mathematical models, and presumably volume 2 will in due course be more directed to applications. Meanwhile, the material on the atmosphere, on rainbows and on radar can be used for group or individual investigations on a rather smaller scale than the Eyam material, though some skill would be needed to avoid mere replication. The book is largely well-written (despite an odd preference for "shall" over "will") and is very attractively presented with few typos that I noticed.

I thoroughly agree with the view that potential physicists should be given open-ended tasks that develop their enthusiasm and intellectual curiosity, and not just be restricted to the content of examination syllabus. French shows conclusively that it can be done, and provides the material and knowledge that would otherwise be stumbling blocks. All school physics or mathematics department with any interest in real education within their subjects should have a copy of this book—and, perhaps above all, those who think they can't do it.

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The one true logic: a monist manifesto, by Owen Griffiths and A. C. Paseau, pp 232, ISBN 978-0-19-882971-3, Oxford University Press (2022).

This book has been written by philosophers for philosophers. As a mathematician with a long-standing interest in mathematical logic, I found the philosophy hard going. I fear that as a result my attention flagged, and as a result the criticisms I make below may reflect my failure to understand what the authors were saying, rather than defects of the book itself.

The authors' aim is to present a logic which captures the semantic consequence relation of *cleaned up* natural languages. They say that a cleaned up language is one 'purged of ambiguity' but nothing more precise than this. This left me wondering whether self-referential sentences such as 'this sentence is false' are allowed in a cleaned up language and, if so, how it would be dealt with in their system of logic.

For the authors a *true logic* consists of a language ℓ and a formalization process which translates a statement s of the cleaned up natural language into a sentence $Form_\ell(s)$ of ℓ so that for all statements s and all sets S of statements, s is a logical consequence of S, if, and only if, $Form_\ell(S) \vDash_\ell Form_\ell(s)$, where $Form_\ell(S) = \{Form_\ell(s) : s \in S\}$.

Here $\vDash_\ell$ is the semantic consequence relation of model theory introduced by Tarski:

$\Sigma \vDash_\ell \sigma$ if, and only if, every model of the set of sentences Σ is also a model of σ. A model of a sentence is a structure which provides an interpretation of the non-logical symbols in such a way that the sentence is true when the logical symbols are interpreted in the standard way.

For the authors of this book, 'the standard way' is as in classical two-valued logic. You might have expected in what is said to be a monist manifesto that detailed arguments would be given for rejecting alternatives to classical logic, and in particular intuitionistic logic. However these are dismissed rather quickly. A lot of weight is put on the argument that while many logicians have studied non-classical logics, 'virtually all logicians use a single (classical) logic in their metatheory'.

The language that the authors propose is an infinitary language, $\mathrm{FTT}_{\infty\infty}$, which has relation symbols of all finite types, and allows infinite disjunctions and quantifications of all cardinal lengths.

To be more precise, the language $\mathrm{FTT}_{\infty\infty}$ has variables X_ρ^t for each finite type t and each ordinal ρ. The atomic formulas are either of the form $X_\rho^t = X_\sigma^t$ or $X_\rho^{(t_1,\ldots,t_n)}\left(X_{\rho_1}^{t_1}, \ldots, X_{\rho_n}^{t_n}\right)$.

If ϕ is a formula, then so also is $\neg\phi$. If κ is a cardinal, and for each ordinal $\rho < \kappa$, ϕ_ρ is a formula, then $\bigvee_{\rho<\kappa} \phi_\rho$ is a formula (with infinite conjunctions defined in terms of negation and disjunction in the usual way). Finally, if κ is a cardinal, and for each ordinal $\rho < \kappa$, $X_r^{t_\rho}$ is a variable then $\exists\left\{X_\rho^{t_\rho} : \rho < \kappa\right\}\phi$ is a formula.

At first sight it is highly implausible that disjunctions and quantifications of any infinite cardinal length are needed to provide a logic adequate to cope with natural language arguments. The authors recognize that their proposal is very radical and give arguments to counter the obvious objections to their idea.

I was not convinced by their arguments. I don't have space to explain in full where I think the authors go astray, but I give two examples which criticize their argument from opposite directions.

As is well known, there is a long-standing philosophical tradition originating with Aristotle that while it is possible to consider *potentially* infinite objects, there is no coherent way to deal with *actual* infinities. For mathematicians this idea broke down in the nineteenth century when real numbers were conceived as being actually infinite objects (for example, Dedekind cuts or equivalence classes of Cauchy sequences) and Georg Cantor introduced his ideas about infinite sets. This abandonment of Aristotle's philosophy was not without controversy, especially as it led to paradoxes and the problem of the status of Cantor's Continuum Hypothesis which remains unresolved.

The authors dismiss objections to their proposal of a language whose formulas are actually infinite objects on the grounds that 'the standard contemporary conception of the infinite in mathematics is absolutist not potentialist'. Undoubtedly this is true of almost all mathematicians, but I would have expected to be given a philosophical defence of using absolute infinities rather than a pragmatic argument based on current mathematical practice. The authors admit that mathematicians use finitary logic in their metatheory even when dealing with absolutely infinite sets. If the practice of mathematicians is used to defend the authors' use of classical logic and actually infinite sets, why is it ignored when it comes to proposing that the language $\mathrm{FTT}_{\infty\infty}$ is needed to analyze natural language arguments?

I also have an issue in the opposite direction. To help to justify the need for their infinitary language the authors conjure up *superhumans* 'who can accomplish infinite tasks in a finite amount of time', and *supersuperhumans* who could utter conjunctions of length κ even for $\kappa > \aleph_0$. However, since their language $\mathrm{FTT}_{\infty\infty}$ only allows for conjunctions of cardinal length, it would be insufficient for *supersupersuperhumans* who could accomplish a proper class of tasks in finite time, and who might therefore wish to consider such propositions as $\bigwedge_{\kappa \in \mathrm{Card}} \kappa < 2^\kappa$, where the conjunction is over the proper class of cardinals.

In the end, I was left wondering what the authors' proposal would accomplish even if the objections to it were considered invalid. I note that the authors themselves

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say on page 66:

'In response to the obvious follow up—*which* is the one true logic?—we have some good news and some bad news. The bad news first: we don't have an exact answer to the question. This book does not contain a systematic argument to show that some specific logic is the true one.'

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Mathematics is beautiful by Heinz Klaus Strick, pp. 366, £24.99 (paper), ISBN 978-3-662-62688-7, £19.99 (eBook) ISBN 978-3-662-62689-4, Springer Verlag (2021)

The subtitle of this book is 'Suggestions for people between 9 and 99 years to look at and explore'. The author has used the material of the book in his work as a schoolteacher and head teacher in Germany to 'loosen up my lessons', but the age span suggests a wider audience which might be attracted by the hundreds of excellent colour illustrations: there is hardly a page without at least one. Thus the choice of material is partly governed by its visual appeal, but there is plenty of algebra in evidence too so this is not a coffee-table book for casual reading. The level of mathematical maturity needed to follow the material varies greatly through the book and the author quotes without proof and uses some moderately hard theorems when they lead to interesting applications—for example 'Descartes' Theorem' about the curvatures (reciprocal radii) k_i of four circles each of which is tangent to the other three: $2\left(k_1^2 + k_2^2 + k_3^2 + k_4^2\right) = (k_1 + k_2 + k_3 + k_4)^2$. Each section concludes with 'suggestions for reflection and investigations' which take the material further or ask leading questions to enhance understanding. The references are nearly all to websites such as Wikipedia and Wolfram Mathworld, though a few printed sources are listed at the end of the book.

Here are a just a few of the topics covered in 17 chapters; I hope this gives an idea of the wide range of the material included. 'Stars' which are obtained from a regular n-sided polygon, by joining every kth vertex for a fixed k, are the basis for the first chapter, with a detailed treatment including the number of components of a star, the number of distinct stars for a given n, the lengths of edges, the total length of a star and the associated angles, regular n-sided figures in the complex plane, a discussion of de Moivre's theorem and solutions of $z^n = 1$. There is also an application to setting up tournament schedules where each team plays each of the others.

Dissection of rectangles into disjoint squares is linked to Fibonacci numbers, the Euclidean algorithm and continued fractions for rational numbers and for square roots of integers. A different chapter covers dissection of rectangles into squares of different sizes, including the famous minimal example of Duijvestijn in 1978 dissecting a square into 21 squares of different sizes. There is an opportunity here to use simultaneous linear equations to determine the sizes of the squares, given the dissection, and also an application to electrical circuits and Kirchhoff's laws.

There is a chapter on areas and perimeters of figures drawn on a lattice, first made up of lattice squares but then more generally, leading to Pick's theorem, that the area of a lattice polygon equals the number of interior lattice points plus half the number of boundary lattice points, minus 1. The theorem is proved in a sequence of elementary steps, more or less following a standard pattern. The next chapter is about rolling pairs of standard dice, constructing histograms for the sum of spots on