent of the assumed viscosity. Therefore, according to the critical Λ value estimated by Spiegel (1966), the solar convection zone is in a large Peclet number regime almost everywhere.

6.1 GRANULES

Granules are bright regions on the solar photosphere surrounded by dark lanes. Their mean horizontal size is $\ell \approx 1.400$ km and they last for some five to ten minutes. Bright regions are associated with upward motion, while dark lanes are associated with downward motion. The average vertical velocity, either up or down, is one kilometre per second and it is well correlated with the horizontal temperature difference, which shows a value of a hundred degrees.

The existence of a strong correlation between velocity and temperature has always been taken as the main argument in favour of the convective nature of the photospheric granules. However such a correlation only implies a very efficient transport of heat and does not tell us anything about the very nature of the driving mechanism. Forced convection is more efficient than natural convection but it is not buoyancy driven. The observation of a cellular structure has also been taken as an argument in favour of the convective origin for the granules, mostly because a turbulent motion, say shear driven, would display a continuous Fourier spectrum with the cut-off length scale being much shorter than the size presumed for the granules.

Two and three-dimensional numerical simulations for modeling granulation have been derived by Nelson & Musman (1977, 1978), Nordlund (1985) and Stein & Nordlund (1989). All these simulations give plausible results, but the geometrical similitude between the structures computed in the latter paper and the photographs of solar granules is so impressive that we can finally believe we understand granules. The pictures obtained from the numerical simulation strongly resemble a granulation photograph. However, in spite of the persistent optical impression of the granules showing a dominant size, their kinetic energy spectrum is smooth, without showing any appreciable peak. The conclusion, therefore, is that no preferred scale exists.

Something similar can be said of the measured spectrum once the resolution of solar observations has been sufficiently increased and low frequency pressure waves have been filtered. Muller & Roudier's (1985, Roudier & Muller 1986) observations show a continuum of sizes for the bright regions, with their transversal dimensions decreasing towards values much smaller than those presumed for granules. The power spectrum measured by the above mentioned authors clearly displays an inertial range, with a slope of $-5/3$ as corresponds to the famous Kolmogorov-Obukhov law - see Zahn's (1987) review. Therefore, observations and numerical simulations seem to agree with the granules being an optical effect associated to a privileged velocity-temperature correlation length.

Indirect evidence for the existence of a privileged correlation length can also be found in Roudier & Muller's (1986) work. By measuring the fractal dimension of the granular structures they revealed a change in dimension near the expected granular size. As discussed by Zahn (1987), at photospheric level and for length-scales close to the granules' size, the Peclet number is order one. Therefore at smaller scales the convective heat transport is expected to be very inefficient. The Nusselt number is $N \approx 1$ but, as discussed in Massaguer, Mercader & Net (1989), velocity and temperature show a good correlation, possibly because they are linearly related. The granules with sizes larger than this are the more corrugated ones - i.e.: those of larger fractal dimension- possibly because of the enhanced turbulent transport. Their Peclet number value is larger than one, heat transport is more efficient, $N \gg 1$, but because of the non-linear relationship between them, temperature and velocity display a reduced correlation factor, and the contrast between hot and cold regions becomes smoother.

6.1.1 *Vortices*

Recent satellite observations made by Brandt *et al.* (1988), once conveniently filtered from low frequency acoustic modes, have shown the presence of a vortical flow. And vortices of size $\ell \approx 5.000$ km, corresponding to approximately three granules, have been identified. Photospheric rotational motions have been reported long before in sunspots. Yet what makes the subject new is the possibility of their being uniformly distributed in the photosphere, possibly in association with granules, and twisting magnetic flux tubes.

6.1.2 *Mesogranules*

November *et al.* (1981) have also found evidence of photospheric structures of sizes intermediate between granules and supergranules, what they have called mesogranules. It is a rather weak motion, as compared to granules and supergranules, with length-scales ranging between 5.000 km and 10.000 km and lifetimes of about two hours. They have been described as associations of granules by Kawaguchi (1980) but their physical origin is still unclear.

6.2 SUPERGRANULES

Supergranules are structures that extend well into the cromosphere. Their horizontal size is $\ell \approx 30.000$ km, approximately twenty times that of a granule, and they last for some twenty hours. The horizontal velocity ranges from three to five hundred metres per second and the vertical motion displays a slightly slower downward motion with the upward one being much slower. Horizontal temperature contrast is very small, with its upper limit being $\Delta T < 1\ ^0K$. As an additional feature, the border of the supergranules is delimited by an intense magnetic field.

The flow described above for a supergranule is very close to what must be expected for cellular convection, and buoyancy driving has never been really questioned. A weak temperature difference across the cell is certainly not in contradiction with it. As, in addition, the presence of a magnetic field concentrated near the border of the cell increases thermal conductivity, such a small ΔT seems not to question a convective origin. More striking is the regularity of the hexagonal pattern observed. Low Prandtl number thermal convection is not the best context for such a regular flow as shear instabilities play a dominant role. Such a geometrical regularity is to be thought of as the result of some additional stabilization effects. As discussed in §5.1, a high-density contrast such as that found in the upper layers of the solar convection zone could be a candidate, but other contributions, like the stabilizing effect of a magnetic field, cannot be ruled out.

The most difficult question about supergranules is that concerning their size.

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Several conjectures can be found in the literature but none of these conjectures seems to be soundly established. Neither laboratory nor numerical simulations support a cellular motion with a size much smaller than the layer depth. Therefore, the possibility of supergranules not being cellular structures has to be taken seriously. In fact, cellular convection is far from being the flow pattern most frequently seen in laboratory experiments. Even in the most classical Boussinesq problem transient bubbles or plumes do constitute the main flow. At this point it may be worth quoting the following description from the experimental work by Castaing *et al.* (1989). *In the central or interior region of the cell we envision motions on many scales up to that of the entire cell, consisting of convective eddies, and of thermals and plumes.*

In non-cellular Boussinesq convection we expect the largest eddies, of sizes comparable to the layer depth, to break into smaller eddies, thus cascading kinetic energy from small to large wavenumbers. In a highly stratified fluid, however, the situation might be different. From a patchy work with materials of very different origin and quality Zahn (1987) was able to build a power spectrum for the solar convection zone. If this spectrum is to be trusted, kinetic energy is being fed up into the motion at the supergranules size, not at the largest scale of motion. Again, much as for the granular flow, the real question about sizes does not concern scales of motion but coherence lengths. So as to produce the largest injection of buoyancy work, velocity and temperature have to be coherent. By writing the buoyancy work fed up to an eddy of size ℓ as

$$E_B \approx \int_{r+\ell}^{r} \rho' v g \, dr$$

we can see that a large injection of buoyancy work requires coherence between ρ' and v, together with a large eddy size. Upper limits for ℓ are difficult to establish. There is no experience yet in modeling turbulence on highly stratified media, so we cannot establish any firm conclusion, but Chan & Sofia's (1989) results support the view that the pressure scale height is a convenient measurement of the coherence length. If so, buoyancy work has to show a maximum for eddy sizes $\ell \approx 50.000$ km, the largest pressure scale height, hence associating the supergranules with the deeper layers of the convection zone.

6.3 GIANT CELLS

The existence of giant cells was conjectured a long time ago by Simon & Weiss (1968). From laboratory experiments there is no doubt that a pattern with a horizontal scale of the order of the depth of the convection zone has to be present, but it is unclear whether it will show up as a cellular motion or not. The so-called torsional waves of Howard & LaBonte (1980) are the only pattern that fits such specifications, so they may be considered as a plausible candidate.

Until very recently there was widespread confidence in the large scale flow of convection in a rotating spherical shell being organized as elongated cells with their axis in the North-South direction, which have been called *banana cells*. Numerical as well as experimental work in fact support this conjecture (Hart *et al.* 1986, Gilman & Miller 1986), but they have never been observed. The observation of the so-called torsional oscillations has challenged this confidence (Snodgrass 1987, Snodgrass & Wilson 1987) much as the migrations of young sunspots (Ribes & Mein 1985, Ribes & Laclare 1988). The former authors describe the whole pattern as a set of toroidal vortices, one on top of the other, like *piled doughnuts*. Each one of these vortices would be a giant cell.

All these large scale flows show a slow equatorward drift along the solar-cycle, so making the magnetic field an obvious candidate for explaining the preference for a doughnut's structure instead of banana cells. In fact, a rotating spherical shell of thermally convecting fluid with an intense magnetic field concentrated in the lower portion of the shell can display such a toroidal pattern (Merryfield 1989).

7. Conclusions

In the present review we have examined the structure of the solar convection zone and the most relevant techniques for its modeling. As much as possible we have tried to avoid *ad hoc* hypotheses, so relying only on numerical results, laboratory experiments and observations. And we have shown how all these three pieces of material converge towards a better understanding of the physics of the solar convection zone.

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