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Article

James Croll, celestial mechanics and climate change

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ABSTRACT: James Croll was a pioneer in studies of the impact of the slowly changing orbital dynamics of the Earth on climate change. His book *Climate and Time in their Geological Relations* (1875) was far ahead of its time in seeking correlations between climate change, the occurrence of ice ages and perturbations to the Earth's orbit about the Sun. The astronomical cycles he discovered are now called 'Milankovitch Cycles' after the Serbian scientist whose research was first published in the *Handbuch der Klimatologie* in 1930. The celestial mechanical and astronomical background to Croll's research is the focus of this essay. The development of the understanding of the impact of perturbations of the elliptical planetary orbits by other bodies in the solar system paralleled new mathematical techniques, many of which were developed in association with celestial mechanical problems. The central contributions of many of the major mathematicians of the late 18th and 19th Centuries, including Euler, Lagrange, Laplace and Le Verrier, are highlighted. Although Croll's contributions faded from view for several generations, his pioneering insights have now been demonstrated to have been basically correct.

KEY WORDS: ellipticity, geological and astronomical dating, ice ages, Le Verrier, obliquity, perturbations by other solar system bodies, planetary orbits, precession of equinoxes.

Until I received the invitation to discuss the astronomical aspects of the pioneering research of James Croll (1821–1890) into the impact of the Earth's orbital dynamics upon climate change, I had not come across this remarkable personality. Many aspects of his research are outside my astrophysical 'comfort zone', but there are three reasons why I agreed to contribute this essay. The first is that, as a native of Dundee, Perth, Croll's home town, was an integral part of my upbringing and I have happy memories of trips there in my youth. Second, I have an interest in the mathematics of celestial mechanics, since it played an important part in the mathematics of the old quantum theory to which astronomers such as Karl Scharzschild made key contributions. Third, I was a good friend of the late Nick Shackleton who contributed so much to establishing the correctness of Croll's insights.

Details of Croll's life and work are described by others in this bicentenary celebration (see also Irons 1896; Imbrie & Imbrie 1979). Always in poor health, he had an inauspicious early career. His scientific interests were stimulated, however, by his appointment in 1859 as janitor at the Andersonian College and Museum, Glasgow. Croll was a self-taught scientist, publishing his pioneering paper *The Physical Cause of the Change of Climate during the Glacial Epoch* in 1864. His research endeavours were strongly supported by Archibald Geikie (1835–1924), the Director of the Geological Survey of Scotland in Edinburgh, where in 1867 Croll was appointed to the post of keeper of maps and correspondence. In 1875, he published his influential book '*Climate and Time in their Geological Relations: a Theory of Secular Changes of the Earth's Climate*' (Croll 1875). The distinction of his research led to the award of an honorary degree by St Andrews University in 1876 and to election to fellowship of the Royal Society of London in the same year. He retired at the age of 59 due to ill health and died in Perth in 1890.

1. Celestial mechanics from the Hipparchus to Le Verrier

Celestial mechanics has an ancient and distinguished history. It was continually refined over the centuries as the most precise of the physical sciences, until the development of modern physics as we know it today following the Galilean and Newtonian revolutions. Prime motivations for these endeavours were originally to keep track of time so that religious festivals could be celebrated on the correct dates and to maintain a track of time and latitude for the purposes of navigation, which will prove to be a significant aspect of this story.

1.1. Ancient history

The great Greek astronomer Hipparchus compiled his famous catalogue of the positions of 850 stars in the northern sky in 127 BC and this work was included in Ptolemy's 13 volume *Great Composition*, known as the *Almagest* in the 2nd Century AD. Hipparchus discovered the precession of Earth's axis of rotation, which we now know is caused by the gravitational influence of the Sun and the Moon acting on Earth's equatorial bulge – the Earth is not a perfect sphere but an oblate spheroid, with an equatorial diameter about 43 km greater than its polar diameter (Pedersen 1993).

During the period 1602–1618, Johannes Kepler (1571–1630) discovered his three laws of planetary motion, establishing that the orbits of the planets are elliptical with the Sun in one focus

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and, crucially, according to the third law, that the period of a planet's orbit about the Sun is proportional to the three-halves power of its mean distance from the Sun. Famously, while working at home at Woolsthorpe, Lincolnshire, during the great Plague of 1665, Isaac Newton (1643–1727) used the third law to derive the inverse square law of gravity, assuming the planetary orbits are circular. In this analysis, he demonstrated that the same force which causes apples to fall towards the centre of the Earth is the same as that which holds the planets in their orbits about the Sun (see Case Study 1, Longair 2020).

Newton had reservations about the general validity of his discovery of the inverse square law of gravity for the following reasons:

- The orbits of the planets are ellipses, not circles.
- What is the influence of the other planets upon the Earth's orbit and on each other?
- Is it correct to place all the mass of the Earth at its centre, as he had assumed in his calculations of 1665?
- He could not account for the details of the Moon's motion.

By 1684, however, he had shown that the orbits of the planets are indeed ellipses under an inverse square law of gravity and that, for spherically symmetric planets, it is entirely correct to place all the mass at a point in its centre so far as celestial mechanics is concerned. But he gave only qualitative answers about the influence of the other planets and the details of the Moon's motion. With the encouragement of Edmund Halley (1656–1742), these results were published in 1687 in Newton's monumental *Principia Mathematica*, along with his three laws of motion or, as he called them more accurately, axioms (Newton 1687).

1.2. Perturbations of the motions of the planets

A central theme of celestial mechanics concerns the perturbations to the orbits and axes of rotation of the planets due the gravitational attraction of other celestial bodies, such as planets, satellites and comets. The understanding of the major gravitational perturbations was of practical importance for the purposes of navigation and for maintaining an accurate track of time. Perturbations to the elliptical orbits of the planets needed more advanced mathematical tools than Newton's laws in their original form. It was known that there is no general closed-form solution to the problem of three bodies interacting under gravity, the intractable three-body problem – the orbits are generally chaotic (Murray & Dermott 2000). The first generally accessible account of the nature of the three-body problem was provided by Leonhard Euler (1707–1783) in his 1749 prize solution for the 'theory of Jupiter and Saturn, explicating the inequalities that these planets appear to cause in each other's motions'.

In its most general form, the shape and orientation in space of a planet's elliptical orbit are described in terms of a planet's *orbital elements* – six of them are required. For the case of the Earth and the other planets, the orbits are taken to be ellipses assuming that the planets can be represented by point masses and in the absence of perturbations. The six elements are taken to be:

(1) the semi-major axis of the ellipse a;
(2) the eccentricity of the ellipse e;
(3) the inclination of the orbit to the plane of the ecliptic i;
(4) the longitude of the ascending node needed to fix a reference point on the Earth's orbit;
(5) the longitude of perihelion of the planet's orbit Ω;
(6) the time at which the planet passed perihelion T.

Superimposed upon these is the *obliquity*, or *axial tilt* ε, of the Earth's equatorial plane to the ecliptic plane, defined by the angle between these planes, with north defined by the right-hand rule. The *longitude of the ascending node* is the longitude at which the Earth's orbit crosses the equatorial plane, corresponding to the epoch of the vernal equinox, providing a reference point on the Earth's orbit about the Sun. The challenge to the celestial mechanicians was that all six orbital elements and the precession of the equinoxes of any planet are perturbed by the presence of all the other planets and bodies in the solar system. This proved to be a major mathematical challenge, but by the late 18th and 19th Centuries, major advances in analysis had been made with contributions from many of the greatest mathematicians, the majority belonging to the French and German schools of mathematical analysis. This can be appreciated from the French and German names of so many of the theorems and functions in analysis and calculus. From Germany, Gauss, Bessel, Jacobi, Dirichlet, Riemann, Kronecker, Weierstrass, Hilbert and Weyl; from France, d'Alembert, Lagrange, Laplace, Legendre, Fourier, Poisson, Cauchy, Liouville, Galois, Hermite, Poincaré, Hadamard and Lebesgue, among many others.

The deviations of the orbits of the planets from perfect ellipses are small and so perturbation theory is the obvious way of evaluating these changes. We will find that the ellipticity of the Earth's orbit e varies from about 0.02 to 0.078. Thus, to obtain accurate predictions about the small perturbations of the planetary orbits, it is convenient to expand the functions describing the orbits to high orders in the ellipticity, e, e^2, e^3, e^4 …. Euler showed that the eccentricities of the planetary orbits are invariant to second-order, e^2, in small quantities. The effects of the presence of the other planets were to be found to higher order in the eccentricity.

Building on the work of Euler, Joseph-Louis Lagrange (1736–1813), in his *Traité de Mécanique Analytique* (Lagrange 1788), transformed mechanics into a branch of mathematical analysis. In the Preface to the Treatise, he wrote: 'One will not find figures in this work. The methods that I expound require neither constructions, nor geometrical or mechanical arguments, but only algebraic operations, subject to a regular and uniform course' (Lagrange 1788, preface, page 1). In 1786, Pierre-Simon Laplace (1749–1827) had also proved that the eccentricities and inclinations of planetary orbits to each other always remain small, constant and self-correcting. These and many of his earlier results formed the basis of his great *Traité de Mécanique Céleste* (Laplace 1799–1825) published in five volumes, the first two in 1799. These contained an extended exposition of perturbation theory and its detailed application to the motion of the Moon and the planets. He demonstrated the need to extend the perturbations to high powers of the eccentricity e. He also resolved anomalies in the motions of Jupiter and Saturn as a result of a resonance between the 2:5 ratio of the periods of their planetary orbits.

In 1795 the French Bureau des Longitudes in Paris, charged with the improvement of nautical navigation, standardisation of time-keeping, geodesy and astronomical observation, was founded with Lagrange and Laplace as the mathematicians, or geometers, among its founding members. Laplace subsequently became head of the Bureau and the Paris Observatory, which was to be responsible for producing formulae and predictions of the perturbations to the motion of the planets, which were to be used by Croll. The reason for the founding of the Bureau was entirely political, to counter the domination of the UK in mastery of the seas and its overseas territories.

William Rowan Hamilton (1805–1865), professor of astronomy at Trinity College, Dublin, Director of the Dunsink Observatory and Astronomer Royal for Ireland, presented his equations in the *Philosophical Transactions of the Royal Society* in 1834 and 1835 (Hamilton 1834, 1835). This developed into the theory of Hamiltonian mechanics and dynamics, in particular, into a pair for first-order dynamical equations replacing the

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second-order equations of the Euler–Lagrange approach:

$$\dot{q}_i = [H,\ q_i] \quad \dot{p}_i = [H,\ p_i]$$

H is the Hamiltonian and q_i and p_i the canonically conjugate spatial and momentum coordinates. The square brackets are Poisson brackets. Whereas the Euler–Lagrange equations require the integration of second-order differential equations, both Hamiltonian equations, being first order, are mathematically simpler. Note that these great papers of Hamilton's came out of his studies of the perturbations of the planetary orbits, part of his responsibilities as Astronomer Royal for Ireland.

In 1834, Carl Jacobi (1804–1851) continued the development of Hamiltonian mechanics, showing how to transform the Hamiltonian formalism from one system of coordinates to another. The resulting *Hamilton–Jacobi equations* were applied to celestial mechanics in 1834. Consequently, coordinate systems that were the most convenient for astronomical orbital problems could be chosen – for example, the use of parabolic coordinates to study perturbations of the planetary orbits. This was, however, distinctly higher mathematics and at the forefront of mathematical innovation.

2. Urbain Le Verrier

Urbain Le Verrier (1811–1877), a central figure in Croll's scientific work, worked at the Paris Observatory for most of his life. In 1845, François Arago (1786–1853), Director of the Paris Observatory, asked him to study the irregularities in the orbit of Uranus. In June 1846 he proved that the irregularities could not result from perturbations by the known planets and postulated that there must be an undiscovered planet beyond the orbit of Uranus. By August 1846, he predicted the orbit of the hypothetical planet.

On 18 September 1846, Le Verrier wrote to Johann Galle (1812–1910) at the Berlin Observatory asking him to search for the planet at its predicted position. Galle stated that it was: '… so novel a thing to undertake observations in reliance upon merely theoretical deductions; and that, while much labour was certain, success appeared very doubtful' (Robertson & O'Connor 2021). Galle searched for the new planet on the same evening he received Le Verrier's letter. He found it that night within one degree of the predicted position. He reobserved it on the following night to be absolutely certain that it was a planet, writing to Le Verrier: 'The Planet whose position you indicated *really exists*' (Robertson & O'Connor 2021).

As one of Le Verrier's colleagues wrote: '… he discovered a star with the tip of his pen, without any instruments other than the strength of his calculations alone' (Robertson & O'Connor 2021). Arago stated: 'In the eyes of all impartial men, this discovery will remain one of the most magnificent triumphs of theoretical astronomy, one of the glories of the Académie and one of the most beautiful distinctions of our country' (Robertson & O'Connor 2021). The discovery of the planet, which was named Neptune, was a magnificent achievement for the science of celestial mechanics, proving the correctness of the application of advanced mathematics to physical problems. This triumph of mathematical analysis helped make the public and governments aware of the importance of scientific research.

In 1847, Le Verrier began the development of the huge project 'of embracing in a single work the whole of the planetary system.' The aim was to construct the theoretical apparatus and tables for the motions of the planets and the determination of their masses taking into account all their mutual gravitational interactions. This work, which he did not complete until a month before his death in 1877, occupies more than 4000 pages of the *Annales de l'Observatoire de Paris.*

Le Verrier began by establishing the expansion of the perturbing forces in their most general form. The expansions extended to the seventh power of the eccentricities, e^7, and inclinations and included 469 terms dependent on 154 special functions. This work was the basis for his later investigations. He then turned to the first four planets, from Mercury to Mars, the theory and tables of which he completed in 1861. Le Verrier's early work on this project, as it was published, was used by Croll (1864, 1875).

In 1854, Le Verrier became Director of the Paris Observatory which, under Arago's leadership, had not performed to the standards that Le Verrier expected. On taking over as director, he immediately insisted on strict discipline, his drive for efficiency making him very unpopular. Staff could only work on the projects assigned to them and all publications went out under the name of the Director. The new regime was strongly opposed by the staff, almost all of whom resigned. As stated by a contemporary: 'I do not know whether M Le Verrier is actually the most detestable man in France, but I am quite certain that he is the most detested' (Robertson & O'Connor 2021). These unhappy circumstances are reflected in the title of James Lequeux's biography: *Le Verrier: Magnificent and Detestable Astronomer* (Lequeux 2015).

Le Verrier hoped that his work would lead to further planetary discoveries analogous to that of Neptune. He already knew by 1843 that the observed motion of Mercury did not agree with theory. In 1859, he showed that the perihelion of Mercury advances at a rate of about 43 arcsec per century, once the perturbing effects of the known planets were taken into account. He suggested that this might be due to the presence of a planet, Vulcan, within the orbit of Mercury. Despite searches, no such planet was observed. No satisfactory explanation was found until 1915, when Albert Einstein showed that, according to general relativity, the perihelion of Mercury should advance at a rate of about 43 arcsec per century because of the distortion of spacetime by the Sun. Le Verrier's analysis turned out to be one of the key observations establishing the correctness of Einstein's general theory of relativity.

3. Croll on Le Verrier

By the time Croll began his studies of the influence of perturbations of the Earth's motion because of the presence of the other planets, the first results of Le Verrier's programme had become available and these were the basis for Croll's research. His motivation was to use the inferred past history of the Earth's motion as a means of providing secure dates for the geological history of the Earth through the dating of the ice ages. In his pioneering works of 1864 and 1867, he described the importance of both the changing ellipticity of the Earth's orbit and the precession of the equinoxes as contributors to climate change and the origin of the ice ages (Croll 1864, 1867a, b). His aims are summarised in the introduction to Chapter 19 of his influential book of 1875, '*Climate and Time in their Geological Relations: a Theory of Secular Changes of the Earth's Climate*' (Croll 1875). Many more details are provided by the other essays in this volume. The introductory paragraph reads as follows:

> GEOLOGICAL TIME — PROBABLE DATE OF THE GLACIAL EPOCH.
>
> If those great Secular variations of climate which we have been considering be indirectly the result of changes in the eccentricity of the earth's orbit, then we have a means of determining, at least so far as regards recent epochs, when these variations took place. If the glacial epoch be due to the causes assigned, we have a means of ascertaining, with tolerable accuracy, not merely the date of its commencement, but the length of its duration.

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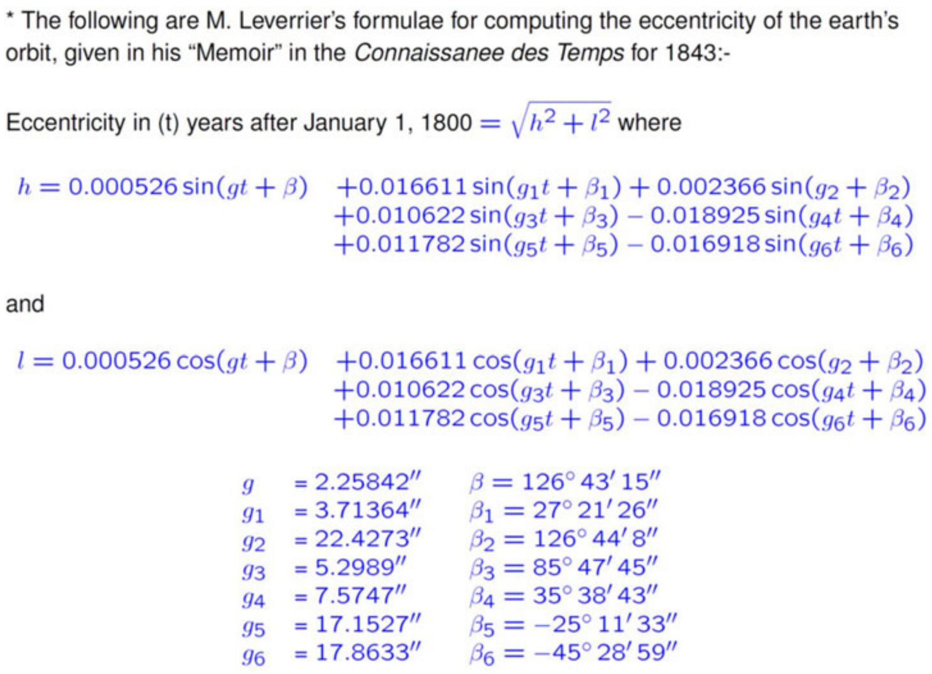

* The following are M. Leverrier's formulae for computing the eccentricity of the earth's orbit, given in his "Memoir" in the *Connaissanee des Temps* for 1843:-

Eccentricity in (t) years after January 1, 1800 = $\sqrt{h^2 + l^2}$ where

$$\begin{aligned} h = 0.000526\sin(gt+\beta) \quad & +0.016611\sin(g_1 t+\beta_1) + 0.002366\sin(g_2+\beta_2) \\ & +0.010622\sin(g_3 t+\beta_3) - 0.018925\sin(g_4 t+\beta_4) \\ & +0.011782\sin(g_5 t+\beta_5) - 0.016918\sin(g_6 t+\beta_6) \end{aligned}$$

and

$$\begin{aligned} l = 0.000526\cos(gt+\beta) \quad & +0.016611\cos(g_1 t+\beta_1) + 0.002366\cos(g_2+\beta_2) \\ & +0.010622\cos(g_3 t+\beta_3) - 0.018925\cos(g_4 t+\beta_4) \\ & +0.011782\cos(g_5 t+\beta_5) - 0.016918\cos(g_6 t+\beta_6) \end{aligned}$$

$g = 2.25842''$ $\quad \beta = 126^\circ 43' 15''$
$g_1 = 3.71364''$ $\quad \beta_1 = 27^\circ 21' 26''$
$g_2 = 22.4273''$ $\quad \beta_2 = 126^\circ 44' 8''$
$g_3 = 5.2989''$ $\quad \beta_3 = 85^\circ 47' 45''$
$g_4 = 7.5747''$ $\quad \beta_4 = 35^\circ 38' 43''$
$g_5 = 17.1527''$ $\quad \beta_5 = -25^\circ 11' 33''$
$g_6 = 17.8633''$ $\quad \beta_6 = -45^\circ 28' 59''$

Figure 1 Le Verrier's expressions for the time-evolution of the eccentricity of the Earth's orbit about the Sun.

> M. Leverrier has not only determined the superior limit of the eccentricity of the earth's orbit, but has also given formulae by means of which the extent of the eccentricity for any period, past or future, may be computed.

Croll used Le Verrier's expression for the variation of the eccentricity of the Earth's orbit about the Sun due to the gravitational perturbations of the planets and includes it as a footnote in his treatise (Croll 1875, p. 312). He used the expressions contained in Le Verrier's Memoir '*Connaisance des Temps*' of 1843 (Le Verrier 1843) (Fig. 1).

Croll used the expression in Fig. 1 to work out the values of the eccentricity of the Earth's orbit every 50,000 years from −3 M years before the present era to 1 M years into the future. The period from −700,000 years to the present is compared with present estimates in Fig. 2. The agreement is remarkably good, attesting to the quality of Le Verrier's and Croll's analyses.

Croll drew particular attention to epochs when the ellipticity of the Earth's orbit had extended maxima at epochs −2.5 M and −1.85 M years in the past and about 1 M years in the future. These were identified as periods when the ice ages occurred, and will occur in the future. Quoting Fleming (2006, p. 37),

> Croll's theory of ice ages took into account both the precession of the equinoxes and variations in the shape of the earth's orbit. It predicted that one hemisphere or the other would experience an ice age whenever two conditions occur simultaneously: a markedly elongated orbit, and a winter solstice that occurs far from the sun. … He assumed only changes in solar insolation were needed, as controlled by the well-established variations in orbital eccentricity and precession of the equinoxes. Later he added changes in the obliquity of the ecliptic. These cosmical factors provided a mechanism for multiple glacial epochs and alternating cold and warm periods in each hemisphere.

This model is presented in the frontispiece to Croll's book (Croll 1875) (Fig. 3). Le Verrier had shown that the maximum eccentricity of the Earth's orbit was 0.07775 and this was the value used by Croll on the coloured frontispiece and also as the upper bound of the eccentricity in Fig. 2. During the period considered by Croll, an eccentricity of 0.0722 was inferred at the epoch −2.4 M years before the present epoch, not much less than the maximum value. At maximum eccentricity, there is a 17% change in the distance from the Sun from perihelion to aphelion, corresponding to a 36% change in insolation.

This is only the beginning of a complex story. Croll fully appreciated that this is only one part of a whole series of climatic interactions between variations in the heat input to the atmosphere and the resulting climatic phenomena. Feedback mechanisms include:

- radiative effects of the ice fields;
- enhanced formation of cloud and fog;
- changes in sea level and the mixing and redirection of warm and cold ocean currents.

These would enhance the climatic changes initiated by small changes in the Earth's orbit and inclination. As Croll wrote:

> The cause of secular changes of climate is the deflection of ocean currents, owing to the physical consequences of a high degree of eccentricity in the earth's orbit … glacial cycles may not arise directly from cosmical causes, they may do so indirectly! (Fleming 2006, p. 47).

These were controversial claims at the time, and it was not until the revival of interest in these topics by Milutin Milanković in 1930 that the theory began to be taken seriously again. The reasons for the eclipse of Croll's theory are summarised by Fleming (2006, p. 50): 'However, because of uncertainties in the timing of ice ages and in the stratigraphic record, and because Croll's theory predicted glaciation in only one hemisphere, the theory was largely disregarded for at least three decades.' The papers in this volume provide many more details of the convoluted history of the reception of Croll's pioneering research, which influenced the thinking of a number of the most distinguished scientists of the age involved in these studies, including Charles Lyell, John Herschel, Charles Darwin and the brothers Archibald and James Geikie.

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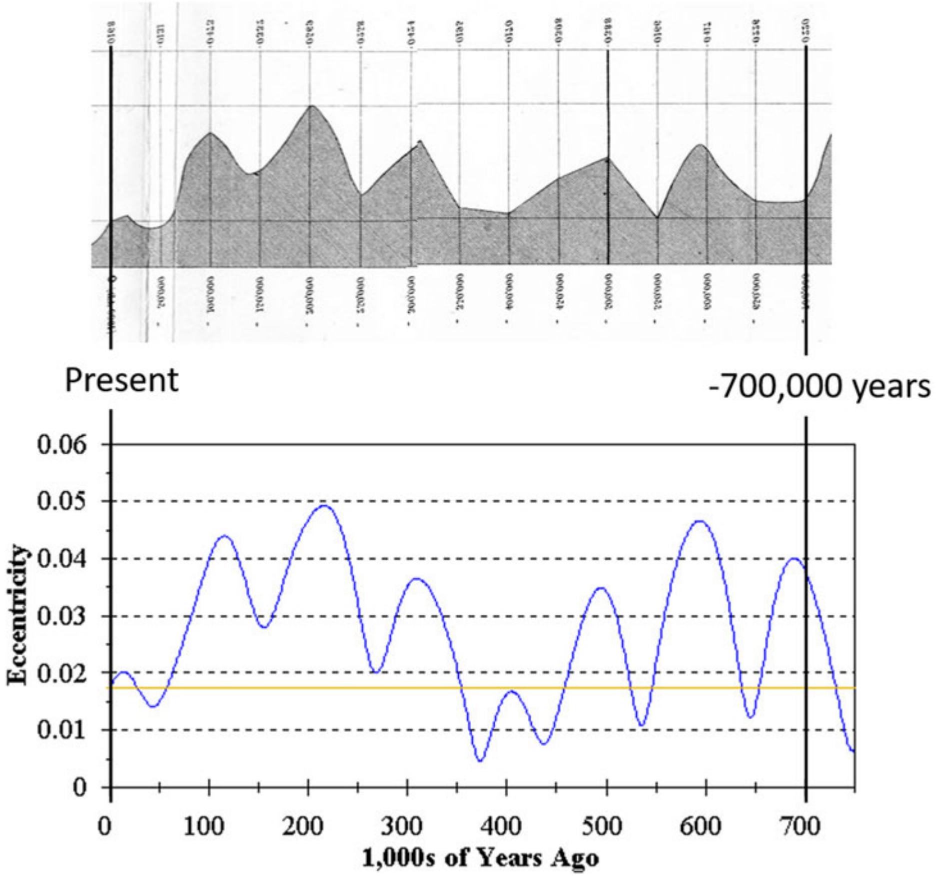

Figure 2 Comparison of Croll's estimate of the variation of the ellipticity of the Earth's orbit about the Sun according to the expression in Fig. 1 (top) with present estimates (bottom) for the last 700,000 years. To make the comparison, Croll's graph has been flipped about the vertical axis. The vertical lines show the present epoch and 700,000 years in the past. (Source: Incredio, courtesy of creative commons: https://commons.wikimedia.org/w/index.php?curid=6930545.)

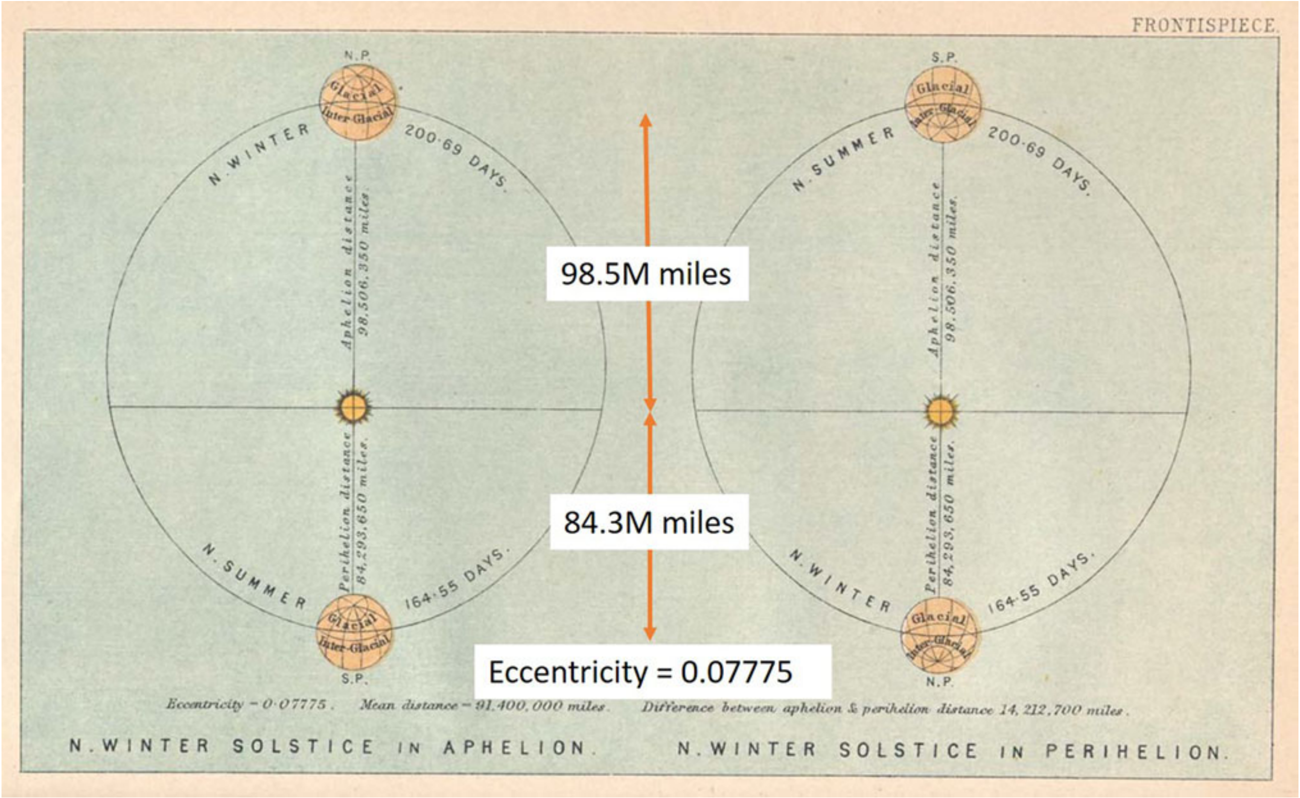

Figure 3 The frontispiece of Croll's book, *Climate and Time in their Geological Relations: a Theory of Secular Changes of the Earth's Climate* (1875). The list of figures in Croll's book states 'Earth's orbit when eccentricity is at its superior limit'. These values refer to the maximum ellipticity predicted by Le Verrier (see Croll's book). The dynamical parameters have been superimposed on the diagram for clarity.

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I conclude this section with a tribute to my colleague Nick Shackleton whose pioneering ice core analyses of 1976 with J. D. Hays and John Imbrie showed convincingly the impact of variations in the Earth's orbital eccentricity, its obliquity and the precession of the perihelion upon climate change in their paper, '*Variations in the Earth's orbit: pacemaker of the ice ages*' (Hays *et al*. 1976). Using Fourier techniques, they showed that the three periodicities with which the Earth's orbit change (100,000 years, 40,000 years and 21,000 years) are all present in the temperature, isotopic and fossil records. Figure 4 illustrates the various perturbations of the Earth's dynamics and their relative amplitudes, as well as recent measurements of indicators of climate change such as carbon dioxide concentration and global temperature measures.

4. The ages of the Sun and Earth

In the second half of the 19th Century, astronomical and geological estimates of the age of the Earth came into conflict (Longair 2006). Geological estimates of the age of the Earth were made by James Hutton (1726–1797) in 1785 and Charles Lyell (1797–1875) in 1830 based on geological evidence. Hutton claimed an age of many millions of years based on erosion and sedimentation rates, while Lyell, a good friend of Croll's, suggested at least 500 million years from sedimentation rates in the oceans. But Croll was cautious. As he wrote in his comprehensive book of 1875,

> At present geological estimates of time are little else than conjectures. Geological science has hitherto afforded no means of estimating the positive length of geological

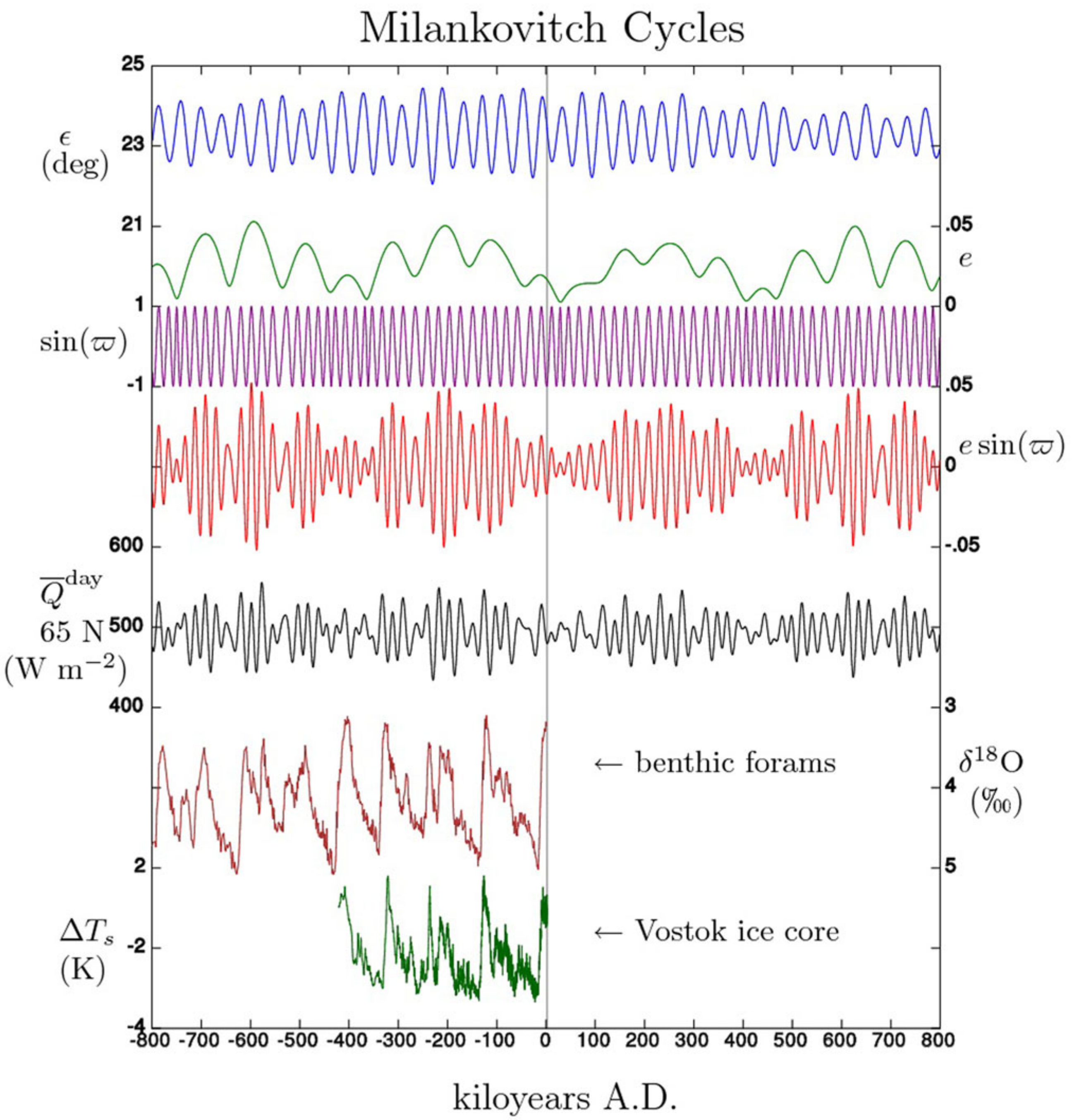

Figure 4 Comparison of the variations of orbital parameters with various measures of the global temperature for the epochs −800 to +800 Kyears. From the top, the seven graphs show: ϵ the obliquity of the Earth's orbit; e the eccentricity of the Earth's orbit; Ω the precession of the perihelion; the combined impact of the variations of the ellipticity and the precession of the perihelion; the mean daily insolation; a measure of the atmospheric mean carbon dioxide variations in the Earth's atmosphere; and the mean global temperature variations. (Source: Incredio, courtesy of creative commons: https://commons.wikimedia.org/w/index.php?curid=6930545.)

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> epochs. Geological phenomena tell us most emphatically these periods must be long but how long they have failed to inform us. (Croll 1875, p. 326)

The origins of the theory of stellar structure and evolution can be traced to the understanding of the first law of thermodynamics in the early 1850s by Rudolph Clausius (1822–1888) and William Thomson, later Lord Kelvin (1824–1907), ‘energy is conserved when heat is taken into account’. Applying the law to the stars, Hermann von Helmholtz (1821–1894) and Thomson proposed that the contraction of the Sun itself would provide an enormous reservoir of potential gravitational energy. The Sun was conceived of as a liquid sphere which gradually contracted and cooled. Energy transport through the Sun was assumed to be by convection.

Kelvin and Helmholtz could then estimate the age of the Sun since its present luminosity is known and its gravitational potential energy can be estimated. This *Kelvin–Helmholtz time-scale* t_{KH} for cooling of the Sun can be estimated by dividing its gravitational potential energy by its present luminosity L,

$$t_{\mathrm{KH}} \approx \frac{E}{L} \approx \frac{GM^2}{LR}$$

where M is its mass and R its radius. For the Sun, this time-scale is about 10^7 years. Thomson used heat flow arguments to estimate the age of the Earth. He knew the temperature gradient in the outer layers of the Earth and estimated how long it would take the Earth to cool by thermal conduction through its surface. He found an age of 20–40 million years – not so different from the Kelvin–Helmholtz time-scale for the Sun.

The astrophysical estimates were considerably shorter than those favoured by the geologists who suggested that the age of the Earth was greater than 100 million years, over 500 million years according to Lyell, but these estimates were subject to some uncertainty, as noted by Croll. Kelvin's changing estimates of age of the Sun and the Earth are set out in some detail by Smith & Wise (1989) in their biography of William Thomson, *Energy and Empire: A Biographical Study*, eventually settling down to values of about 20 to 40 million years.

Everything changed with the discovery of radioactivity in 1895 by Henri Becquerel (1852–1908). Here was a continuing source of energy for the interior of the Earth and the Sun and so the cooling estimates could no longer be maintained. Even more remarkably, in 1904, Ernest Rutherford (1871–1937) used the relative abundances of long-lived radioactive and stable isotopes of heavy elements such as uranium to estimate the age of the Earth. His estimate was at least 700 million years, much to the relief of the geologists. Rutherford announced his first results in 1904 in his Friday Evening Discourse at the Royal Institution. In his reminiscences, he remembers:

> I came into the room which was half dark and presently spotted Lord Kelvin in the audience and realised that I was in for trouble with the last part of my speech dealing with the age of the Earth where my views conflicted with his. To my relief, Kelvin fell asleep, but as I came to the important point, I saw the old bird sit up, open an eye and cock a baleful glance at me! Then a sudden inspiration came and I said Lord Kelvin has limited the age of the Earth, provided no new source was discovered. That prophetic utterance refers to what we are now considering tonight, radium! Behold! the old boy beamed upon me (Eve 1939, p. 107).

The first person to investigate the internal structure of the Sun as a gaseous, rather than a liquid, body was J. Homer Lane (1819–1880). During the American Civil War, he made the first calculations to determine whether or not the Sun's surface properties were consistent with it being considered to be a sphere of a perfect gas. In 1869, he reversed the calculations and, assuming the material of the Sun to be a perfect gas, attempted to reproduce its surface properties and to determine the variation of its density, temperature and pressure with radius. This programme was not particularly successful. Nonetheless, he was the first person to adopt the correct equations of hydrostatic equilibrium and mass conservation for the stars.

In the late 1870s, similar calculations were carried out by Augustus Ritter (1826–1908) at Aachen, who identified the initial phase of evolution of the star as the contraction of a perfect gas sphere, which then cooled according to the Kelvin prescription. The culmination of these early physical models for stars was the treatise by Robert Emden *Gaskugeln*, published in 1907. Emden (1826–1908) went further than Lane and Ritter by introducing polytropic solutions, which allowed a much wider range of stellar models to be investigated. The equation of state of the gas was assumed to be of power-law form $p = \kappa\rho^\gamma$. Emden showed that the solutions result in stars with a boundary at a finite radius. These theories did not gain the recognition they deserved because astronomers argued that the physical conditions inside the stars and the process of energy generation were unknown.

The solution of the problems of understanding the energy source of the Sun had to await the possibility of nuclear energy generation following Einstein's discovery of the mass-energy identity $E = mc^2$ and accurate estimates of the mass deficit in the conversion of hydrogen to helium, proposed by Eddington (1882–1944) in 1920. This process ensured that the lifetime of the Sun would be about 10^{10} years. Likewise, internal heat sources in the Earth removed the second of Kelvin's arguments. In a letter to me in 1993, William McCrea (1904–1999) wrote:

> (People) don't realise that before, say, 1916 astronomers simply had no idea what the inside of a star was like, and had no idea how to find out anything about this. The speed at which Eddington transformed the situation was incredible.

5. Conclusion

There can be no doubt about the pioneering nature of Croll's research. I have concentrated upon the astronomical aspects of his contributions, but it is apparent that the care he took in evaluating the perturbations of the Earth's orbit applied equally to his geological, atmospheric and glaciological studies. Given the nonlinearity and complexity of understanding the relation between the changes of the Earth's orbit and climate change, it is not surprising that these were controversial areas, and the data were not really available during Croll's lifetime to advance his insights.

Croll is not alone in being far ahead of this time for the depths of his insights to be fully appreciated. It is inspiring to see his hard life and pioneering work attracting much greater appreciation, demonstrating how inspiration and persistence can overcome significant adversity.

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