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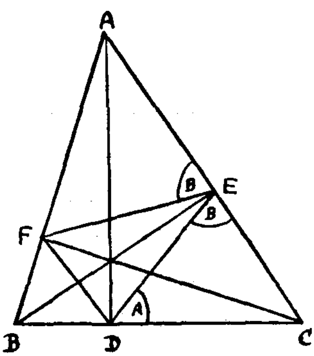
Geometrica Proof of a Trigonometrical Identity.—The following proof of the identity

$$1 - \cos^2 A - \cos^2 B - \cos^2 C - 2 \cos A \cos B \cos C = 0$$

where A, B, C are the angles of a triangle differs from that given by Dr Bell in the last number of *Mathematical Notes* and from that given in "New Trigonometry for Schools" by Lock and Child in that the area of the triangle is made use of.



DEF is the pedal triangle of ABC. $\triangle DEC$ is similar to $\triangle ABC$, CE and CB being corresponding sides.

$$\therefore \quad \frac{\triangle DEC}{\triangle ABC} = \frac{CE^2}{BC^2} = \frac{a^2 \cos^2 C}{a^2} = \cos^2 C$$

$$\therefore \quad \triangle DEC = S \cos^2 C$$

where S denotes the area of ABC.

Similarly $\triangle AFE = S\cos^2 A$

and $\triangle BFD = S\cos^2 B$

$$\therefore \quad \triangle DEF = S(1 - \cos^2 A - \cos^2 B - \cos^2 C).$$

But $$\frac{FE^2}{CB^2} = \frac{\triangle AFE}{\triangle ABC} = \cos^2 A$$

$\therefore$ $FE = a\cos A$. Similarly $DE = c\cos C$, and angle

$$FED = 180° - 2B.$$

$$\begin{aligned}\therefore \quad \triangle DEF &= \tfrac{1}{2}FE \,.\, ED \sin FED \\ &= \tfrac{1}{2}a\cos A \,.\, c\cos C \,.\, \sin(180° - 2B) \\ &= \tfrac{1}{2}ac\cos A\cos C \,.\, 2\sin B\cos B \\ &= \tfrac{1}{2}ac\sin B \,.\, 2\cos A\cos B\cos C \\ &= S \,.\, 2\cos A\cos B\cos C.\end{aligned}$$

The result follows by equating the two values found for $\triangle DEF$.

A. G. BURGESS.

Proof of some Triangle Formulae. — Let I be the incentre of $\triangle ABC$, and let the excentre opposite A be I_1. Draw perpendiculars IF and I_1F_1 to AB. $\angle IBI_1 = 90°$.

$$\therefore \quad \angle FBI = 90° - \angle F_1BI_1 = \angle F_1I_1B.$$

Hence $\triangle FBI$ is similar to $\triangle F_1I_1B$.

$$\therefore \quad \frac{IF}{FB} = \frac{BF_1}{F_1I_1}$$

$$\therefore \quad IF \,.\, F_1I_1 = FB \,.\, BF_1.$$

Again $\angle AI_1B = \frac{1}{2}(180° - B) - \frac{A}{2} = \frac{C}{2}$

$\therefore$ $\triangle BAI_1$ is similar to $\triangle IAC$.

$$\therefore \quad \frac{AI}{AC} = \frac{AB}{AI_1}$$

$$\therefore \quad AI \,.\, AI_1 = AB \,.\, AC.$$