embeds into the Turing degrees. We will show that in a certain extension of ZF (which is incompatible with ZFC), this holds for all partial orders of height two, but *not* for all partial orders of height three. Our proof also yields an analogous result for Borel partial orders and Borel embeddings in ZF, which shows that the Borel version of Sacks' question has a negative answer.

We will end the thesis with a list of open questions related to Martin's conjecture, which we hope will stimulate further research.

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Justin Miller, ***Intrinsic density, asymptotic computability, and stochasticity***, University of Notre Dame, Notre Dame, IN, USA, 2021. Supervised by Peter Cholak. MSC: Primary 03D32, 03D30. Keywords: Intrinsic density, asymptotic computation, stochasticity, randomness.

Abstract

There are many computational problems which are generally "easy" to solve but have certain rare examples which are much more difficult to solve. One approach to studying these problems is to ignore the difficult edge cases. Asymptotic computability is one of the formal tools that uses this approach to study these problems. Asymptotically computable sets can be thought of as almost computable sets, however every set is computationally equivalent to an almost computable set. Intrinsic density was introduced as a way to get around this unsettling fact, and which will be our main focus.

Of particular interest for the first half of this dissertation are the intrinsically small sets, the sets of intrinsic density 0. While the bulk of the existing work concerning intrinsic density was focused on these sets, there were still many questions left unanswered. The first half of this dissertation answers some of these questions. We proved some useful closure properties for the intrinsically small sets and applied them to prove separations for the intrinsic variants of asymptotic computability. We also completely separated hyperimmunity and intrinsic smallness in the Turing degrees and resolved some open questions regarding the relativization of intrinsic density.

For the second half of this dissertation, we turned our attention to the study of intermediate intrinsic density. We developed a calculus using noncomputable coding operations to construct examples of sets with intermediate intrinsic density. For almost all $r \in (0,1)$, this construction yielded the first known example of a set with intrinsic density r which cannot compute a set random with respect to the r-Bernoulli measure. Motivated by the fact that intrinsic density coincides with the notion of injection stochasticity, we applied these techniques to study the structure of the more well-known notion of MWC-stochasticity.

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Cheng Peng, ***On Transfinite Levels of the Ershov Hierarchy***, National University of Singapore, Singapore, 2018. Supervised by Yue Yang. MSC: 03D28, 03D55. Keywords: Turing degree, Ershov hierarchy.

Abstract

In this thesis, we study Turing degrees in the context of classical recursion theory. What we are interested in is the partially ordered structures $\mathcal{D}_\alpha$ for ordinals $\alpha < \omega^2$ and $\mathcal{D}_a$ for notations $a \in \mathcal{O}$ with $|a|_o \geq \omega^2$.

The dissertation is motivated by the Σ_1-elementary substructure problem: Can one structure in the following structures $\mathcal{R} \subsetneqq \mathcal{D}_2 \subsetneqq \cdots \subsetneqq \mathcal{D}_\omega \subsetneqq \mathcal{D}_{\omega+1} \subsetneqq \cdots \subsetneqq \mathcal{D}(\leq \mathbf{0}')$ be a Σ_1-elementary substructure of another? For finite levels of the Ershov hierarchy, Cai, Shore, and Slaman [***Journal of Mathematical Logic***, vol. 12 (2012), p. 1250005] showed that $\mathcal{D}_n \not\preceq_1 \mathcal{D}_m$ for any $n < m$. We consider the problem for transfinite levels of the Ershov hierarchy and show that $\mathcal{D}_\omega \not\preceq_1 \mathcal{D}_{\omega+1}$. The techniques in Chapters 2 and 3 are motivated by two remarkable theorems, Sacks Density Theorem and the d.r.e. Nondensity Theorem.

In Chapter 1, we first briefly review the background of the research areas involved in this thesis, and then review some basic definitions and classical theorems. We also summarize our results in Chapter 2 to Chapter 4. In Chapter 2, we show that for any ω-r.e. set D and r.e. set B with $D <_T B$, there is an $\omega+1$-r.e. set A such that $D <_T A <_T B$. In Chapter 3, we show that for some notation a with $|a|_o = \omega^2$, there is an incomplete $\omega+1$-r.e. set A such that there are no a-r.e. sets U with $A <_T U <_T K$. In Chapter 4, we generalize above results to higher levels (up to ε_0). We investigate Lachlan sets and minimal degrees on transfinite levels and show that for any notation a, there exists a Δ^0_2-set A such that A is of minimal degree and $A \not\equiv_T U$ for all a-r.e. sets U.

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Alejandro Poveda, ***Contributions to the Theory of Large Cardinals through the Method of Forcing***, Doctorate program in Mathematics and Computer Science—Universitat de Barcelona, Barcelona, Spain, 2020. Supervised by Joan Bagaria. MSC: Primary 03Exx, Secondary 03E50, 03E57. Keywords: Large Cardinals, Forcing, Prikry-type forcings, Singular Cardinal Combinatorics.

Abstract

The dissertation under comment is a contribution to the area of Set Theory concerned with the interactions between the method of Forcing and the so-called Large Cardinal axioms.

The dissertation is divided into two thematic blocks. In Block I we analyze the large-cardinal hierarchy between the first supercompact cardinal and Vopěnka's Principle (Part I). In turn, Block II is devoted to the investigation of some problems arising from Singular Cardinal Combinatorics (Part II and Part III).

We commence Part I by investigating the Identity Crisis phenomenon in the region comprised between the first supercompact cardinal and Vopěnka's Principle. As a result, we generalize Magidor's classical theorems [2] to this higher region of the large-cardinal hierarchy. Also, our analysis allows to settle all the questions that were left open in [1]. Finally, we conclude Part I by presenting a general theory of preservation of $C^{(n)}$-extendible cardinals under class forcing iterations. From this analysis we derive several applications. For instance, our arguments are used to show that an extendible cardinal is consistent with "$(\lambda^{+\omega})^{\mathrm{HOD}} < \lambda^+$, for every regular cardinal λ." In particular, if Woodin's HOD Conjecture holds, and therefore it is provable in ZFC + "There exists an extendible cardinal" that above the first extendible cardinal every singular cardinal λ is singular in HOD and $(\lambda^+)^{\mathrm{HOD}} = \lambda^+$, there may still be no agreement at all between V and HOD about successors of regular cardinals.

In Part II and Part III we analyse the relationship between the Singular Cardinal Hypothesis (SCH) with other relevant combinatorial principles at the level of successors of singular cardinals. Two of these are the Tree Property and the Reflection of Stationary sets, which are central in Infinite Combinatorics.

Specifically, Part II is devoted to prove the consistency of the Tree Property at both κ^+ and κ^{++}, whenever κ is a strong limit singular cardinal witnessing an arbitrary failure of the