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Convection and the Relevant Problems of Stellar Structure, Evolution and Oscillations

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Abstract. The main results of stellar structure, evolution and oscillations given by our non-local statistical theory of convection are summarized in this paper. A main difficulty of convection theories, i.e. supersonic convection, is also discussed.

1. Introduction

Convection causes transport of energy, momentum and matter, so it seriously affects the internal structure of stars and their properties of evolution and pulsation. The most popularly applied theory is the so-called Mixing Length Theory (Vitense 1953, hereafter MLT). The original MLT is a local expression for stellar convection. Convection is, by nature, a fluid instability caused by the thermal structure of stars. Therefore it is surely a non-local phenomenon. Spiegel (1963) and Ulrich (1970) developed the non-local versions of the MLT. Their nonlocal convection theories were used to the construction of the solar atmosphere model. A comparison of the theoretical limb-darkening with the observed one shows that the theoretical temperature distribution of the solar atmosphere is too gentle (Travis & Matsushima 1973). It is due to the fact that their theories overestimate the convective energy transport in the overshooting zone and the most upper region of the convection zone. The MLT is not a hydrodynamic theory, but is a phenomenological theory. The main defect of the phenomenological theory is that it cannot give a precise description of the dynamical behaviors of turbulent convection. This defect becomes more serious and intolerable for the problems of non-local convection and time-dependent convection. A possible approach for turbulent convection is the direct simulation based on the hydrodynamic equations. It will require an extremely huge amount of computing power. Such a powerful computer is not available yet. Another simpler hydrodynamical approach is the two- or three-dimensional simulations (Nordlund 1985; Stein & Nordlund 1998), however, they are all large-eddy approximations. They were used to simulate the structure of the solar granulation and got some interesting results. Although the numerical simulations have got some significant successes, they still cannot be applied to theoretical calculations of stellar structure and

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evolution owing to the costly computing and the rather coarse grids. For the theoretical calculations of stellar structure and evolution, one really wishes to have a practical theory of convection which is exact enough to describe the reality and simple enough to be used. A statistical theory of correlation functions seems to be appropriate for this goal.

Although the non-local convection theory on a more sound hydrodynamical basis has made significant progress over the past two decades (Xiong 1979, 1981, 1989, 1997; Canuto 1993; Grossman et al. 1993; Grossman 1996; Ventura et al. 1998), the MLT is still the most popular theory of convection used in theoretical calculations of stellar structure and evolution. We are not trying to give a thorough review of stellar convection, in stead, we will make a brief summary of the main results of our theory in researches of the stellar structure, evolution and oscillations (sections 2-4). In the last section we will discuss briefly the difficulty of supersonic convection in convection theories.

2. Influence of non-local convection on the structure of the convective envelope of stars

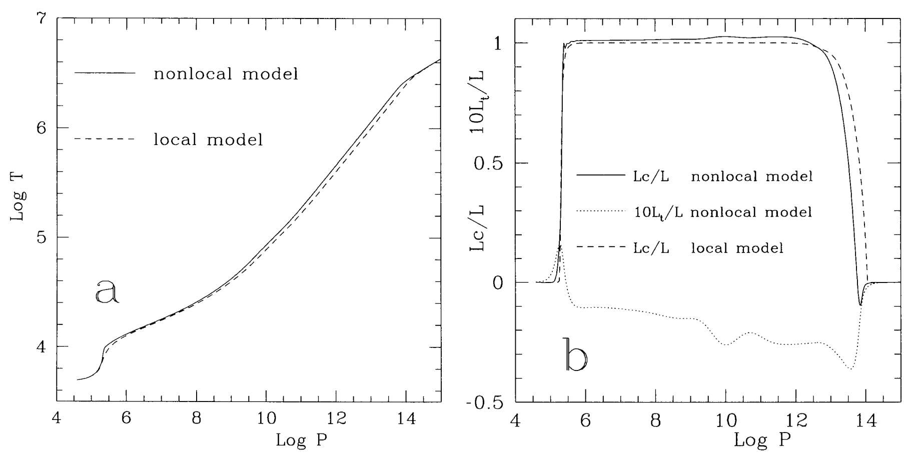

Figure 1. The structures of a local (dashed lines) and a non-local (solid lines) models of solar convective envelope using the same convective parameters. a) $\log T$ vs. $\log P$; b) fractional convection flux L_c/L and the fractional turbulent kinetic energy flux L_t/L (dotted line) vs. $\log P$.

The plausibility of the popular application of the local MLT can be understood as the following: 1) The local MLT is optimum in regard to the straightforwardness in physical picture and simplicity for application; 2) The local MLT is still a good approximation for construction of stellar models, if the convective overshooting and time-dependent convection are not special concerns. In the deep interior of stars the convective energy transfer is very efficient, so the temperature gradient approaches the local adiabatic one disregarding for the

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treatment of convection. The effects of non-local convection are significant only in the narrow zones around the boundaries of the convection zone. Figure 1 depicts a local (the dashed line) and a non-local (the solid line) convective envelope models of the Sun. It is clearly shown that they are closer to each other, however, the non-local model possesses a more shallow convection zone compared with the local one with the same convective parameter. This is due to the fact that the depth of a convection zone is determined mainly by the superadiabatic convection zone on the top. Owing to the non-local properties of convection, the turbulent energy flux diffuses into the stable zone outside the convectively unstable zone. As a result, in the superadiabatic convection zone the convective energy flux will be smaller and the temperature gradient will be larger for non-local model in comparison with those of the local model. By tuning the convective parameters, we can always construct a non-local model which has the same depth of convection zone as that of the local model with a slightly smaller convective parameter. Figure 2 shows a pair of models of the local and non-local convective envelopes of the Sun. The depth of convection zone is $r_c/R_\odot \approx 0.71$. Figure 2b draws the relative differences of squared sound speed and density between the non-local and local models of the Sun vs. the depth (radius). The differences between them are within 1%. Such differences are impossible to be determined for any distant star.

3. Convective Overshooting and Stellar Evolution

Historically, the MLT was always the only choice for treatment of convection in the theoretical calculations of stellar structure and evolution. According to the local MLT, convection is determined by the local temperature gradient at the same place of turbulent fields. It is a convectively unstable zone while the temperature gradient is greater than the adiabatic one; otherwise, it is a convectively stable zone. Therefore, there is a sharp boundary between the convective and radiative zones. However convection cannot stop suddenly at the boundary of convection zone. The convective elements will penetrate into the radiative zone owing to their inertia of motion. The extent of penetrating from the convectively unstable zone into the adjacent stable one is the so-called overshooting distance. Convective overshooting is a long standing problem in stellar evolution theory. While we discuss this problem, we must answer the following two questions

1. How to define the boundaries of a convection zone?

2. What is the structure of overshooting zone?

In fact, these two questions are ultimately linked with each other. The definition for the boundary of a convection zone depends very much on how one understands the structure of the overshooting zone. In the community of stellar evolution and structure researches, convective and overshooting zones was understood on the basis of local stability analysis using the local MLT. Most people believe such a nearly adiabatic model of overshooting zone. In the convectively unstable zone, the temperature gradient ∇ is close to and slightly higher than the adiabatic value ($\nabla - \nabla_{ad} \gtrsim 0$). In the overshooting zone adjoined to it, the tem-

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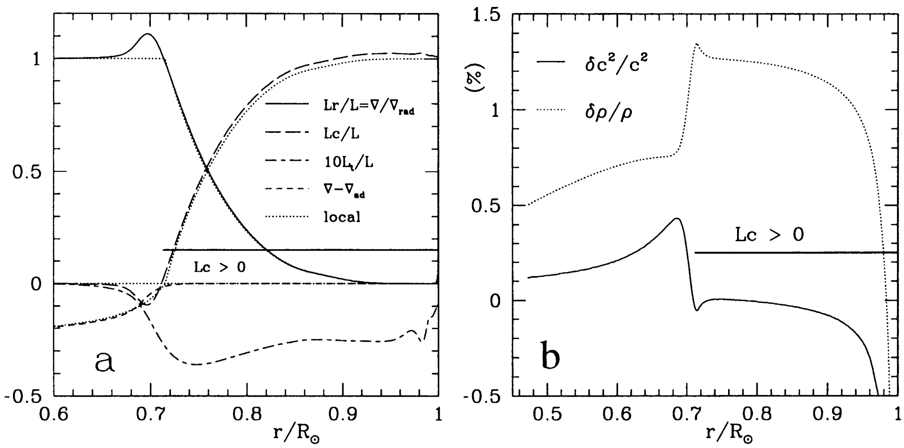

Figure 2. a) The fractional radiative, convective and turbulent kinetic energy fluxes L_r/L, L_c/L, L_t/L, and the super-adiabatic temperature gradient versus the fractional radius $r/R_\odot$, for a non-local convection model of the solar convective envelope. The dotted lines are the corresponding ones of a local convection model with the same depth of the convection zone as in the non-local model. The horizontal black line shows the convection zone. b) The relative differences in the squared sound speed (solid line) and density (dotted line) between the non-local and the local convection models. The horizontal black line shows the convection zone.

perature gradient is near and slightly less than the adiabatic one ($\nabla - \nabla_{ad} \lesssim 0$). Passing through a very thin transition layer, the nearly adiabatic temperature gradient jumps to the radiative one, and in the mean time the convective velocity becomes zero (Shaviv & Salpeter 1973, Maeder 1975, Bressan et al. 1981, Zahn 1991, Monteiro, et al. 2000). Hence, this phenomenological model predicts a near discontinuity of temperature gradient (and therefore the derivative of sound speed) at the bottom of the overshooting zone. The discontinuity of the derivative of sound speed will depend on the overshooting distance. Gough and Sekii (1993) analyzed observational data of solar oscillations, and they concluded that there is no convincing evidence for overshooting. Other authors supposed that the overshooting zone must be extremely narrow. In literature, the upper limit is in the range 0.25–0.05 Hp (Roxburgh and Vorontsov 1994, Monteiro et al. 1994, Christensen-Dalsgaard et al. 1995, Basu et al. 1997). This nearly adiabatic stratification model of overshooting zone, however, is never true. They think convective heat transport is efficient enough to establish an adiabatic stratification. In this phenomenological model of the overshooting zone, in fact, an implicit assumption, i.e. the turbulent velocity and temperature are almost fully correlated, is normally included (Petrovay & Marik 1995). However, this is not true. The results of our complete non-local model and three-dimensional simulations of compressible convection (Singh et al. 1995) show that the correlation coefficients of turbulent velocity and temperature decrease very quickly towards the lower boundary of convection zone. Grossman (1996) has also argued that this model is not self-consistent. Our non-local statistical theory of convection gave a self-consistent model of overshooting zone. Within our theoretical framework, convection has already become subadiabatic far away from the lower boundary of the convection zone, and the convective energy flux in the overshooting zone is negative. As a result, the overshooting zone is subadiabatic and super-radiative (Xiong & Deng 2001b). The temperature gradient gradually approaches to the radiative one. There is never a near discontinuity of temperature gradient as that predicted by the phenomenological mixing-length model. Therefore, the statement that no significant discontinuity in the derivatives of sound speed has been found at the bottom of the solar convection zone does not mean that either there is no overshooting or the overshooting zone is extremely narrow (Basu et al. 1997). There are several ways for defining the boundary of the convection zone, among which the best choice seems to be the place where convective energy flux becomes zero. Passing through such a boundary, the convective flux F_C changes its sign, giving $F_C > 0$ for convection (unstable) zone and $F_C < 0$ for overshooting zone. Such a definition possesses two merits:

1. In practice, it is fairly easy to determine the place for F_C to change its sign;

2. Such a definition of the boundary of convection zone is compatible with the usual one of the local convection theory. The structure of such a non-local model is very close to the local one with the same depth of convection zone (refer to Fig. 2).

The differences shown in Fig. 2b are very similar to the results of the relative differences of squared sound speed and density between the Sun and the reference (local convection) model deduced by the helioseismic inversion (Basu et al. 1997). Therefore, it can be expected that if our nonlocal convection model is used to replace the SSM of Christensen-Dalsgaard et al. as the reference model of inversion, the relative differences of squared sound speed and density mentioned above would be eliminated, at least they would be reduced considerably. As the convective energy flux is negative and the temperature gradient is super-radiative in the lower overshooting zone in our non-local convection theory, the temperature and sound speed are higher there. Hence, this is a natural result for our non-local theory of convection. It is not needed to assume a sudden increase of the opacity at the bottom of convection zone (Baturin & Ajukov 1996), or to introduce an unknown mixing below the convection zone (Gough et al. 1996). These differences can be naturally explained by using our theory. Our non-local convection model seems to be supported by helioseismology in this respect.

Overshooting has very important influence on stellar evolution. The vast majority of theoretical treatments of convective overshooting are used with the simple ballistic description (Maeder 1975; Bressan et al. 1981). However, This treatment is not self-consist, as mentioned above. In fact overshooting and the thermal structure of star are closely coupled. Our non-local convection theory for chemically inhomogeneous stars forms a set complete and self-consist equations of stellar structure (Xiong 1981). It has also been applied to evolutionary calculations of massive stars (Xiong 1986). The theoretical evolutionary tracks

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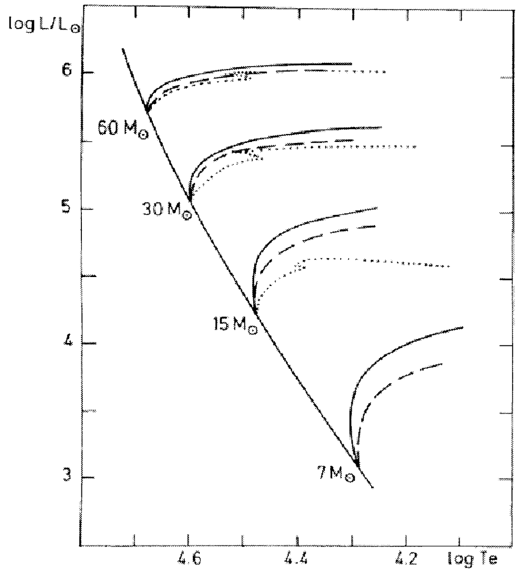

Figure 3. The theoretical evolutionary tracks in the framework of our non-local convection theory for the convection parameters $c_1 = c_2 = 1/2$ (black lines) and $c_1 = c_2 = 1/3$ (dashed lines). The dotted lines are the results of local convection theory (Meader, 1981a,b).

are shown in Figure 3 in which the solid and dashed lines are for different convection parameters. For a comparison, the models computed with local MLT by Maeder (1981) are also drawn. It is clear from Figure 3 that the luminosities and the width of the main sequence band in our models are both larger than those of the local theory. This can be used to explain the mass contradiction of Cepheids, and it is also favorable to resolve the contradiction between the observed and theoretical distribution of luminous stars in the HR diagram (Chiosi & Maeder 1986, Deng et al. 1996). Figure 4 draws the hydrogen profile during the evolution of a 60 $M_\odot$ star. It can be seen that the non-local convection mixing of elements penetrates deeply into the radiative zone. A gradient zone of molecular weight is formed automatically in the overshooting zone.

One important conclusion coming out from our non-local theory of convection is that there is not a sole universal overshooting distance. For different physical quantities, the effective overshooting distances can be different by several times. It is not feasible to use directly the overshooting distance given by helioseismology, or the e-folding length of the turbulent velocity field of solar atmosphere, as the overshooting distance in stellar evolution calculations. Actually, the effective non-local mixing distance of chemical compositions, namely the overshooting distance in the usual sense of stellar evolution, is much larger than the two distance scales mentioned above (Xiong 1986).

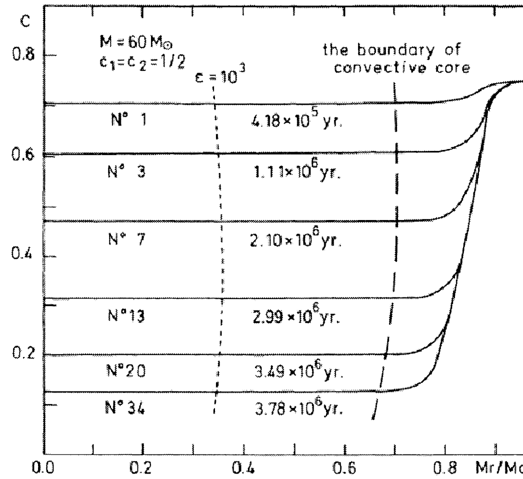

Figure 4. The hydrogen profiles at various ages for the models of a 60 solar mass star in our non-local convection theory. The dashed line is the boundary of the convective core.

4. Time-dependent Convection and pulsational stability of stars

For low temperature stars having extended convective envelopes, convection, instead of radiation, becomes the principal manner of energy transfer. Therefore, it controls the pulsational stability for such stars. Convection intervenes the pulsational stability of stars through convective energy transfer (thermodynamic coupling), turbulent pressure and turbulent viscosity (dynamic coupling). To deal with such couplings, a time-dependent theory of convection is needed, and this has been investigated by many authors (eg. Unno 1967, Gough 1977, Xiong 1977, 1989, Kuhfuss 1986, Stellingwerf 1982, Gehmeyr & Winkler 1992).

We developed a non-local time-dependent theory of convection (Xiong 1989), which was used to calculate the nonadiabatic oscillations of stars. the coupling between convection and oscillation is treated carefully. Our results not only give correctly the red edges of the pulsational instability strips of RR Lyr and δ Scuti (Xiong, Cheng & Deng 1998, Xiong & Deng 2001a), but also predict the existence of a Mira instability region beyond Cepheid instability strip. Such luminous red variables are pulsationally unstable at their fundamental and the first overtones, while being stable for all modes higher than the second overtone, (Xiong, Deng & Cheng 1998). For the stars in the middle and lower parts of red giant branch (RGB), pulsational instability can happen for some modes higher than the second overtone, their fundamental and the first overtone are stable (Xiong, Cheng & Deng 1998, Xiong &Deng 2001a). Pulsational instability shifts to high order overtones for still lower luminosities. For the Sun, all radial p-modes in $11 \leq n \leq 23$ are pulsationally unstable, while the low order ($n \leq 10$) and high order ($n \geq 24$) p-modes are pulsationally stable. The coupling between convection and pulsation plays a key role for stabilizing the low order p-modes and exciting the intermediate order p-modes. However, the normal coupling be-

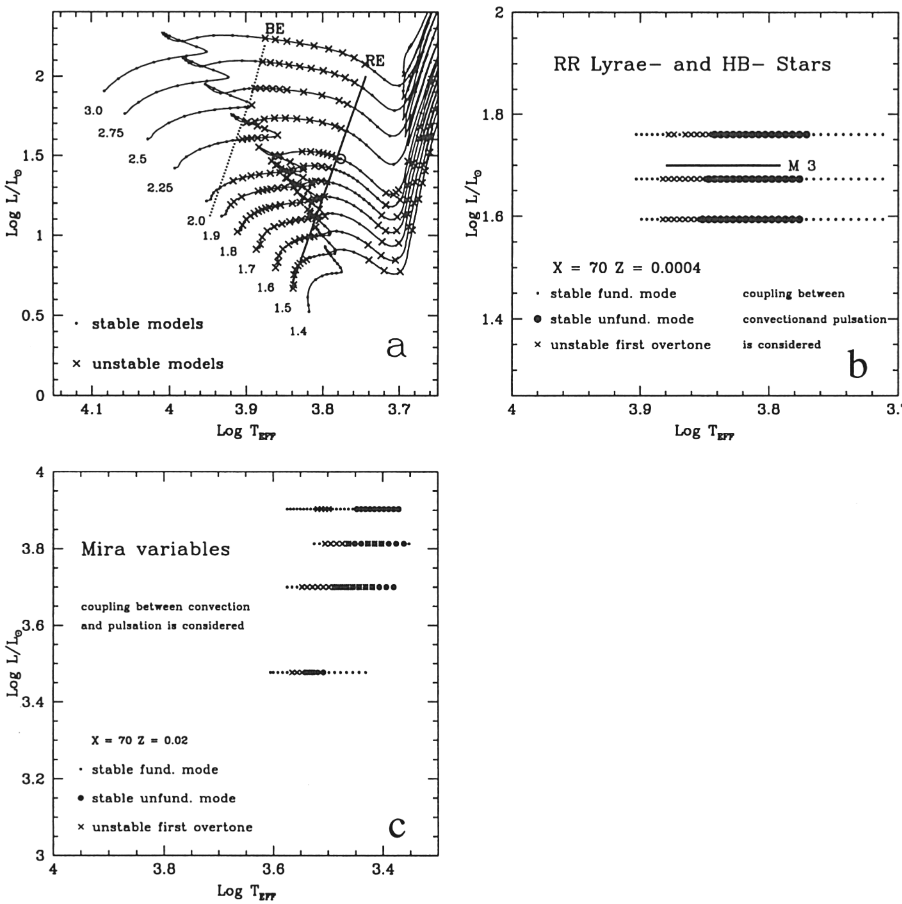

Figure 5. a) Theoretical pulsationally unstable modes denoted by crosses (at least one unstable mode from the fundamental to 8th overtone) and the stable modes denoted by small dots in the HR diagram for 1.4 - 3.0 solar mass stars. b) HB stars. The horizontal black line shows the observed pulsationally unstable strip for the globular cluster M3. c) luminous red variables. In the panels b and c the large solid dots and crosses show pulsationally unstable fundamental mode and first overtone respectively, and the small dots are pulsationally stable models.

tween convection and oscillation is not enough to interpret all the observational properties of the solar 5-minutes oscillations. Therefore, it is a straightforward conclusion that there must be some alternative excitation mechanism besides the ordered coupling between convection and oscillation. The stochastic excitation caused by turbulence is the most likely one (Xiong, Cheng & Deng 2000).

Various types of variable stars possess distinctively pulsational properties, which can all be explained by using the complicated properties of the coupling between convection and oscillation. Turbulent viscosity always works as a damping factor. The turbulent pressure normally lags behind density variation, therefore it is an excitation factor. On the contrary, the convective energy transfer in the deep interior of convection zone is a damping mechanism. In the zones of radiative flux gradient, the radiative and convective energy fluxes are modulated by the pulsational motions, and this gives rise to a so-called modulation excitation mechanism (Xiong, Cheng & Deng 1998). The impact of the above four factors of excitation and damping varies for different stellar parameters such as mass, luminosity and effective temperature. The interplay of these factors in turn gives rise to the various pulsational properties of different types of low temperature variable stars. Figure 5 a-c show the theoretical pulsational instability strip for Delta Scuti, RR Lyr and Mira variables.

5. Supersonic convection

When applied to the main sequence dwarf stars, our theory yields reasonably good results. The theory is marginally applicable to the low luminosity giants. However, the calculation of the envelope models becomes more and more difficult for luminous stars, and sometimes the numerical scheme ends up with no convergence. Figure 6 a-c draw the turbulent Mach numbers $< w'/Cs >$ and relative temperature fluctuations $< T'/T >$ vs. depth for three series of stellar envelope models with different luminosities, which are calculated by the usual MLT. It can be seen that the convection already becomes supersonic for red-yellow giants and super giants, because there are very low density and very large temperature gradient in their surface superadiabatic convection zone. Our theory can only be applied to the subsonic convection. The direct reason of the above-mentioned numerical difficulty, we think, is due to supersonic convection. Supersonic convection is a challenge not only for our non-local convection theory, but also for all exsiting theories. In fact, the usual MLT is also valid only for subsonic convection. The formalism of the MLT also fails for supersonic convection (Deng & Xiong 2001). The supersonic convection in red-yellow giants and supergiants is only a numerical consequence of extrapolation of the current theories of stellar convection. The physical background of such phenomenon is not yet clear. The properties of subsonic and supersonic convections are surely distinctively different. The development of a convection theory covering from subsonic to supersonic convection is very difficult. The envelope models of red- and yellow giants and supergiants, in our opinion, are very uncertain at present time, so the evolutionary and pulsational modeling for such stars are greatly handicapped by such a problem.

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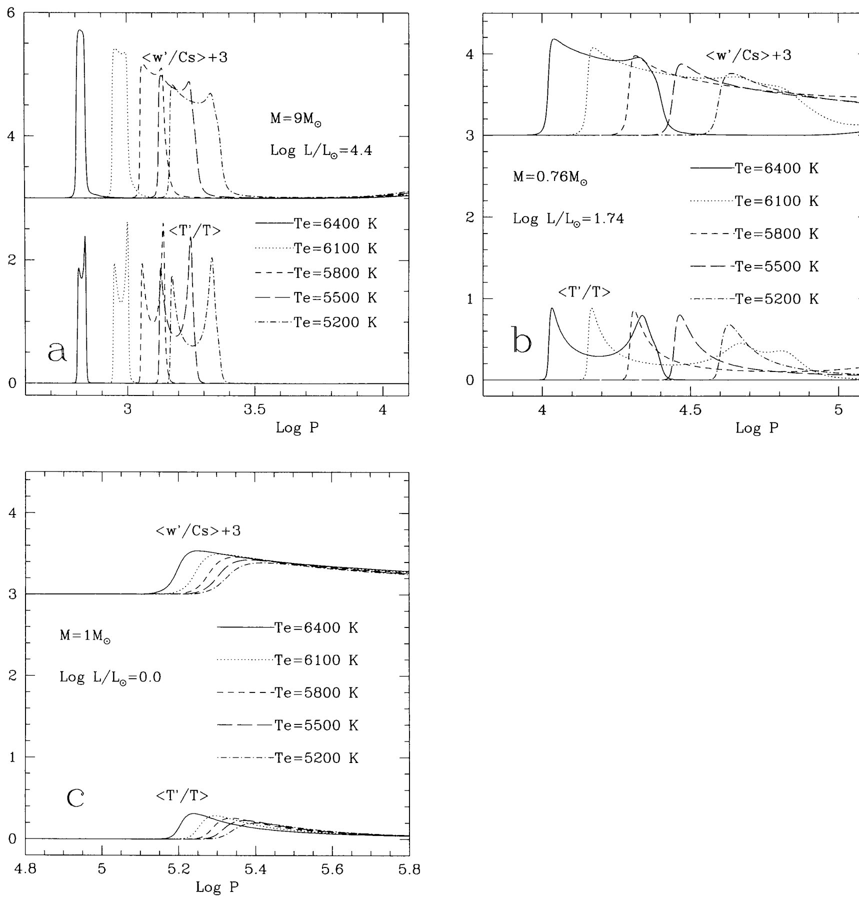

Figure 6. The turbulent Mach numbers $< w'/Cs >$ and relative temperature fluctuations $< T'/T >$ vs. depth (log P). for a) Cepheid models; b) H-B stars; c) solar-type stars.

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6. Discussion

LOBEL: What are the supersonic convection velocities you compute for the yellow luminous starts?
XIONG: Subsonic convention is an implicit hypothesis for all existing theories of stellar convection, which will give velocities of convective motion higher than the local sound speed when applied to yellow-red giants and supergiants. Supersonic convection may not be real in all cases; instead, it can be due to the extrapolation of the current theories of stellar convection. This will greatly affect our understandings of the nature of yellow-red giants and supergiants.

ZAHN: Your turbulent model involves several adjustable parameters. How are they chosen?
XIONG: There are two free parameters in our theory, which are linked with the dissipation and diffusion processes of turbulent convection respectively. The calibration manners for the mixing-length of the Mixing-Length-Theory such as solar evolutionary calibration, the depth of the solar convection zone deduced from helioseismic inversion, and Li abundance of the Sun and solar type stars etc, can all be applied to constrain the two parameters.

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