elementary theorems of the theory of probability; as a consequence, they differ from the truth-tables of other many-valued systems in that the truth-value of a connection—e.g. of a conjunction—appears as a function of the truth-values of the two connected members and of a third argument, the "degree of coupling" or the relative probability of the second member with respect to the first. It is, however, at least doubtful whether Reichenbach thus really establishes a language which is governed by a many-valued logic; for he seemingly wants to maintain in his language the rules of inference and the valid formulas of the sentential calculus. Thus it seems to be more adequate to say that by the establishment of the concept of weight Reichenbach introduces an important new semantical (or, perhaps, syntactical) concept, which, however, does not generalize the concept "true," but the concept "verified," and therefore does not conflict with the assumption of a two-valued logic.

Finally, Reichenbach expounds his theory of induction, which cannot be outlined here.

CARL G. HEMPEL

ANDRZEJ MOSTOWSKI. *O niezależności definicji skończoności w systemie logiki* (On the independence of the definitions of finiteness in a system of logic). ***Dodatek do Rocznika Polskiego Towarzystwa Matematycznego*** (supplement to the ***Annales de la Société Polonaise de Mathématique***), vol. 11 (1938), pp. 1–54.

This paper, which is the author's doctor's thesis, contains a succession of very valuable and interesting results from the domain of metalogic. As subject of his research the author has chosen the formalized system of ***Principia mathematica***, based on a simplified theory of types, and enlarged by adding the axiom of infinity; more precisely, he considers the system T_∞ outlined by the reviewer in 285*13*. However, all the results obtained are—according to the author—applicable also to other kindred formal systems, in particular to the formalized system of Zermelo (cf., e.g., Skolem 247*8* and Quine II 51.)

Mostowski's principal topic is the question of relationships of inference between certain definitions of the notion of a finite class. He shows the impossibility of proving the equivalence of different known definitions of finiteness without using Zermelo's axiom of choice. This concerns primarily the three following definitions (in which "reflexive" and "inductive" are used in the sense of ***Principia mathematica***):

(1) *A class is finite if and only if it is inductive.*

(2) *A class is finite if and only if it is not reflexive.*

(3) *A class is finite if and only if the class of all its subclasses is not reflexive.*

The author proves that no one of the sentences expressing the equivalence of two of these definitions is provable on the basis of the system T_∞ (provided, of course, this system is consistent). Since, however, the equivalence of these definitions can be proved after enlargement of the system T_∞ by addition of the axiom of choice, the author obtains as a side result *an exact proof of the independence of the axiom of choice from the axioms of the system* T_∞. This result can be considerably strengthened; it appears that even such a sentence as,

(4) *Every non-inductive class is a sum of two mutually exclusive non-inductive classes,* which is a very weak consequence of the axiom of choice, is not derivable from the axioms of the system T_∞.

The proof consists in applying with due care the classical method of (so called) proof by interpretation. The necessity of care is due to: (1) the fact that here the primitive concepts of logic itself and not as usual the primitive terms of an axiomatic system based on logic are interpreted; (2) the circumstance that the system dealt with is based on an infinite number of axioms. The most essential point of the proof is apparently the proper interpretation of the universal and existential quantifiers (i.e. of such expressions as "for every x" and "for certain x"), and therefore the author—following the reviewer—calls the method of proof used the *method of relativization of quantifiers*. The conception of this method comes from the reviewer, who applied it to different methodological problems in 285*19* and in a joint paper with Lindenbaum (I 115).

The results concerning the definitions (1)–(3) answer the questions put in the appendix to the reviewer's 285*5*; the problem of the independence of the sentence (4) is due to Chwistek (220*15*). These results, as well as the theorem concerning the independence of the axiom

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of choice, are connected with papers of Fraenkel on the same subject (2692, 26927). According to the author, his proofs contain the principal idea of Fraenkel, "but the way of carrying out this idea, which in Fraenkel's papers gave rise to serious doubts, has been entirely altered." The work dealing with this last question was done in collaboration with Lindenbaum, and a number of remarks on the same topic are contained in a paper of Lindenbaum and Mostowski, *Über die Unabhängigkeit des Auswahlaxioms und einiger seiner Folgerungen*, shortly forthcoming in the ***Comptes rendus des séances de la Société des Sciences et des Lettres de Varsovie***. ALFRED TARSKI

ALFRED ERRERA. *Sur les démonstrations de non-contradiction*. ***Travaux du IX^e Congrès International de Philosophie***, VI ***Logique et mathématiques***, Actualités scientifiques et industrielles 535, Hermann et C^ie, Paris 1937, pp. 121–127.

These remarks concern in the main a method for establishing the consistency of a set of axioms, the underlying logic being assumed to be both consistent and sufficiently strong. Unfortunately the author proves no actual set to be consistent by the method in question, and the single example referred to concerns the relation of Euclidean to Lobachevskian geometry. Let p be Euclid's axioms without the axiom of parallels, let q be that axiom, and let the entire set be so chosen that the Lobachevskian parallel axiom can be written as $\sim q$. We require two rules: one such that, given an interpretation of $p.q$, we can specify within it an interpretation satisfying $p.\sim q$, and another such that, given an interpretation satisfying $p.\sim q$, we can specify within it an interpretation of $p.q$. If both rules can be found, then the two sets of axioms are equivalent with respect to consistency, and, moreover, from the consistency of p follows that of both sets. If a set of axioms $p_1, p_2, \cdots, p_n$ can be so arranged that each successive axiom is independent of the preceding axioms in this way, then given the consistency of p_1, that of the entire set is assured. (In one place Errera speaks as if the consistency of p_1 itself could be proved by his method and in another as if it could not.)

In order to establish the consistency of $p.q$ on the hypothesis that p is consistent, it is not necessary, as Errera seems to say it is, that we have two rules; indeed, it is neither necessary nor sufficient. If r implies $\sim q$ but is a stronger condition, and if we can represent $p.q$ in any interpretation of $p.r$ and conversely, then the two sets are equivalent with respect to consistency; but the consistency of neither set follows from that of p. If, however, r is identical with $\sim q$, then one rule, which provides a representation of $p.q$ within any system satisfying $p.r$, is enough.

However desirable it may be to have a method which enables us on occasion to show that if a set of assumptions is consistent, the addition of a certain further assumption will not introduce inconsistency, Errera's procedure seems to be subject to serious limitations. Let $p_1, p_2, \cdots, p_n$ be a set of axioms arranged in the manner required. In general, p_1 will admit of a finite representation; but in many important cases there comes a point at which the axioms are no longer finitely representable. Let q be the axiom whose introduction makes necessary an infinite representation, and let p be the preceding axioms. Then $p.q$ cannot be finitely represented, whereas $p.\sim q$ can be, since any finite system satisfying p must satisfy $\sim q$. And if we are to show that the consistency of $p.q$ follows from that of p, we must of course show that *every* system satisfying $p.\sim q$ provides a representation of $p.q$. C. H. LANGFORD

JACQUES PICARD. *Les normes formelles du raisonnement déductif*. ***Revue de métaphysique et de morale***, vol. 45 (1938), pp. 213-254.

This paper attempts to classify ordinary deductive arguments on the basis of the formal logical principles involved in them. E.g., subsumptive syllogisms and relational arguments are shown to be special cases of the principle of relative product: $xRy.ySz . \supset . x\, R|S\, z$. A similar reduction and generalization of immediate inferences and of the opposition of propositions to, and in terms of, fundamental logical relations is attempted. The author explains the principles of simplification, composition, substitution, abstraction, and mathematical induction, indicating in regard to the latter Poincaré's objections and Russell's