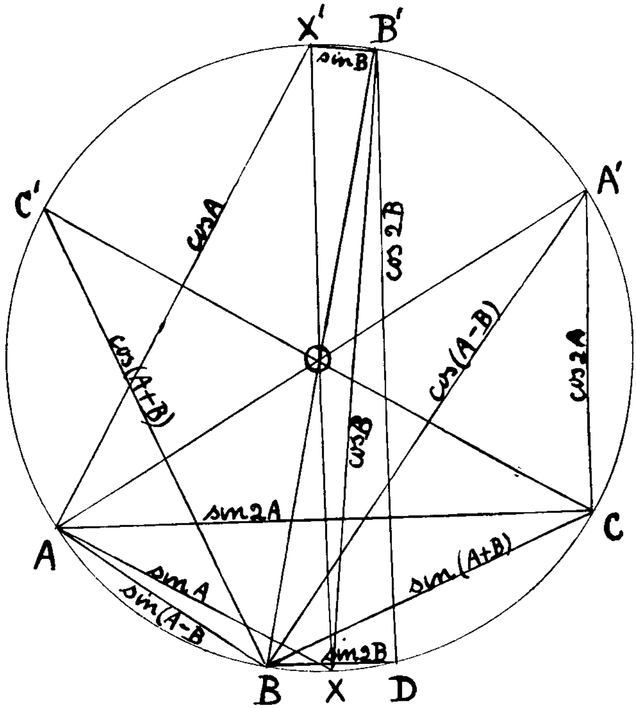
Ptolemy's Theorem and certain Trigonometrical Formulæ.—Let $XOX' = 1$ be the diameter of a circle,

$$\angle AOX = \angle COX = 2A, \quad \angle BOX = \angle DOX = 2B.$$



Then the lines representing the sines and cosines of the angles,

$$2A, \ 2B, \ A, \ B, \ A+B, \ A-B$$

are marked in the figure.

BXCX′ is a cyclic quadrilateral.

∴ by Ptolemy's Theorem, $BC \,.\, XX' = CX \,.\, BX' + B'C \,.\, BX$

$$\text{i.e. } \sin(A+B) \,.\, 1 = \sin A \,.\, \cos B + \cos A \,.\, \sin B.$$

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In like manner we derive from

ABXX′, $\sin(A-B)=\sin A\cos B-\cos A\sin B$.

C′BXX′, $\cos(A+B)=\cos A\cos B-\sin A\sin B$.

A′XBX′, $\cos(A-B)=\cos A\cos B+\sin A\sin B$.

AX′A′X, $1=\cos^2 A+\sin^2 A$.

AXCX′, $\sin 2A=2\sin A\cos A$.

XCA′X′, $\cos 2A=\cos^2 A-\sin^2 A$.

ABDC, $\sin 2A\sin 2B=\sin^2(A+B)-\sin^2(A-B)$.

ADCB′, $\sin 2A\cos 2B=\sin(A+B)\cos(A+B)+\cos(A-B)\sin(A-B)$

B′A′CD, $\cos 2A\cos 2B=\cos^2(A+B)-\sin^2(A-B)$.

etc.

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