Proceedings of the Edinburgh Mathematical Society (1987) **30**, 325–339 ©

BOOK REVIEWS

CONWAY, J. H., CURTIS, R. T., NORTON, S. P., PARKER, R. A. and WILSON, R. A., ATLAS *of finite groups: maximal subgroups and ordinary characters for simple groups* (Oxford University Press, 1985), xxxiii + 252 pp., £35.

At last the monster task is complete and the ATLAS of finite simple groups has appeared in print! This book, which brings together a wealth of detail for the study of finite simple groups begins with eight chapters of introduction. The first three chapters are a survey of the finite simple groups; the classical groups; the Chevalley and twisted Chevalley groups. There then follow four chapters on how to read the ATLAS, the four chapters describing the subtopics of the "constructions" sections, information about subgroups and their structure, the map of an ATLAS character table and character tables. These chapters describe the uniform system of notation used in the tables which follow. The notation was given careful consideration by the authors. Finally there is a chapter of concluding remarks. A careful study of this introductory material is essential before proceeding to the tables.

The major part of the book consists of tables for the finite simple groups. These tables cover all of the sporadic simple groups. As far as the infinite families of groups are concerned the tables "go as far as a reasonable person would go, and then go a step further". In practice this means that there are, for example, tables for the groups $L_2(q)$, $q \leqq 32$; $L_3(q)$, $q \leqq 9$ and $U_3(q)$, $q \leqq 11$. A typical ATLAS consists of the following four parts: the order of G, its Schur multiplier and outer automorphism groups; various constructions for G, or related groups or concepts; the maximal subgroups of G and its automorphic extensions; the compound character table, from which the ordinary character tables for various extensions of G can be read off.

Although there are a few minor errors in the introductory chapters, the computational group theory "grapevine" would suggest that the necessary very high degree of accuracy in the tables has been attained. The book contains a valuable bibliography. Given the large page size it would be difficult to say that it is a must for the bookshelf. Nevertheless for the amount of information that it contains, for the reasonable price and for the care with which it has been written the ATLAS will prove an extremely interesting and useful purchase for any mathematician involved in the study of finite simple groups.

C. M. CAMPBELL

PRIESTLEY, H. A., *Introduction to complex analysis* (Oxford University Press, 1985), 197 pp., £8.50.

It is a daring venture to add to the plentiful supply of texts on this subject. Dr. Priestley sets out to cover the topics usually included in a first course, stating that "advanced and specialized topics have been ruthlessly excluded". One exception, however, is permitted: with an eye to the reader more interested in applications, she includes an introduction to Fourier and Laplace transforms.

The book is clearly written, with a pleasing style. The reader is well supplied with motivation and signposts. He is also warned to expect "brevity". At times, this is perhaps overdone: certain concepts, such as convexity and the length of a path, are accorded almost no discussion, and some students (at least outside Oxford) will find the handling of analytic details rather too abrupt. The author certainly presupposes successful completion of a first course on real analysis, but does not assume any prior knowledge of multivariate or metric space analysis.

My strongest reservations concern the handling of Cauchy's theorem. The reader is offered the theorem at two "levels", both depending on notions like homotopy, the Jordan curve theorem and triangulation of a polygon. These notions are presented hastily, with only outline proofs, in a way that gives the student no time to get used to them. It is unsatisfactory—and unnecessary—to have the central theorem resting on such shaky foundations. There is indeed a case for presenting Cauchy's theorem at different levels, but one of them should be a version that is free of dependence on these topological notions. As is by now widely recognised, the version involving a closed path in a star-shaped set satisfies this condition, and is sufficient for all the applications required in a first course.

There is a plentiful supply of exercises, and the technical presentation by Oxford University Press is superb.

G. J. O. JAMESON

ROSENBLUM, M. and ROVNYAK, J., *Hardy classes and operator theory* (Oxford University Press, USA, 1985), xii + 161 pp., £37.50.

This book treats a mature and substantial branch of classical function theory, centring on analytic functions on a disc or half-plane, using the insights and techniques of the theory of operators on Hilbert space. This particular conjunction of ideas has had a recent triumph with the proof of the Bieberbach conjecture by L. de Branges (the second-named author's sometime collaborator), but the present monograph addresses itself to quite other matters, barely touching on coefficient problems. The two main strands of the book are interpolation problems and factorization questions—for instance, does a given positive function on $\mathbb{R}$ admit a representation as the boundary value function of $|g|^2$ for some analytic function g in the upper half-plane? Such questions arise in a diversity of investigations of a "pure mathematical" nature, and they have been long studied. However, much of the impetus for present developments comes from applications. Factorization problems play a central role in prediction theory and in several areas requiring the solution of linear integral equations on a half-line, while variants of the interpolation problem for complex functions solved by Nevanlinna and Pick are highly topical in the design of control systems.

Classical studies concentrated on scalar analytic functions, whereas many of the applications deal in matrix- or operator-valued functions. Refinements of the old results are thus required, and operators are inescapably involved. However, even for scalar functions operator theory plays an important role: it is implicit in the early classical work. A bounded analytic function acts by multiplication on a suitable Hilbert space of analytic functions, and so is accessible to the highly developed concepts and techniques of operator theory. This approach does to some extent unify the theory, though it has to be used in conjunction with older methods: the book contains plenty of solid hard analysis.

Although the authors are aware of the applications, their own interest is in the fundamental underlying mathematical issues. They have made many contributions to the field themselves, and the choice of material in the book reflects the predilections evident in their research. They present a wealth of material from Nevanlinna–Pick and Loewner interpolation theory, the factorization of Toeplitz operators, Nevanlinna and Hardy classes of operator-valued functions, inner and outer functions and the factorization of operator functions. The style of the book is rigorous and economical to the point of terseness, and is marked by a consistent adherence to the strict logical progression from general to particular. Theorems are stated and proved in the fullest possible generality, with more concrete conclusions being relegated to somewhat hurried sections of "Examples and addenda". This policy allows the authors to cover a great deal of ground, but does not always bring out the aesthetic appeal of the results. It therefore places a burden on the reader's knowledge and experience and makes the book most suitable for a mathematician who is familiar with the scalar theory and wishes to learn about the analogue for operator-valued functions, rather than one who is trying to build up intuition from scratch. There is not much motivation, but there are historical notes pointing to a substantial bibliography.

There is one feature of the authors' treatment which, I believe, sets them apart from Schur, Nevanlinna and the many mathematicians who apply the theory: they place a strong emphasis on