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Editorial

This issue is dedicated to nonsmooth dynamics, particularly the widening applications where dynamics is modelled by nonsmooth differential equations. The modeling of electrical or mechanical switches as nonsmooth can be traced throughout the (at least) 90-year history in which nondifferentiable terms, such as sign or step functions, have been turning up in differential equations. In recent decades, nonsmooth models have found increasing use in areas like contact mechanics, climate modeling, and the life sciences, among others, with a wealth of new theory and novel dynamical phenomena discovered along the way. Our aim here is to give just a partial snapshot of the current landscape of research topics in the field. We open with Paul Glendinning's article extending a classic phenomenon of nonlinear dynamics — a form of chaos introduced by Leonid Pavlovich Shilnikov — to nonsmooth systems. The scenario introduces a fundamental notion of nonsmooth dyamics, that of trajectories (in this case the crucial homoclinic orbit) that can 'slide' along a discontinuity. Shilnikov's scenario is moreover shown to occur naturally as an equilibrium hits a discontinuity, helping with a fundamental yet complex problem, namely that of extending the notion of boundary equilibrium bifurcations beyond systems of two variables.

A nonsmooth dynamics volume could not be compiled without a foray into contact mechanics, and our next two papers show this well aged topic to be still very much alive. An article by Peter Varkonyi extends recent breakthroughs in resolving the Painlevé paradox to systems of multiple contacts. Painlevé's problem describes an indeterminacy that arises in the rigid body contact when an inclined rod is pushed along a surface, resulting in behaviours like jamming and the peculiar 'impact without collision'. Here Peter finds how certain of these phenomena extend to multiple contacts, develops the methodology to study them, and reveals new sources of singularity and indeterminacy along the way.

The next article shows how such ideas are being put to use in real technological challenges, as Vahid Vaziri, Marcin Kapitaniak and Marian Wiercigroch present experimental data showing stick-slip vibrations in a drillstring. A careful modeling of the sliding dynamics (motion along a discontinuity surface) is used to control these vibrations, and this is put into practice directly in an industrial-level experimental apparatus.

From there we turn to the life sciences. Roderick Edwards presents some of the novel phenomena arising from nonsmooth dynamics in gene regulatory models, along with the methods being used to tackle them. The article shows how a system encountering a discontinuity can be seen to pass through, or to pause for an arbitrary time, under slight changes in the system or its dynamical model, and indeterminacy again plays an important role. Our second living application could not be more 'every day', as the sleep-wake cycle is modelled by Matthew Bailey, Anne Skeldon and Gianne Derks. With seemingly simple models harking back to theoretical analyses 3 of 'maps with gaps' on the circle, the authors reveal novel phenomena organized by Arnold tongues arising from

border collision bifurcations. Stephen Coombes, Yi Ming Lai, Mustafa Sayli, and Rüdiger Thul then take us into the brain itself, in a rich problem modeling firing activity in the cerebral cortex, looking at synchronization on a switching network studied by means of saltation matrices and Floquet multipliers — a glimpse of a wide range of methodologies applied to a complex nonsmooth system.

In Julie Leifeld's article we see that the climate continues to provide equal measures of challenge and opportunity for nonlinear dynamics modeling, a place to stretch our understanding at the same time as illustrating novel phenomena. Here Julie uses a model of ocean convection to challenge the current theoretical knowledge of boundary equlibrium bifurcations — equilibria hitting discontinuity surfaces — resulting in changes of stability and the birth of oscillatory behaviour.

Oscillatory or periodic behaviour turn up in a number of these papers, unsurprisingly. An approach to studying oscillatory behaviour in complex systems, and in phase-randomizing applications to medicine for example, has been the study of asymptotic phase — essentially the phase a trajectory will converge upon as it approaches a periodic attractor. This allows us to ask what happens to the relative phase of oscillations if they are perturbed from their steady oscillations, a problem addressed here for nonsmooth system by an article by Park, Shaw, Chiel and Thomas. Bernard Brogliato closes the volume with a different approach to nonsmooth systems, the description of discontinuities in terms of complementarity conditions, discussing existence, uniqueness, and stability for extensions to Moreau's general formalism of unilateral constraints.

We hope you find the range of topics and challenges here stimulating.

From the guest editors Mike Jeffrey and Petri Piiroinen