NONLINEAR DYNAMO IN A DISK GALAXY

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Abstract. A nonlinear thin-disk galactic dynamo model based on α-quenching is proposed. Assuming that the mean helicity depends on the magnetic field strength averaged across the disk, we derive a universal form of nonlinearity in the radial dynamo equation. We discuss the evolution of the regular magnetic field in the Milky Way and the Andromeda Nebula. It is argued that the reversals of the regular magnetic field in the Galaxy are a relic inherited from the structure of the seed field. We also briefly discuss the role of the turbulent diamagnetism and the effects of galactic evolution on the dynamo.

Key words: Nonlinear galactic dynamos – Magnetic field reversals – Nonlinear transients

1. Introduction

The radial structures of the regular axisymmetric magnetic field in the Milky Way and Andromeda Nebula are remarkably different, even though these two galaxies are similar from the astronomical viewpoint. The analysis of Beck (1982) showed that the regular magnetic field in M31 is directed uniformly between the galactocentric radii of 6 and 18 kpc (see also Ruzmaikin *et al.*, 1991). Meanwhile, it is well known that the field in the Milky Way has opposite directions in the local spiral arm and the Sagittarius arm (Simard-Normandin and Kronberg, 1980); there are indications of additional reversals (Vallée *et al.*, 1988; Agafonov *et al.*, 1988; see also Vallée, 1991). To study the origin of the reversals, we have developed a nonlinear model of mean-field dynamo in a thin disk based on α-quenching.

At early stages of magnetic field evolution, the field growth is described by the eigensolutions of the kinematic dynamo. The leading eigenfunction of the dynamo is sign-constant in a thin disk (see Ruzmaikin *et al.*, 1988), *i.e.*, possesses no reversals. Thus, the reversals observed in the Milky Way imply that higher dynamo modes play a more important role in the Galaxy than simple kinematic considerations would suggest. This can happen if nonlinear dynamo effects had become pronounced before the leading eigenfunction could become dominant. In turn, this indicates that either the seed field was relatively strong or the dynamo was sufficiently efficient. Since the dynamo efficiency depends on the thickness of the galactic magnetoionic layer, the observed radial structure of the regular magnetic field can be used to obtain constraints on the strength and structure of the galactic seed field and also on the geometric shape and thickness of the magnetoionic layer. We apply our nonlinear dynamo model to the Milky Way and the Andromeda Nebula. A more detailed discussion of the results will be published elsewhere.

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2. Nonlinear Thin-Disk Dynamo

The generation of the regular magnetic field $\mathbf{B}(\mathbf{r}, t)$ in a turbulent conducting medium is governed by the following equation which represents the induction equation averaged over turbulent pulsations:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B} + \alpha \mathbf{B} - \beta \nabla \times \mathbf{B}), \qquad (1)$$

where $V = \boldsymbol{\Omega} \times \mathbf{r}$ is the galactic rotation velocity, α is related to the mean helicity of interstellar turbulence and β is the magnetic diffusivity (see, e.g., Krause and Rädler, 1980). Below we consider only axisymmetric solutions of eq. (1). We restrict ourselves to the thin-disk dynamo model discussed in detail, e.g., by Ruzmaikin *et al.* (1988). In a thin disk, solutions of eq. (1) can be represented in the form

$$\mathbf{B} = Q(r)\mathbf{b}(z; r), \qquad (2)$$

where $Q(r)$ describes the field distribution along the radius. The field distribution across the disk is included into the vector function $\mathbf{b}$, called the local solution, which parametrically depends on r and is normalized by the condition $\int_{-h}^{+h} \mathbf{b}^2 dz = 1$, where integration is carried out across the disk, $|z| \leq h$.

The nonlinear stage of the magnetic field evolution can be described using the concept of α-quenching by prescribing the mean helicity coefficient α as a function of the field strength B. As long as the z-distribution of magnetic field in a thin disk is established much faster than the radial one, it seems possible to assume that α depends on the magnetic field *averaged over the disk thickness*:

$$\alpha(r, \mathbf{B}) = \alpha_0(r)\mathcal{F}[B_0^{-2}(r)\textstyle\int_{-h}^{+h} B^2(r, t)dz] \equiv \alpha_0(r)\mathcal{F}[Q^2(r)/B_0^2(r)], \qquad (3)$$

where α_0 is the hydrodynamic helicity coefficient, $\mathcal{F}(x)$ is some function restricted by the inequality $d\mathcal{F}/dx \leq 0$, B_0 is the characteristic (equilibrium) field strength at a given radius, and the last equality in (3) is a consequence of eq. (2).

For the *ansatz* (2), eq. (1) splits into two one-dimensional problems for $\mathbf{b}$ and Q which can be solved consecutively; $Q(r)$ is governed by the following equation:

$$\frac{\partial Q}{\partial t} - \gamma(r, Q^2)Q - \lambda^2 \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial Q}{\partial r}\right) = 0, \qquad (4)$$

with the boundary conditions $Q(0) = Q(r_{\max}) = 0$, where $\gamma(r, Q^2)$ is obtained as an eigenvalue of the local problem together with $\mathbf{b}$ as an eigenvector. We solve numerically the Cauchy problem for eq. (4) using well-known solutions of the local equations discussed, e.g., by Ruzmaikin *et al.* (1988).

Consider the form of the function $\gamma(r, Q^2)$ in eq. (4). The dynamo number D in the Galaxy and M31 is close to its critical value $D_{\rm cr}$ (except for the very central parts which we do not consider here) (Krasheninnikova *et al.*, 1990). Near the generation threshold, the local growth rate scales as $\gamma(r, Q^2) \propto |D - D_{\rm cr}|^{1/2}$ (see Kvasz *et al.*, 1992). As long as γ vanishes for a certain threshold value of α corresponding to $D_{\rm cr}$, and α is assumed to be a monotonically decreasing function of Q^2, it is clear that γ vanishes at a certain value of Q^2 approximately equal to B_0^2 and becomes negative at larger values of Q^2. Therefore,

$$\gamma(r, Q^2) \approx \gamma_0[1 - Q^2(r)/B_0^2(r)] \tag{5}$$

near the generation threshold, where $\gamma_0 \equiv \gamma(r, 0)$ is the local growth rate. We stress that the form (5) by no means represents a consequence of any particular choice of the function $\mathcal{F}$ in eq. (3) but directly follows from the fundamental property of a threshold nature of the dynamo action.

3. The Turbulent Diamagnetism in Galactic Dynamos

According to the modern understanding, diffuse ionized warm gas in the Milky Way is represented by two components: a relatively thin layer with the scale height $\simeq 500$ pc, the midplane number density 0.1 cm^{-3} and the r.m.s. velocity 10 km s^{-1} (Lockman, 1984; Cordes *et al.*, 1985), and also a recently discovered (Reynolds, 1989) extended component with the scale height 1500 pc, the midplane number density $\simeq 0.03$ cm^{-3} and the r.m.s. velocity of at least 20 km s^{-1} that, possibly, grows with z (see a review in Dickey and Lockman, 1990). If the Reynolds layer is more or less homogeneous, the dynamo can be active in both layers but, as we shall show now, the disk magnetic field is restricted to the thinner layer due to the turbulent diamagnetism. To verify that the dynamo action is feasible in both layers, one may estimate the dynamo number,

$$D \simeq (3\Omega h/v)^2. \tag{6}$$

For $h_1 = 500$ pc, $v_1 = 10$ km s^{-1} and $h_2 = 1500$ pc, $v_2 = 20$ km s^{-1}, with the subscripts 1 and 2 referring to the thin and thick layer respectively, the dynamo number is about the same in both layers (provided $\Omega_1 = \Omega_2$) and slightly exceeds the dynamo threshold $|D_{\text{cr}}| \approx 10$. However, when $\beta \simeq \frac{1}{3}lv$ considerably varies, say, with the height above the midplane, the turbulent diamagnetism becomes important (Zeldovich, 1956). This effect can be described as a transport of magnetic field at the velocity $-\nabla\beta$. Since β grows with z (see below), the magnetic field is transported towards the midplane thereby facilitating the dynamo activity in the disk. We stress that this transport is not associated with any bulk motion of matter.

Let us estimate the diamagnetic transport velocity for the Milky Way. For obvious symmetry reasons, $\partial\beta/\partial z = 0$ at $z = 0$. Therefore, the diamagnetic effect can be neglected in a sufficiently thin layer. The turbulent scale l at the height $z = 1500$ pc can be tentatively estimated from mass conservation for a turbulent cell, $n_1 l_1^3 \simeq n_2 l_2^3$, where n is the gas number density. Taking typical values $l_1 = 100$ pc, $n_1 = 0.1$ cm^{-3} and $n_2 = 0.03$ cm^{-3}, we obtain $l_2 \simeq 170$ pc. With $v_1 = 10$ km s^{-1} and a conservative estimate $v_2 = 20$ km s^{-1} we derive $\beta \simeq 10^{26}$ $\text{cm}^2\ \text{s}^{-1}$ at $z \leq 500$ pc and $\beta \simeq 3.5 \times 10^{26}$ $\text{cm}^2\ \text{s}^{-1}$ at $z = 1500$ pc. Thus, $\Delta\beta/\Delta z \simeq -1$ km s^{-1} and, during the field regeneration time $\gamma^{-1} \simeq 8 \times 10^8$ yrs, the field can be transported over the distance $\gamma^{-1}\Delta\beta/\Delta z \simeq 800$ pc comparable to the Reynolds layer scale height. This implies that the vertical scale of the field must be considerably smaller than that of the Reynolds layer.

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To estimate the steady-state scale height of the regular magnetic field, h, we consider the balance of the diamagnetic and diffusive transport velocities at the dynamo time scale, $-\nabla\beta(h) = \frac{1}{2}\sqrt{\beta(h)\gamma}$, where the diffusive velocity on the right-hand side is defined as $(d/dt)\sqrt{\beta t}|_{t=1/\gamma}$. For a Gaussian distribution $\beta = \beta_0 \exp(z^2/h_\beta^2)$, choosing $h_\beta = 1200$ pc to obtain $\beta(1500\ \mathrm{pc})/\beta(500\ \mathrm{pc}) = 3.5$, and using the estimate $\gamma \simeq \beta/h^2$, we obtain $h \simeq \frac{1}{2}h_\beta \approx 600$ pc. Hence, the *regular* magnetic field concentrates within a layer of the half-thickness of about 500–600 pc.

In agreement with above arguments, the analysis of the Faraday rotation measures of extragalactic radio sources and pulsars shows that the half-thickness of the layer containing the regular magnetic field in the Milky Way is as low as 400–500 pc (Ruzmaikin and Sokoloff, 1977; see also Ruzmaikin *et al.*, 1988, Sect. IV.3). Polarization observations of NGC 891 (Sukumar and Allen, 1991) indicate that the scale height of the ***disk*** component of magnetic field is considerably smaller than the Hα scale height as observed by Dettmar (1990) and Rand *et al.* (1990). It is notable that the synchrotron emission distributions in the Milky Way and other spiral galaxies are described in terms of a thick and thin disks, with the half-thickness of the thin one being several hundred parsecs (Beuermann *et al.*, 1985).

4. Results

Our simulations imply that the reversals in the present-day radial structure of the magnetic field in the Milky Way are inherited from those in the seed field. In other words, the reversals are interpreted as *nonlinear transient structures* in galactic dynamos. We analyzed the scenarios of magnetic field evolution for the seed fields either produced within the galaxy by interstellar turbulence as a chaotic field or trapped by the protogalaxy as a regular one.

Even though our model is somewhat idealized, it permits a detailed comparison with observation, so that we achieved certain constraints on the geometric shape and thickness of the ionized disks in the Milky Way and the Andromeda Nebula. Firstly, the ionized disk thickness must increase with r, which distinguishes the ionized disk from the HI layer in both galaxies. Secondly, the presence of the regular magnetic field in the Galaxy and M31 and, on the other hand, the existence of magnetic field reversals in the former and magnetic ring in the latter imply the following bounds for the half-thickness of the magnetoionic disk in these galaxies: $350 \lesssim h \lesssim 1500$ pc at $r = 10$ kpc in the Galaxy and $350 \lesssim h \lesssim 450$ pc at $r = 10$ kpc in M31, if the seed field for the dynamo is produced within the galaxies by interstellar turbulence. These limits are, of course, model-dependent but they indicate that the the layer containing the regular magnetic field in these two galaxies cannont be as thick as the Reynolds layer.

When the seed field strength is weaker, the linear growth stage is longer, and the reversals have a smaller chance to survive. In order to obtain a lower estimate on the seed field strength, we choose the models of both the disk and the seed field in which the reversals are most persistent and find the weakest seed field consistent with the reversal in the inner Galaxy. The reversals survive for a longer period if the seed field is regular. The reversals are more persistent, when the dynamo is stronger, *i.e.*, when the dynamo number given by eq. (8) is larger. In the early

Galaxy, Ω was, probably, smaller than now whereas h and v were larger. Therefore, it is difficult to say without a detailed analysis, whether the dynamo was stronger or weaker in the young Galaxy; thus, we tested both possibilities.

As a preliminary model, we consider γ_0 in eq. (5) to be time-dependent,

$$\gamma_0 = \gamma_0(r)[A \exp(-t/t_0) + 1],$$

with a certain time scale t_0 and the degree of time variation A ($\gamma_0(r) \equiv \gamma(r, Q)|_{Q=0}$ is taken the same as above). For $A = 0$, which is the model discussed above, we obtain the lower limit on the seed field in the galactic disk as $B_s \gtrsim 2 \times 10^{-7}$ G. If 10^{10} yrs ago the dynamo was three times weaker than now, $A = -2/3$, and settled to the present-day intensity rather rapidly, $t_0 = 8 \times 10^8$ yrs, the lower estimate for the seed field is approximately the same as that for a steady galaxy: $B_s \gtrsim 5 \times 10^{-8}$ G. This estimate is slightly sensitive to the value of t_0: for $t_0 = 1.6 \times 10^9$ yrs (3.2×10^9 yrs), $B_s \gtrsim 8 \times 10^{-8}$ G (1.5×10^{-7} G). If, otherwise, the dynamo in the young Galaxy was stronger than now, say, $A = 2$, we have $B_s \gtrsim 1.5 \times 10^{-8}$ G for $t_0 = 8 \times 10^8$ yrs, $B_s \gtrsim 8 \times 10^{-9}$ G for $t_0 = 1.6 \times 10^9$ yrs, and $B_s \gtrsim 3 \times 10^{-9}$ G for $t_0 = 3.2 \times 10^9$ yrs. For even stronger early dynamo, $A = 9$, the result is insensitive to the value of t_0: the magnetic field grows so rapidly that nonlinear dynamo effects fix its radial structure early enough and, afterwards, it evolves in a slow, quasi-steady fashion. For $A = 9$, we obtain $B_s \gtrsim 3 \times 10^{-10}$ G. We conclude from these experiments that the presence of the reversal in the inner Galaxy implies that the seed magnetic field in the galactic disk hardly could be weaker than $10^{-9} - 3 \times 10^{-10}$ G.

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