should be borne by the $5n$ individuals already assured is

$$5n \times \frac{\frac{ve}{A}}{n\left(5+\frac{v}{A}\right)} = \frac{5ve}{5A+v};$$

thus reducing the surplus to $(V+s)-\left(v'+\frac{5ve}{5A+v}\right)$. This sum, however (which for brevity call β), being the present value of the entire profits which can ever be derived from the $5n$ policies, would, if it were all distributed forthwith, cut off the possibility of any further bonus being allotted to these members; but it is wished at present only to give such portion of this surplus as will leave a sufficiency for an *equal bonus at least*, to be divided every m years in future. The number of prospective bonuses is $\frac{v}{mA}$; hence it follows, that the sum which may *now* be looked upon as divisible surplus is $\beta \div \left(1+\frac{v}{mA}\right)$. In deducing the expression $\frac{5ve}{5A+v}$, I have supposed the $5n$ persons to be all alive at the present time; if, however, the decrements should be sufficiently numerous to render such a step necessary, a correction might be applied.

Some of the preceding operations, it is true, are in a mathematical sense only approximative; but the discrepancies arising therefrom being of small moment, and all "on the safe side," a greater nicety of calculation would in actual practice be but labour thrown away. Any modifications which might be deemed prudent, such as assuming that the future expenses will increase progressively in a certain ratio, could easily be made, the foregoing being intended as a mere indication of the general method to be pursued.

I am, Sir,
Your obedient Servant,
SAMUEL YOUNGER.

Engineers' Assurance Office, 345, *Strand*,
18*th January*, 1854.

NOTE.—Our correspondent's object might perhaps be more easily obtained as follows:—Let the amount of premiums and interest received since the last division be denoted by S, the total expenses in the same interval by E, and the "loading" of the premiums per pound be ϕ. Then the surplus fairly divisible in the case supposed will be nearly $\phi S - E$ (*see* vol. ii., page 334, of this *Journal*). Of course, the correctness of such a proceeding will depend on the accuracy with which the premiums for the risk have been assumed.—ED. *A. M.*

ASSURANCES ON ONE LIFE AGAINST ANOTHER, DURING THEIR JOINT DURATION, AND FOR n YEARS LONGER.

To the Editor of the Assurance Magazine.

SIR,—In the second volume of the *Magazine*, page 95, Mr. Peter Hardy gives a formula for determining "the present value of a reversion of £1 payable on the death of A, provided he dies before another life, B, or within n years after him." The expression is

$$\underset{\mathrm{I}}{\overset{\quad 1}{\overline{\mathrm{A}n}}}\,\mathrm{I}\mathring{\mathrm{i}} + \left(\underset{\mathrm{I}}{\overset{\quad 1}{\overline{\mathrm{A}_n\mathrm{B}}}}\,\mathrm{I}\mathring{\mathrm{i}}\right)\frac{a_n}{ar^n};$$

and for the convenience of those who may not be familiar with Mr. Hardy's notation, he deduces the following rule:—"To the value of a temporary assurance on the life of A, add the value of a reversion contingent on B surviving a life n years older than A, multiplied into the present value of £1 payable if A lives n years."

The formula for an insurance of this description was first given, I believe, by Mr. David Jones, in his *Treatise on Life Annuities*, Art. 230; and certainly, anything more formidable in appearance than the expression he arrives at is not to be met with in the rest of that work. The object of my troubling you on this subject is to draw the attention of those who use the Carlisle 3 per cent. Tables to the facility with which such questions may be worked out, according to Mr. Hardy's formula, by the survivorship tables of Messrs. Gray, Smith, and Orchard, and the more recent actuarial tables of Mr. William Thomas Thomson.

Take, for example, the ages of 35 and 60; and suppose it were required to find the single and annual premium for an assurance of £1 upon 35 failing before 60 or within 3 years thereafter—

Thomson, Table 2, Single Deaths, Age 35.—

Whole life assurance . . . . . = ·43397
Deferred assurance, 3 years = ·40444

$\underset{\mathrm{I}}{\overset{\quad 1}{\overline{\mathrm{A}3}}}\,\mathrm{I}\mathring{\mathrm{i}}$ = Temporary assurance for 3 years = ·02953

$\frac{a_3}{ar^3}$ = ·88647 Thomson, Table 1, Single Lives, age 35.

$\overset{\quad 1}{\overline{\mathrm{A}_3\mathrm{B}}}\,\mathrm{I}\mathring{\mathrm{i}}$ = ·14503 Gray, Smith, and Orchard, Table 5, 38 against 60.

$\left(\overset{\quad 1}{\overline{\mathrm{A}_3\mathrm{B}}}\,\mathrm{I}\mathring{\mathrm{i}}\right)\times\frac{a_3}{ar^3}$ = ·14503 × ·88647 = ·128564.

$\underset{\mathrm{I}}{\overset{\quad 1}{\overline{\mathrm{A}3}}}\,\mathrm{I}\mathring{\mathrm{i}} + \left(\overset{\quad 1}{\overline{\mathrm{A}_3\mathrm{B}}}\,\mathrm{I}\mathring{\mathrm{i}}\right)\frac{a_3}{ar^3}$ = ·02953 + ·128564 = ·15809 = Single premium.

Annual premium payable during the joint lives $= \frac{\cdot 15809}{10\cdot 41} = \cdot 01518$

I now proceed to work out, for the sake of comparison, the same question according to Jones, and by his tables.

The formula is, when $m_1 - t$ is greater than $m - 1$,

$$\frac{\mathrm{M}_m - \mathrm{M}_{m+t}}{\mathrm{D}_m} + \frac{r^{t+1}\left(\mathrm{N}_{m+t-1,\,m_1-1} - \mathrm{N}_{m+t,\,m_1-1}\right) + r^t\left(\mathrm{N}_{m+t-1,\,m_1} - \mathrm{N}_{m+t,\,m_1}\right)}{2\,\mathrm{D}_{m,\,m_1}}.$$

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Here $m=35$	$r^{t+1}=r^4=\cdot 8885$
$m_1=60$	$N_{m+t-1,m_1-1}=N_{37,59}=33088097\cdot 7$
$t=3$	$N_{m+t,m_1-1}=N_{38,59}=32592121\cdot 7$
$M_m=M_{35}=826\cdot 9604$	$r^t=r^3=\cdot 9151$
$M_{m+t}=M_{38}=770\cdot 6867$	$N_{m+t-1,m_1}=N_{37,60}=30326495\cdot 8$
$D_m=D_{35}=1905\cdot 566$	$N_{m+t,m_1}=N_{38,60}=29876452\cdot 1$

$$2D_{m,m_1}=2D_{35,60}=2\times 3315526\cdot 4=6631052\cdot 8$$

And the above formula becomes, by substitution,

$$\frac{56\cdot 2737}{1905\cdot 566}+\frac{\cdot 8885\times 495976+\cdot 9151\times 450043\cdot 7}{6631052\cdot 8}$$

$=\cdot 02953+\cdot 12856=\cdot 15809$, the single premium, as before.

It will be seen that the expression given by Mr. Hardy, when the tables of Mr. Thomson, and Messrs. Gray, Smith, and Orchard, are used, affords the means of ascertaining the premium for risks of this description much more readily than the method of solution given by Jones. Insurances of the kind have become very frequent of late; and it is impossible for the practical actuary to estimate too highly the services of those who are at the trouble of pointing out the most concise methods of solution, and who supply us with such valuable auxiliary tables as those which Mr. David Jones, Mr. William Thomas Thomson, and Messrs. Gray, Smith, and Orchard, have had the courage to compute.

I am, Sir,
Yours faithfully,
ROBERT TUCKER.

Lombard Street,
25*th February*, 1854.

Postscript.—By the time this letter appears in the *Magazine*, many of your readers will have heard with regret of the death of Mr. William Orchard, one of the writers above alluded to. The loss at so early an age of one who had already done so much and had given promise of still greater things, must be deplored by all who are interested in actuarial pursuits. Mr. Orchard was one of the earliest applicants for admission to the Institute; he was likewise among the first of the Associates who came forward for examination; and it will be in the recollection of the members, that he passed with such great credit as to induce the Examiners to recommend him for election as a Fellow, in conjunction with Mr. Bailey and Mr. Porter. Although Mr. Orchard was not at that time so well known by his writings, he was much to be commended for undergoing that voluntary examination, which perhaps was scarcely necessary, to place beyond a doubt his fitness to become an actuary. He was a self educated mathematician, and owes the reputation he has earned entirely to his own unaided exertions and persevering industry. It is understood he has left behind him in an unfinished state some valuable contributions to the theory of life contingencies, which by directions in his will are to be confided to an intimate friend, and it is to be hoped they will some day be made public.