A Giant Step: from Milli- to Micro-arcsecond Astrometry
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Relativistic astrometry and astrometric relativity

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Abstract. The interplay between modern astrometry and gravitational physics is very important for the progress in both these fields. Below some threshold of accuracy, Newtonian physics fails to describe observational data and the Einstein's relativity theory must be used to model the data adequately. Many high-accuracy astronomical techniques have already passed this threshold. Moreover, modern astronomical observations cannot be adequately modeled if relativistic effects are considered as small corrections to Newtonian models. The whole way of thinking must be made compatible with relativity: this starts with the concepts of time, space and reference systems.

An overview of the standard general-relativistic framework for modeling of high-accuracy astronomical observations is given. Using this framework one can construct a standard set of building blocks for relativistic models. A suitable combination of these building blocks can be used to formulate a model for any given type of astronomical observations. As an example the problem of four dimensional solar system ephemerides is exposed in more detail. The limits of the present relativistic formulation are also briefly summarized.

On the other hand, high-accuracy astronomical observations play important role for gravitational physics itself, providing the latter with crucial observational tests. Perspectives for these astronomical tests for the next 15 years are summarized.

Keywords. gravitation, relativity, astrometry, celestial mechanics, reference systems, time

1. Introduction

The tremendous progress in technology, which we have been witnessing during the last 30 years, has led to enormous improvements of accuracy in the disciplines of astrometry and time. A good example here is the growth of accuracy of positional observations in the course of time: during the 25 years between 1988 and 2013 we expect the same gain in accuracy (4.5 orders of magnitude) as that realized during the whole previous history of astrometry, from Hipparchus till 1988 (over 2000 years). Observational techniques like Lunar and Satellite Laser Ranging, Radar and Doppler Ranging, Very Long Baseline Interferometry, high-precision atomic clocks, etc., have already made it possible to probe the kinematical and dynamical properties of celestial bodies to unprecedented accuracy. Microarcsecond astrometry projects like Gaia (Lindegren 2008) and SIM (Shao 2008) will open fascinating possibilities for obtaining important physical information on celestial objects using their astrometry.

The goal of this review is to stress that the fascinating potential possibilities of high accuracy astronomical observations can only be realized if data modeling and analysis are made fully compatible with general relativity. It is not sufficient to consider the relativistic effects as small corrections to some Newtonian picture. The basic concepts of a reference system, moment of time, simultaneity, etc. are fundamentally different from their Newtonian counterparts. Although the relativistic (at least post-Newtonian) data modeling is rather simple conceptually, it requires a different way of thinking compared to that of a typical Newtonian physicist or astronomer.

2. Relativistic astrometry

In 2000 the International Astronomical Union has adopted the standard general-relativistic framework for modeling the high-accuracy astronomical observations. This framework allows one to construct a standard set of building blocks for relativistic models, a suitable combination of which can be used to formulate a model for any given type of astronomical observations. The IAU has adopted two reference systems defined in the mathematical language of general relativity. These reference systems are the Barycentric Celestial Reference System (BCRS) and the Geocentric Celestial Reference System (GCRS). The BCRS is the fundamental reference system covering the solar system and observed sources. The center of the BCRS lies in the barycenter of the Solar system. The word "celestial" in the name of BCRS is used to underline that the BCRS does not rotate with the Earth and that remote sources (e.g., quasars) can be assumed to be at rest with respect to the BCRS in some averaged sense. The BCRS is used to model the dynamics of the solar system as a whole and to describe light propagation between the source and the observer. The coordinate time of the BCRS is called Barycentric Coordinate Time (TCB). The Parameterized Post-Newtonian (PPN) version of the BCRS valid for certain class of metric theories of gravity has been also considered by a number of authors.

The GCRS is constructed in such a way that the gravitational fields generated by other bodies are reduced to tidal potentials and are thus effaced as much as it is possible according to general relativity. The coordinate time of the GCRS is called Geocentric Coordinate Time (TCG). Its scaled version, called Terrestrial Time (TT), is a physical model of TAI. The GCRS is a reference system physically suitable for modeling of physical processes in the vicinity of the Earth (e.g. Earth rotation or motion of an Earth's satellite).

The theory of the local reference systems like GCRS can be applied to any massive or massless bodies of the solar system. In particular, a GCRS-like reference system can be constructed for an observing satellite (Klioner 2004). That reference system is physically adequate to model any physical processes occurring within or in the immediate vicinity of a satellite (for example, the process of registration of incoming photons and the rotational motion of satellite).

Using these standard reference systems any kind of astronomical observations can be modeled in the way depicted on Figure 1. A suitably chosen reference system allows one to derive four building blocks. The equations of motion of the observed object, the observer and the electromagnetic signal relative to the chosen reference system should be derived and a method to solve these equations should be found. The equations of motion of an object and the observer and the equations of light propagation enable one to compute positions and velocities of the object, observer and the photon (light ray) with respect to that particular reference system at a given moment of the coordinate time, provided that the positions and velocities at some initial epoch are known. As the last step the observable quantities should become relativistic definitions. This part of the model allows one to compute a coordinate-independent theoretical prediction of observables starting from the coordinate-dependent quantities mentioned above. These four components can now be combined into relativistic models of observables. The models give an expression for relevant observables as a function of a set of parameters. These parameters can then be fitted to observational data using some kind of parameter estimation scheme. The

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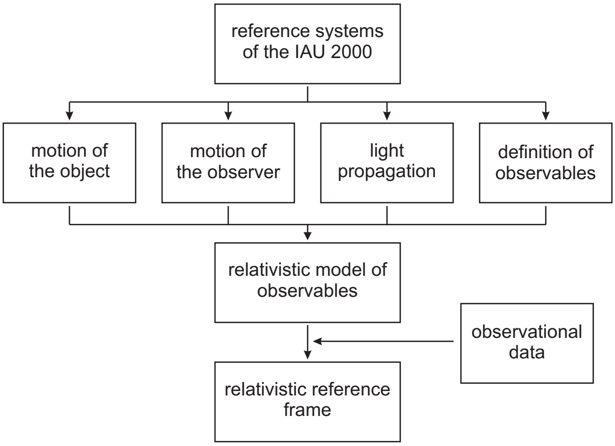

Figure 1. General scheme of relativistic modeling of astronomical observations.

sets of certain estimated parameters appearing in the relativistic models of observables represent astronomical reference frames.

Most of the relativistic models used in practice are fully compatible with the IAU framework sketched above. These are the models for VLBI, SLR, LLR, pulsar timing, time keeping and time transfer, GPS, and the models used for the development of solar system ephemerides. Some of these models will soon need some refinement because of the expected increase of accuracy. The only operational model which is not compatible with the IAU framework is that of the rotational motion of the Earth (and other planets). The rotational motion of celestial bodies is currently modeled in a purely Newtonian framework with the main relativistic effect - the geodetic precession - added in a non-consistent way. This situation must be improved in the nearest future. Some recent progress in relativistic modeling of Earth rotation can be found in (Klioner *et al.* 2007).

3. Towards four-dimensional solar system ephemerides

One of the questions which triggers a lot of difficulties and controversies among astronomers are relativistic time scales and their interrelation. Since the relativistic framework involves several relativistic reference systems (at least, BCRS and GCRS), several coordinate time scales must be considered (at least *TCB* and *TCG*). One important implication of relativity is that the transformation of time moments from one reference system to another is only possible if the position of the event is specified. So to say, without knowing "where" one cannot know "when". This is a natural consequence of the four-dimensionality of the coordinate transformations in relativity. In general the transformations of coordinate times T and t of two relativistic reference systems $(t, \boldsymbol{x})$ and $(T, \boldsymbol{X})$, $\boldsymbol{x}$ and $\boldsymbol{X}$ being the spatial coordinates, is a part of four-dimensional transformations between these two reference system:

$$\begin{aligned} T &= T(t, \boldsymbol{x}), \\ \boldsymbol{X} &= \boldsymbol{X}(t, \boldsymbol{x}). \end{aligned} \tag{3.1}$$

The details of this transformation for BCRS and GCRS are given, e.g., by Soffel *et al.* (2003). An important *special case* of this transformation is to specify a particular spatial location (maybe as a function of time in order to calculate the transformation along some worldline): for any t one has $\boldsymbol{x} \equiv \boldsymbol{x}_{\rm o}(t)$. In this case T becomes a function of t only:

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$T = T(t,\boldsymbol{x}) = T(t,\boldsymbol{x}_\mathrm{o}(t)) = T(t)$. Trajectory $\boldsymbol{x}_\mathrm{o}(t)$ may be chosen to coincide with the coordinates of a observing astrometric satellite like Gaia or SIM, or with the geocenter. The latter case is especially important for practice. Let us consider it in more detail. The well-known differential relation between $T = TCG$ and $t = TCB$ at the geocenter reads

$$\frac{dT}{dt} = 1 + \frac{1}{c^2}\,\alpha(t) + \frac{1}{c^4}\,\beta(t) + \mathcal{O}(c^{-5}). \tag{3.2}$$

Here $\alpha(t)$ and $\beta(t)$ are known functions of time t. There functions involve masses, positions and velocities of the massive bodies of the solar system and can be computed as

$$\alpha = -\frac{1}{2}\,v_E^2 - \sum_{A\neq E}\frac{GM_A}{r_{EA}}, \tag{3.3}$$

$$\begin{aligned}\beta = {} & -\frac{1}{8}\,v_E^4 + \left(\beta - \frac{1}{2}\right)\left(\sum_{A\neq E}\frac{GM_A}{r_{EA}}\right)^2 + (2\beta-1)\sum_{A\neq E}\left(\frac{GM_A}{r_{EA}}\sum_{B\neq A}\frac{GM_B}{r_{AB}}\right)\\ & + \sum_{A\neq E}\frac{GM_A}{r_{EA}}\Bigg(2(1+\gamma)\boldsymbol{v}_A\cdot\boldsymbol{v}_E - \left(\gamma+\frac{1}{2}\right)v_E^2 - (1+\gamma)v_A^2\\ & \qquad + \frac{1}{2}\boldsymbol{a}_A\cdot\boldsymbol{r}_{EA} + \frac{1}{2}(\boldsymbol{v}_A\cdot\boldsymbol{r}_{EA}/r_{EA})^2\Bigg),\end{aligned} \tag{3.4}$$

where capital latin subscripts A, B and C enumerate massive bodies, E corresponds to the Earth, M_A is the mass of body A, $\boldsymbol{r}_{EA} = \boldsymbol{x}_E - \boldsymbol{x}_A$, $r_{EA} = |\boldsymbol{r}_{EA}|$, $\boldsymbol{v}_A = \dot{\boldsymbol{x}}_A$, $\boldsymbol{a}_A = \dot{\boldsymbol{v}}_A$, a dot signifies time derivative with respect to *TCB*, and $\boldsymbol{x}_A$ is the BCRS position of body A. The PPN parameters β and γ (both equal to 1 in general relativity) are given here for completeness, and normally can be put to 1 for practical calculations. Introducing two functions $\Delta t(t)$ and $\Delta T(T)$

$$T = t + \Delta t(t), \tag{3.5}$$

$$t = T - \Delta T(T). \tag{3.6}$$

one gets two ordinary differential equations for $\Delta t(t)$ and $\Delta T(T)$

$$\frac{d\Delta t}{dt} = \frac{1}{c^2}\,\alpha(t) + \frac{1}{c^4}\,\beta(t) + \mathcal{O}(c^{-5}), \tag{3.7}$$

$$\frac{d\Delta T}{dT} = \frac{1}{c^2}\,\alpha(T-\Delta T) + \frac{1}{c^4}\,\left(\beta(T-\Delta T) - \alpha^2(T-\Delta T)\right) + \mathcal{O}(c^{-5}). \tag{3.8}$$

Initial conditions for these two differential equations are given by the IAU definitions of *TCB* and *TCG*: $TCB = TCG = 32.184$ s on 1977, January 1, $0^h\ 0^m\ 0^s$ TAI at the geocenter: for $JD_{TCB} = 2443144.5003725$ one has also $JD_{TCG} = 2443144.5003725$. Any reasonable numerical integrator can be used to integrate these two differential equations with these initial conditions and using any given solar system ephemerides. Moreover, the numerical integration of $\Delta t(t)$ and $\Delta T(T)$ can be performed simultaneously with the equations of motion of the solar system which are routinely integrated during the process of ephemeris construction. Eq. (3.2) and, correspondingly, Eqs. (3.7)–(3.8) can be modified in an obvious way to relate any reasonable pair of the time scales *TCG*, *TCB*, *TT* and *TDB*, the last two being fixed linear functions of the first two (Soffel *et al.* 2003; IAU 2006). For *TDB* the initial conditions have to be selected according to IAU (2006): for $JD_{TT} = 2443144.5003725$ one has $JD_{TDB} = 2443144.5003725 - 6.55\times10^{-5}/86400$.

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It is natural to include the results of the numerical integration of (3.7)–(3.8) into the standard distribution of the ephemerides. Functions $\Delta t(t)$ and $\Delta T(T)$ can be easily presented in the same Chebyshev polynomial representation as the rest of the ephemeris data. Each ephemeris has slightly different numerical values of $\Delta t(t)$ and $\Delta T(T)$ exactly in the same way as each ephemeris has its own numerical values of positions and velocities of the solar system bodies. This naturally leads to four-dimensional space-time ephemerides of the solar system as it has been implied by general relativity from the very beginning.

4. Limitations of the current relativistic framework

The standard relativistic framework of the IAU will have to be extended when more accurate data models will be required. Currently the relativistic framework is formulated in the post-Newtonian approximation of general relativity. This means that the relativistic effects are taken into account to the lowest order in c^{-2}. In principle, the extension to the next order, the post-post-Newtonian approximation, is straightforward, although not all aspects of the formalism are understood with the same level of details as in the post-Newtonian approximation. However, the post-Newtonian approximation scheme just operates with analytical orders of magnitude of various terms and not with their numerical magnitude. Taking into account all terms of order c^{-4} we would do a lot of unnecessary work since only a few of those terms are numerically important. A lot of work should be done to identify which post-post-Newtonian terms should be accounted for and which can be safely neglected in various situations.

Another restriction of the current formalism is the assumption that the solar system is isolated. This means that all gravitational fields generated outside of the solar system are ignored. Those external gravitational fields are only interesting if they produce time-dependent effects (the time-independent part is absorbed by the source positions just like secular aberration). The main time-dependent effects of this kind is the gravitational light deflection caused by (a) weak microlensing on the stars of the Galaxy (Belokurov & Evans 2002), and (b) lensing on gravitational waves (both primordial ones and those from compact sources). All these effects can be taken into account by a simple additive extension of the standard model since at the required accuracy the external gravitational fields can be linearly superimposed on the solar system gravitational field. The only exception could be the effects of cosmological background, but a preliminary study by Klioner & Soffel (2005) shows that even here the coupling of the local solar system fields and the external ones can be neglected.

5. Astrometric relativity

Since the formulation of general relativity in 1915, astronomical observations have played a very important role for testing this theory. Three of the four classical tests of General Relativity are based on astronomical observations. Although General and Special Relativity have been tested with a good precision, some ideas in the field of gravitational physics suggest that a deviation from general relativity maybe expected at the level of 10^{-5} to 10^{-8} (see, Damour, Piazza & Veneziano(2002) and references therein). This level is still beyond the possibilities of the available tests. On the other hand, independent of these arguments it is clearly the basic principle of natural sciences to test the suggested theories as accurately as possible.

It is expected that by 2020 the main PPN parameters β and γ will be measured with 6-7 meaningful digits. Considering that at such a high level of accuracy interpretation

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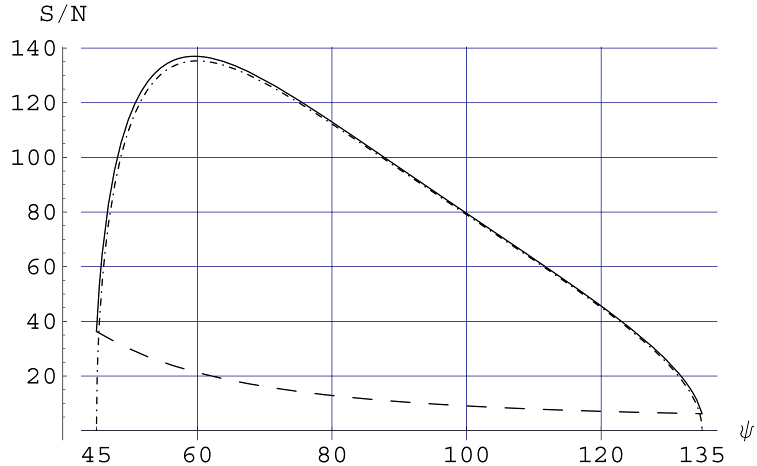

Figure 2. Signal-to-noise ratio in the Gaia observations of the gravitational light deflection from the Sun as function of the angular distance ψ (in degrees) between the Sun and the observed source for bright stars ($V = 13$ and brighter). Lower (dashed) line is the signal-to-noise ratio in the across scan data, the dot-dashed line is that in the along scan data and the solid line is the total signal-to-noise ratio. The across scan data contribute significantly only close to the maximal and minimal values of ψ where the along scan observations are insensitive to the gravitational light deflection due to the Sun.

of astronomical data becomes increasingly complicated, special care should be taken to avoid and/or understand all possible sources of systematic errors. It is important to realize that a detection of any (even very small) violation of General and Special Theories of Relativity in the weak field of solar system would have profound consequences for physics and astrophysics as a whole. For example, our understanding of physical properties of black holes and gravitational radiation may need a revision.

The most stringent relativistic test which can be expected from optical astrometry in the next 15 years is the measurement of the PPN parameter γ by astrometry mission Gaia. The expected accuracy here is $10^{-6} - 5 \cdot 10^{-7}$. Due to high sensitivity of observations in a wide range of angular distances from the Sun (see Fig. 2), it can be expected that besides γ (that gives just the amplitude of a particular light deflection pattern) it will be possible to verify that light deflection indeed follows the general-relativistic deflection law for different angular distances. That latter test is beyond the scope of conjunction experiments like Cassini, BepiColombo or LATOR that can provide estimates of γ in a very narrow range of angular distances.

The PPN parameter β will be measured by Gaia from observations of asteroids with an accuracy of $< 10^{-4}$ (Hestroffer *et al.* 2007), but the most stringent test of β will be done by BepiColombo with expected accuracy of $2 \cdot 10^{-6}$ (Milani *et al.* 2002).

Besides these "major" tests, a series of other tests are possible (Klioner 2007). Let us give a few examples.

• Three components of the gravitational light deflection due to giant planets will be measured (Anglada *et al.* 2007): (1) the monopole gravitational deflection (with a precision of 10^{-3} for Jupiter), (2) the deflection due to translational motion of the planets (with a precision of $2 \cdot 10^{-3}$ for Jupiter), and (3) the deflection due to the quadrupole gravitational field of Jupiter (with a precision of > 0.08).

• Gaia will be able to measure the acceleration of the solar system's barycenter with respect to remote sources (quasars) with an accuracy of about $1.8 \cdot 10^{-11}$ ms^2 (Mignard

& Klioner 2007). An acceleration causes a specific change of secular aberration and thus can be measured as a specific pattern of proper motions. The amplitude of that pattern is related to the absolute value of acceleration. The acceleration of the solar system relative to the galactic center, as expected from the standard orbit of the solar system, should produce a proper motion of up to 4.5 μasyr^{-1}. Gaia will be able to measure this with the accuracy of 10%. This will allow one to verify, for the first time, the accuracy claims of the binary pulsar tests of relativity that assume the value of the acceleration from the standard model of the Galaxy.

• Another pattern in the proper motions allows one to constrain (Gwinn *et al.* 1997) the energy flux of gravitational waves with frequencies $\omega < 3 \cdot 10^{-9}$ Hz (see Mignard & Klioner (2007) for further details).

• Finally, one more group of tests deals with specially selected, relativistically interesting objects. An outstanding example here is given by compact binaries with one component being a black hole candidate. Combining usual Doppler measurements of these objects with Gaia astrometry one can derive the mass of the invisible companion without any further, model-dependent assumptions (Fuch & Bastian 2005). For example, for the well-known system Cyg X-1 the astrometric wobble of the visible companion is expected to attain 25 μas and can be measured by Gaia. Not only Gaia, but also ground-based interferometers have a good chance since narrow-field differential astrometry with those interferometers seems to be sufficient to detect the wobble with a well-known period.

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