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Toward inertial reference frames with the SIM observatory

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Abstract. The SIM Lite Observatory is expected to provide a global astrometric reference frame surpassing the 1-μas accuracy threshold in some spherical harmonics. A range of time-varying physical distortions of the reference frame will become observable as large-scale perturbations of the proper motion field. I consider the main sources of the apparent and physical motion of reference objects, such as the aberration of light caused by the acceleration of SIM, long gravitational waves and hypothetical rotation of the Universe, and present some estimates of the astrometric sensitivity to these effects. I argue that a global solution and covariance analysis is of crucial importance for the SIM mission to differentiate the inevitable accidental and systematic zonal errors from real physical phenomena.

Keywords. astrometry, reference systems, gravitational waves, large-scale structure of universe

1. Introduction

The standard of inertiality of reference frames has been rising along with the remarkable progress in the accuracy and the density of astrometric systems. The SIM Lite Observatory will approach, for the first time, the level of 10^{-12} rad in accuracy, which poses new problems in the characterization of observable effects related to the accelerating motion of the instrument and the systemic motion of the frame objects. A number of key projects with SIM rely on the rigidity and non-rotation of the SIM reference frame. For example, the dark matter potential of the Milky Way halo can be mapped from ultra-precise measurements of the proper motions of hyper-velocity stars or satellite dwarf galaxies (Unwin *et al.* 2008), but this method requires the residual rotation and other large-scale distortions of the reference frame to be much less than the magnitude of the effects to be measured. Besides the measurement errors, which are accurately estimated in a global astrometric solution (Makarov & Milman 2005), the proper motions of distant quasars will include a number of relativistic effects, making them significantly nonzero.

2. Categories of global reference frame perturbations

An astronomical reference frame is realized through a catalog of angular coordinates of a set of reference objects. Even the most distant astronomical objects are not fixed in position on the celestial sphere; as their angular coordinates change with time, the reference frame undergoes perturbations at various spatial scales. Not all of the predictable perturbations are observable with astrometric techniques. For example, the *physical* motion of extragalactic reference frame objects includes the following categories:

- Rotation of the Universe
- Peculiar motion of quasars and galaxies.

A number of relativistic effects and cosmological phenomena result in the *apparent* motion of reference objects, i.e.

- Spacetime ripples (gravitational waves)
- Gravitational deflection of light
- Acceleration of the Milky Way
- Acceleration of the Sun (secular aberration)
- Acceleration of the observer.

Rotation of the Universe. A generalization of the Friedman model of a homogeneous universe includes a systemic rotation of matter besides the isotropic expansion (Gamow 1946 and Gödel 1949). A global vorticity of the observable matter may result in a number of interesting phenomena, but for an astrometric mission, it mostly results in a rigid rotation of the reference frame. Most of the estimates of the present-day rate of rotation are bounded to $\sim 10^{-13}$ (Li 1998) – 10^{-14} (Hawking 1969) rad s^{-1}, but for certain Bianchi models, rates of up to 10^{-12} can be realized (Ciufolini & Wheeler 1995). The latter estimate is close to the sensitivity of SIM (a few parts in 10^{-12}). However, an astrometric instrument can only measure the angular distances between celestial objects, and any unitary transformation of the reference frame is not directly observable. Speculatively, a global systemic rotation of distant quasars can be detected with respect to an ideal local gyroscope. A mechanical gyroscope matching the required astrometric accuracy is not feasible, but the orbital motion of SIM around the Sun, and the motion of the Sun around the Galactic barycenter, are the substitutes that can be considered. The orbital plane of any free-falling body will slowly tilt with respect to the extragalactic reference frame. This tilt is observable with SIM as a drift in the dipole direction of the global aberration pattern.

Relic gravitational waves. The existence of stochastic, long gravitational waves has not been proven by experiment, but it is consistent with the generic inflationary models (Grishchuk 2006). A monochromatic, single-polarization gravitational wave propagating through the local part of the Universe will cause the apparent direction to a quasar in the transverse direction to oscillate with the frequency of the wave, whereas a quasar observed in the wave direction will not change its position (Fig. 1). On the limited time scale of the SIM mission (5 years), the former quasar will have a net proper motion with respect to the latter quasar. A global grid of quasars will be affected by a smooth pattern of apparent proper motions, represented by the second- and higher-order vector spherical harmonics. The magnitude of these proper motions depends on the amplitude of the relic gravitational waves and on their power spectrum. Different frequencies and modes of polarization may be present today, but most theoretical estimations seem to yield a low energy density $\Omega_{\rm GW} h^2 < 10^{-9}$ (Smith *et al.* 2006). At a signal-to-noise ratio of 10, SIM is expected to be sensitive to energy densities of order 10^{-6}. Given these numbers, the pulsar timing method holds better prospect of the actual experimental discovery of relic gravitational waves. It is noted, however, that compared to other planned and operating experiments in gravitational wave astronomy, such as LIGO, LISA and BBO, SIM will be sensitive to much longer waves, just shortward of the expected Big Bang nucleosynthesis mode ($\nu_{\rm BBN} \approx 10^{-11}$ s).

Aberration. The free fall of the Solar System onto the center of the Galaxy makes the stellar aberration pattern vary with time. The resulting proper motion field is represented by the three first-order electric harmonics (Kopeikin & Makarov 2006). The magnitude of this secular aberration effect is ~ 4 μas/yr, and it will be confidently measured by SIM. Subsequently, this perturbation can be calibrated out of the observed proper motion field of grid stars, and thus, the reference frame can be brought to the Galactic barycenter standard of rest. But the Galaxy is probably involved in a nonlinear motion due to, for

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example, the gravitational attraction to the other members of the Local Group. If this acceleration is not much smaller than 6 mm s^{-1} yr^{-1}, which is the expected acceleration of the Sun toward the Galactic center, it should result mainly in an offset of the axis of the proper motion dipole from the Galactic center. Other short-term variations in the reference frame positions due to the deflection of light by Solar System bodies, and higher-order relativistic effects, such as the coupling terms between stellar aberration and parallax (Klioner 2003), should be carefully subtracted from the data prior to this analysis.

3. Description of reference frame distortions

The time-varying component of large-scale reference frame distortions is represented by a series of vector spherical harmonics on a unit sphere

$$\mu(\lambda,\beta) = \sum_i e_i\,\mathbf{E}_i + m_i\,\mathbf{M}_i, \tag{3.1}$$

where μ is the proper motion field of reference frame objects, which is a function of the angular coordinates (λ,β), and $\mathbf{E}_i$, $\mathbf{M}_i$ are the electric and magnetic vector spherical harmonics, respectively. The coefficients of the expansion e_i and m_i include the genuine distortions of physical origin, as well as positionally correlated astrometric errors of proper motions, which are for historical reasons called "zonal errors". The latter can be systematic or accidental by origin. The physical effects of scientific interest mostly reside in the low-order vector harmonics:

$$\mu = \sum_{k=-1}^{1} e_1^k\,\mathbf{E}_1^k + \sum_{k=-1}^{1} m_1^k\,\mathbf{M}_1^k + \sum_{k=-2}^{2} e_2^k\,\mathbf{E}_2^k + \sum_{k=-2}^{2} m_2^k\,\mathbf{M}_2^k + \ldots. \tag{3.2}$$

The first term in the right-hand part, which is the set of three first-order electric harmonics or dipoles, includes the effects of secular aberration. The second term, the first-order magnetic harmonics, represents the net rotation of the reference system as a whole. The third and the fourth terms, the second-order harmonics, include most of the perturbation caused by long gravitational waves (Gwinn *et al.* 1997).

It is well known that a global astrometric solution in its canonical form, which includes positions, parallaxes and proper motions of grid stars, is deficient in rank by 6. The 6 indefinite parameters are a rigid rotation and a rigid spin of the reference frame, i.e.,

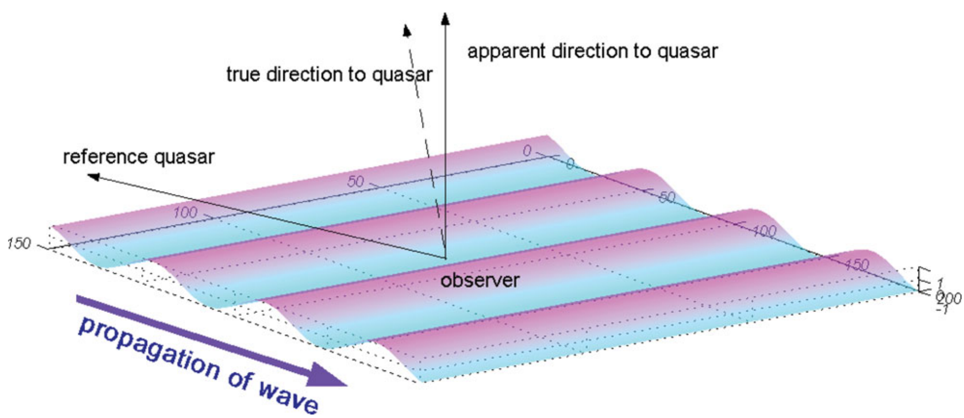

Figure 1. Astrometric effects of a propagating gravitational wave.

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the first-order magnetic harmonics of positions and proper motions. The deficiency of rank can be circumvented by constraining the solution for suitably chosen "one and a half" stars in the grid catalog, or by constraining the systemic spin (in the case of proper motions) to zero for the system of grid quasars. The latter method will carry over any accidental or systematic errors of the proper motion harmonics into the reference frame, as well as any genuine rotation of the reference frame, unless we can find other locally inertial systems or bodies available to observation.

4. Galactic velocity field

Hipparcos was not sensitive enough to observe quasars or AGN, and its reference frame is composed of Galactic stars. The stars in the local part of the Galaxy are involved in the differential rotation, azimuthal shear, asymmetric drift, warp, and local dilation or contraction. Each of these patterns is larger in magnitude than the cosmological and relativistic effects considered in this paper. The presence of dynamical streams, associations and the natural dispersion of velocities further complicate the effort to detect more subtle effects at the 1-mas level. Today, some of the phenomena in the local velocity field remain poorly understood or outright mysterious. A fit to the linear Ogorodnikov-Milne model of the local velocity field, reveals at least one unexpected, statistically significant magnetic term (Makarov & Murphy 2007). This term may be related to the Galactic warp, but it has the opposite sign with respect to a static warp model. Furthermore, when a higher-order vector harmonic model is fitted to the data, and all the "known" Ogorodnikov-Milne terms are subtracted, the residual proper velocity field displays a fascinating pattern (Fig. 2), characterized by a few overlapping multipoles and two conspicuous foci, which are close to, but not quite coincident with, the direction of the solar motion with respect to the Local Standard of Rest. Is this field real, or just a

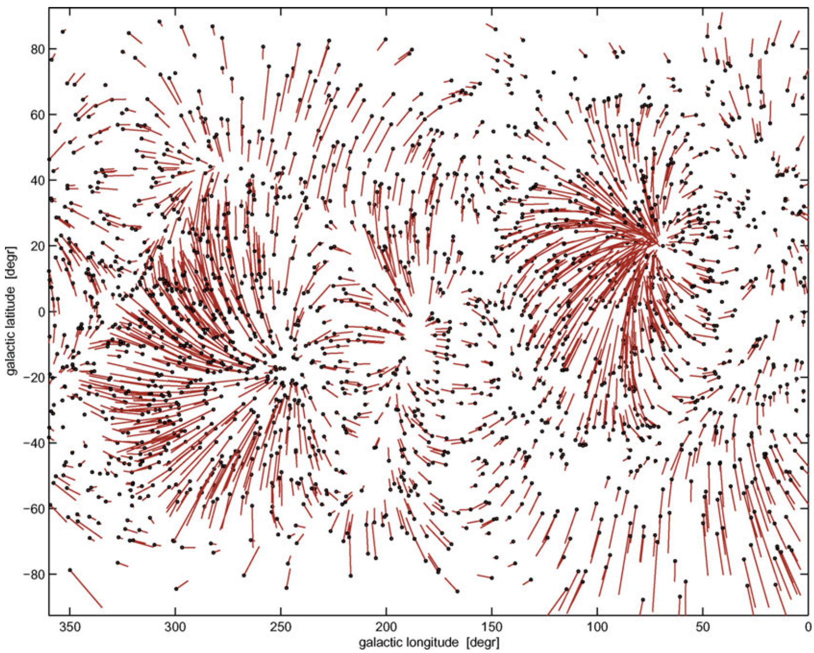

Figure 2. Residual pattern of higher-order proper motion harmonics in the Hipparcos data.

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realization of zonal error within the expected variance? It is impossible to answer this question without a rigorous covariance analysis of the zonal error propagation.

5. Benefits of a global solution

A global astrometric problem in the perturbation form can be presented as a system of linearized condition equations

$$\mathbf{A}\,\mathbf{x} = \mathbf{y}, \tag{5.1}$$

where $\mathbf{A}$ is the global design matrix, $\mathbf{x}$ is the vector of unknowns, including the complete set of astrometric parameters for all grid objects, instrument calibration parameters and the attitude corrections, and $\mathbf{y}$ is the vector of observations in the differential form "observed minus calculated". A global solution is a one-step, direct Least-Squares adjustment, yielding $(\mathbf{A}^{\mathrm{T}}\mathbf{A})^{-1}$, the global covariance matrix of all mission unknowns. At first glance, such a solution is intractable due to the large size of the design matrix. Although it is block-structured and sparse, the number of unknowns (of order 10^6 for SIM and 10^7 for JMAPS) makes it impossible to store the entire normal matrix in fast memory, let alone to invert it by standart routines. However, analysis of zonal errors is simplified by the fact that we are interested in a finite set of low-order harmonics. The covariance matrix of a set of harmonics can be computed exactly, and by virtue of their near orthogonality, the covariances with the higher-order terms can be safely neglected. Once the covariance of the harmonics for all 5 astrometric parameters is known, the confidence levels of accidental perturbations in the parameters of interest, (e.g, the residual spin) are readily derived. A full global solution simulation and covariance analysis has been performed for SIM, resulting in an accurate estimation of the zonal error propagation.

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