# ON MAGNETIC MIXING AND ROTATION IN STELLAR EVOLUTION

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## 1. INTRODUCTION

In this paper some thoughts and problems are presented from the viewpoint that the evolution of stars may play a key role in generating magnetic fields which, in turn, may affect the mixing of nuclearly processed elements from the stellar interior to the surface. The relevant parameter is stellar rotation which, upon interaction with convective turbulence driven by thermal instabilities, leads to the generation of magnetic fields. A possible connection to Bidelman's hypothesis on the evolutionary status of Ap stars is also discussed in the context of a post-core-helium-flash hypothesis.

## 2. PROCESSING OF MAGNETIC FIELDS IN STELLAR EVOLUTION?

Historically, the assumption of magnetic flux conservation during stellar evolution has been the simplest so perhaps the most adequate approach to the problem. Despite theoretical complications, this view may eventually be abandoned because of increasing observational evidence for non-conservation of flux and steady advances in the theory of stellar dynamos (cf. Weiss, 1971). From the latter one infers that the fates of B-fields and rotation must be intertwined. Of fundamental importance in this respect is the existence of thermal instabilities associated with shell helium burning as shown by Schwarzschild and Härm (1965), and with nuclear ignitions in electron degenerate cores. The former has been recently emphasized by Levy and Rose (1974) who considered the B-fields for the problem of dissipating angular momentum from the main sequence to white dwarfs and neutron stars. In turn,

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the latter is inexorably connected with mass loss precesses which may sometimes proceed during the late stages of stellar evolution at rather high rates as shown by Weidemann (1977).

To estimate the strengths of the B-fields, one may adopt the approximate formula of Malkus (1959) which gives $B^2 \sim 4\pi\rho \langle v \rangle \Omega r$, for a density $\rho$, turbulent velocity $\langle v \rangle$, angular velocity $\Omega$ at distance r from the center. Assuming conservation of angular momentum in the radiative zones, one has $\Omega(r) = \Omega_o r_o^2/r^2$; for a double shell burning red giant $\Omega_o$ and $r_o$ would be referred to the inner edge of the envelope convective zone and r to the intershell convective zone (ICZ). Taking typical physical values from the shell-flash computations of Schwarzschild and Harm (1967, hereafter SH), one obtains $B^2 \sim \pi \times 10^{22} \Omega_o$ or $B \sim 2 \times 10^8$ G for $\Omega_o \sim 7 \times 10^{-7}$ s$^{-1}$ (~3 revolutions/yr) at $r_o \sim 3 \times 10^{11}$ cm and $\rho \sim 10^3$ gcm$^{-3}$, $\langle v \rangle \sim 5 \times 10^4$ cm s$^{-1}$ and $r \sim 2 \times 10^9$ cm for the ICZ of a double shell burning red giant. For thermal instabilities in electron degenerate cores one may expect even higher values, especially for cases of rapid core rotation.

An indirect argument can be based on a hypothesis of flux ejection or destruction during dynamical collapse. It is well known that during the collapse of a protostellar cloud the conservation of magnetic flux is violated by many orders of magnitude. The usual explanation is that significant flux loss must occur (cf. Mestel and Weiss 1974). Similarly, one may expect the dynamic collapse to the neutron star phase to lead to B-fields significantly greater than $10^5$-$10^8$G in the presupernova giant for pulsar fields of $10^{11}$-$10^{14}$G (cf. Glasser and Kaplan 1975, Sarazin and Bahcall 1977). In any case, one might expect an upper limit to the field from the requirement that the pressure be not very different in the presence and absence of B-fields, i.e., $B^2/8\pi \lesssim 10^{17}$-$10^{18}$ erg cm$^{-3}$, or $B \lesssim 1.5 \times 10^9$-$5 \times 10^9$G in the red giant preceeding the dynamical transition to the end phase of evolution. If one thinks of stellar B-fields being returned to the interstellar medium (cf. Michel and Yahil 1973), the above numbers indicate flux violation during dynamical ejection as well. In this case flux generation would have to occur if one ascribed a stellar origin to fields in the interstellar medium, particularly for fields in the Crab Nebula (cf Woltjer 1972).

Independently of any theories of dynamo generated B-fields, the observations of white dwarf (WD) fields in excess of $10^8$G (cf. Angel 1977, Tapia 1977) indicate that such fields must have been generated in the pre-WD red giant, not too long before the onset of the dynamical planetary nebula ejection, since otherwise such fields would have floated upward from the surface of the red giant core (cf. Parker 1974). Thus, the epoch of the onset of the final dynamical instability relative to the previous shell thermal instability may be responsible for the strength of the

B-field on the subsequent WD or neutron star. The small number of WD's with fields over $10^8$G might be consistent with this, if the B-fields in the red giant progenitor can move upward fast enough. Direct observations of fields of ~400 G on the Cepheid W Sgr by Wood and Weiss (1977) are consistent with an origin in strong interior fields that floated to the surface. The only way such high WD or redgiant fields could come from the main sequence via flux conservation is from Ap stars. While this possibility cannot yet be excluded, the converse inference that the fields of the magnetic main sequence stars (MSS) may have originated in interior stellar dynamos, much like the fields of the magnetic WD's and red giants, should be considered within the context of Bidelman's hypothesis discussed in Section 4.

Finally, the complicated effects of anomalous abundances in low-mass stars (cf. Dickens and Bell 1976, Bessell and Norris 1976, Auer and Demarque 1977, and references therein) suggest that a partial mixing mechanism is at work. In light of the difficulty of bringing nuclearly processed material to the surfaces of low-mass stars via ordinary convective theories (Sweigart 1974), the idea of transport by buoyant magnetic bubbles is an attractive one, especially if future studies will succeed in combining it with the complications of s-process nucleosynthesis recently investigated by Truran and Iben (1977).

## 3. THE MAGNETIC STIRRER APPROXIMATION

The author's interest in this subject was originally stimulated by the magnetostrictive force on the interface of media with different magnetic susceptibilities and two remarkable theorems by Woltjer (1958) and Cowling (cf. Ferraro and Plumpton 1966) on force-free fields. However, the magnetic susceptibility of any Boltzmann plasma is of the same magnitude as either the spin paramagnetic or the Landau diamagnetic susceptibility. This makes the magnetostrictive forces negligible compared to the magnetic pressure forces. A number of investigations (Parker 1955, Gurm and Wentzel 1967, Finzi and Wolf 1968, Kippenhahn 1974) have shown that the effect of the latter force is to make a magnetic tube of flux buoyant, so that it will rise.

Consider a prominence-like magnetic perturbation at the upper edge of a convective (dynamo) zone, so that the magnetic tube of force is lifting material of mean molecular weight $\mu'$ into a radiative zone of mean molecular weight $\mu$. In most stellar cases of interest $\mu'>\mu$ due to nuclear reactions. Then the conditions of pressure and thermal equilibrium of the prominence with its surroundings imply that

$$(\rho-\rho')/\rho = (P_B/P)(\mu'/\mu)-(\mu'-\mu)/\mu, \tag{1}$$

where primes denote "inside" quantities, $P$ is the outside gas pressure and $P_B = B^2/8\pi$ is the magnetic pressure. According to Archimedes' principle, if $\rho>\rho'$ the magnetic strand will be buoyant.

There are a number of possibilities for detaching such a buoyant prominence: a) cutting of the lines of force by convective elements from below, b) dynamical instabilities (cf. Landquist 1951, Parker 1955), c) aerodynamic drag due to differential rotation in the radiative zone above and d) the buoyancy overcoming the tension anchoring the prominence when its length $L$ increases to the extent that $Lg(\rho-\rho')>B^2/4\pi$ (Parker 1955), where $g$ is the local acceleration due to gravity. Combining this with condition (1) above shows that the magnetic buoyancy will rip out the prominence when $L \gtrsim 2\ H_p$, where $H_p$ is the pressure scale height.

It appears that c) and d) together can easily lead to the formation of detached toroidal magnetic bubbles with size $\sim H_p$ and some thickness $D$. The bubble will rise initially on a rapid, dynamic time scale. However, the adiabatic cooling of the bubble will decrease the inside temperature $T'$ relative to $T$ outside. Thereafter, the bubble rises quasistatically, with inside and outside densities nearly equal ($\rho\approx\rho'$) at a rate controlled by the time scale on which the bubble can be heated. Following Parker (1974), one obtains in this case an approximate rising velocity

$$v \sim 5\gamma^2(\gamma-1)^{-2}\, F/(g^2\rho^2D^2\varepsilon)P[(P_B/P)(\mu'/\mu)-(\mu'-\mu)/\mu], \qquad (2)$$

where $\gamma$ is the adiabatic exponent, $F$ the energy flux and $\varepsilon = |\nabla T-\nabla T_{ad}|/|\nabla T_{ad}|$, with $\nabla T$ the actual, and $\nabla T_{ad}$ the adiabatic temperature gradients. One notes that the bracket in Eqn. (2) is a small quantity by virtue of Eqn. (1) and the condition $\rho\approx\rho'$; what is important is the algebraic sign of this bracket which shows that the condition for the bubble to keep rising is that $P_B/P \approx (\mu'-\mu)/\mu'$, which at the outer edge of the ICZ of a double shell burning star becomes $P_B/P\approx 0.1$. From the flash computations of SH one then has $B\sim 8.7\times10^8$, $3.8\times10^8$ and $1.4\times10^8$G for the 1st, 7th and 13th He -shell flash cycles of a 1 $M_\odot$ star, respectively. These are in the appropriate range for the observations discussed in the last section. If the bubble rises, it rises on the thermal adjustment or Kelvin-Helmholtz time scale $\tau_{KH}\sim 3c_p\kappa\rho^2D^2/(8acT^3)$, where $\kappa$ is the opacity, $c_p$ the specific heat, $a$ the radiation constant and $c$ the speed of light (Kippenhahn 1974). Whether the rising bubble slows down or speeds up depends on what happens to $P_B/P$ during the ascent; Gurm and Wentzel (1967) found this ratio to be approximately constant (as can also be deduced from a dimensional argument and flux conservation), while in a slightly different analysis Kippenhahn (1974) deduced that the bubbles must slow down. In any case, in the absence of detailed numerical calculations, it appears that the magnetic toroids will rise 1-2 pressure scale heights, which is all one needs to produce mixing of hydrogen-rich material into the ICZ.

The transport of ICZ material to the surface can be estimated by considering that during any flash cycle there will be $N{\sim}f4\pi r^2/LD$ prominences being formed, where f is a "magnetic activity" number of order unity. These contain an amount of mass $N\rho LD^2$, so $M{\sim}f4\pi r^2\rho DN_f$ will be transported upward after $N_f$ flashes. With $r{\sim}3x10^9$cm, $\rho{\sim}100$ $gcm^{-3}$, $D{\sim}10^7$cm ($\sim 0.1H_p$), $N_f{\sim}100$ one has $M{\sim}10^{31}g{\simeq}0.005M_\odot$, quite a sufficient amount.

In the limit of this approximation, one might consider the rising toroids in the differentially rotating radiative zone as rigid stirrers. The toroids must then rotate with the curl of the velocity field. If there is negligible spin down so that $\Omega{=}\Omega_o\ r_o{}^2/r^2$, the toroids will stir with a period of a few days, so that one may expect the hydrogen-helium interface above the ICZ to be well mixed. The differential rotation is thus converted into turbulence; however, the magnetic mixing (and dynamo) can be effective only if significant differential rotation can be maintained. This can be achieved if two conditions are met: 1) the H and He burning shells should "burn" through an amount of mass comparable to that between the ICZ and the envelope convection zone during the span of a few flashes and 2) the spin-down in the "fresh" radiative zone should be negligible during this time. For the $1M_\odot$ star of SH, condition 1) is met after 13 cycles of relaxation oscillations. However, the spin-down criterion is less well understood. If the Ekman layer below the envelope convection zone is not turbulent (*cf.* Goldreich and Schubert 1967, Howard, Moore and Spiegel 1967); the spin-down time for the radiative region below it would be $\tau_s{\sim}r_o/(\nu\Omega)^{\frac{1}{2}}{\sim}10^6$-$10^7$yrs, and is approximately constant for double shell burning stars of practically all masses (0.6 - 7 $M_\odot$); this is because the morphology as a function of radial distance from the center is insensitive to the total mass. On the other hand, the amount of mass $\Delta M$ in the radiative zone above the ICZ decreases with increasing stellar mass (*cf.* Paczynski 1974), as does the time needed to burn through a given amount of mass. The time scale in which the H-shell burns through the mass $\Delta M$ for the 1 $M_\odot$ model of SH is $\tau_e$ ${\sim}5x10^6$ yrs. Since $\tau_e$ increases with decreasing total mass, comparison with $\tau_s$ indicates that magnetic mixing might be effective for stars down to below 1 $M_\odot$ and may be applicable to globular cluster stars.

More appropriately, the problem of magnetic fields and Eddington-Sweet currents should be treated together. The principal virtue of the magnetic-stirrer approximation is that it can be adapted for use in simple, one-dimensional, Henyey-type evolutionary calculations.

## 4. ON BIDELMAN'S HYPOTHESIS FOR Ap STARS

Bidelman (1960a,b) suggested that the peculiar A stars, metallic-line stars and possibly other sharp-line stars of classes O,B,A, may be in a phase of evolution analogous to the horizontal-branch stars found among Population II stars. After many conferences on the magnetic and peculiar stars, the complexity of the current state of affairs is well indicated by the fact that none of the handful of theories proposed have been rejected and there is a strong possibility that several processes may be responsible for these phenomena (*cf*. Cameron 1971). Fowler Burbidge, Burbidge and Hoyle (1965) have shown that the internal nucleosynthesis may be consistent with Bidelman's hypothesis for sufficiently complicated evolutionary histories. The theoretical HR diagrams of Fig. 1 illustrate the complications and ambiguities in the evolutionary status of stars falling near the intersections of main sequences and horizontal branches.

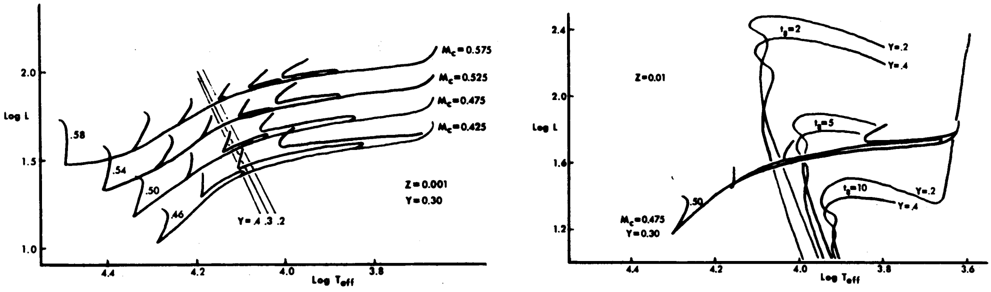

Figure 1. Two theoretical HR diagrams for horizontal-branches (HB) main sequences and isochrones from Sweigart and Gross (1976) and Ciardullo and Demarque (1977). On the left are shown HB sequences for helium and heavy elements (Y,Z) = (0.30, 0.001) and core masses $M_c$ = 0.425, 0.475, 0.525, 0.575 $M_\odot$, each with evolutionary tracks for masses M = 0.46(0.04)0.62, 0.50(0.04) 0.62, 0.54(0.04)0.70, 0.58(0.04)0.78 $M_\odot$, respectively, increasing from the left. Three zero-age main sequences are shown for Y=0.4, 0.3, 0.2 and Z = 0.001. On the right, the HB sequence is for ($M_c$,Y,Z) = (0.475, 0.30, 0.01) and M = 0.50(0.04)0.62 $M_\odot$, and three pairs (Y = 0.2, 0.4) of isochrones with Z = 0.01 are shown for ages $t_8$ = 2,5,10, in units of $10^8$ yr. The luminosity L is in solar units.

What do the zero-age HB models falling on the main sequence have in common? In Fig. 1 they *all* have the same inhomogeneity parameter q = $M_c$/M≈0.85. The importance of q for the theoretical HR diagram has been particularly emphasized by Giannone (1967); his more approximate computations gave q≈0.87 for HB stars along the main sequence. The HB models computed by Rood (1970) and Gross (1973) are in general agreement with this constant-q hypothesis.

In the context of the mixing by magnetic bubbles discussed above, the Bidelman hypothesis must be connected with the details of the thermal instabilities during the preceeding core helium flash. Rapid internal rotation of the degenerate helium core can then provide magnetic dynamo mechanisms to create buoyant magnetic strands that will bring both magnetic fields and nuclearly processed matter to the surface of the star. The implied spread in $M_c$ for the constant-q, HB models can also be produced by the internal rotation as shown by Mengel and Gross (1976) who found $M_c$ as large as 0.6 for a rotation rate of $6x10^{-4}$ $s^{-1}$ for solid-body rotation on the main sequence. In more realistic calculations, the approximation of solid-body rotation would have to be replaced by rapid internal rotation on the main sequence. Even higher core masses will result if the energy of rotation is included into the equations, since this becomes comparable to the thermal energy near the core flash (*cf.* Tuominen and Musylev 1974). Within this context of higher core masses, one might be able to represent stars like HR 7129 (Wolff and Wolff, 1976) which is a helium variable with a large magnetic field ($\sim 6x10^3$G), as an extension of the group of magnetic Ap stars to higher temperatures ($\log T_{eff} \sim 4.3$). The dissipation of rotation in the core-flash dynamo could also explain the origin of the magnetic fields and slow rotation of Ap stars.

Mass loss on the red giant branch (*cf.* Fusi-Pecci and Renzini, 1975) will also affect the value of $q = M_c/M$. This is further complicated by the effect of an influx of magnetic energy into the outer envelope as a result of the rising magnetic strands. It is not evident how the influx of magnetic energy might change the adiabatic exponents (*cf.* Kippenhahn 1970), thereby altering the mean rate of mass loss. Is the mass-loss rate coupled to the internal rotation, as is the mixing to the surface? The correlation between total mass and $M_c$ implied by a constant-q hypothesis is the weakest part in the Bidelman hypothesis, but this possibility cannot be dismissed and needs theoretical and observational investigation. The possible importance of fast rotating cores for explaining the gaps on the giant branch of M15 has been emphasized by Demarque, Mengel and Sweigart (1972). Magnetic mixing in the core flash may also be important in the Balmer-jump explanation of gap 1 of Newell (1973) proposed by Auer and Demarque (1977). It is not yet possible to say whether Newell's BC stars are related to the $q \approx 0.85$ phenomenon.

The general importance of rotation in theoretical stellar evolution in the context of magnetic fields and differential rotation has been discussed by Kippenhahn (1973), with particular emphasis on the variation of rotation along the zero-age main sequence. In this context, it is important to note that the constant-q hypothesis with magnetic mixing will work only if the main sequence progenitors of the peculiar stars were among the fast main sequence rotators, but their main sequence masses were sufficiently small for degenerate electron cores to form. The latter limit is about 2.5 $M_{\odot}$ (Iben, 1967) and might be higher for MSS with rapid internal rotation. This would imply that Bidelman's hypothesis can be correct, only if Ap stars were found predominantly in clusters which also have normal (fast rotating) A stars. This necessary (not sufficient) condition for Bidelman's hypothesis seems to be supported by observations.

## 5. CONCLUSION

This investigation has emphasized the interconnections between the magnetic enrichment of the ISM through stellar dynamos, which must exist if the high magnetic fields of some white dwarfs and pulsars are to be explained within the context of "conventional" stellar evolution. The approximation of mixing by magnetic toroids is evidently a crude one and, although the numbers fall into place, much work in magnetohydrodynamics remains to be done. Bidelman's hypothesis needs to be thoroughly investigated, especially in regard to the interior nucleosynthesis and the constant-q hypothesis. The upper mass limit to electron-degenerate helium ignition for MSS with rapid internal rotation has to be determined from theoretical computations, and the masses of Ap stars which are members of binaries, the rotation of main sequence stars in globular clusters, time variations of red giant peculiarities, etc. need to be observed.

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