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On monopole and confinement

Though our knowledge of confinement is, at present, quite poor, it is necessary to discuss briefly the approach of monopoles where some activities have been investigated recently for understanding the mechanism of confinement.[1] One of the most favoured mechanism of confinement is the one due to monopole condensation [620], where one has to see if the monopole condensation occurs in the confined phase but not in the deconfined one. Dual-superconductivity mechanism of confinement assumes the formation of an Abrikosov-type tube between heavy quarks introduced into the vacuum via the Wilson loop,[2] while the tube itself is a classical solution of the equations of motion of the Higgs-type model Lagrangian of the action:

$$S_{\text{eff}} = \int d^4x \left[|D_\mu \phi|^2 + \frac{1}{4} G^2_{\mu\nu} + V(|\phi|^2) \right] , \tag{46.1}$$

where ϕ is a scalar field with a non-zero magnetic charge, $G_{\mu\nu}$ is the field strength tensor built from the dual-gluon field B_μ, D_μ is the covariant derivative, and $V(|\phi|^2)$ is the potential energy:

$$V(|\phi|^2) = \frac{m^2}{2} |\phi|^2 - \frac{\lambda}{4} |\phi|^4 , \tag{46.2}$$

ensuring that $\langle \phi \rangle \neq 0$ in the vacuum. If $m^2 < 0$, the potential has the typical Mexican shape and $|\phi|^2 = m^2/\lambda$. However, the relation of these effective fields to the fundamental ones of QCD is not yet clear, which is the main limitations of the use of this effective theory. However, there is not, at present, any answer to this question, and the answer can only come from the data, which are, at present, lattice measurements. This lack of understanding concerns the nature of non-perturbative field configurations defined as monopoles in non-Abelian gauge theories. The few knowledge one has is that monopoles are intrinsically $U(1)$ configurations. However, it is not a priori clear which $U(1)$ subgroup of e.g. $SU(2)$ is to be taken for classifying the monopoles. If one takes the most successful *maximal Abelian projection* [617–619], and associates a conserved magnetic charge to any operator in the adjoint representation, we still have very little understanding of the field configurations describing

[1] For reviews, see e.g. [617–619].

[2] The energy of the flux tube is proportional to the length of the flux implying that an infinite energy is needed for dissociating at infinite distance a monopole–antimonopole pair.

monopoles in this projection, and in particular on the monopole size. Lattice measurements indicate that magnetic charges condense in the confined phase, and is independent of the specific choice of the Abelian projection [619]. On the other, a lattice measurement of the monopole size gives the radius [621]:

$$R_{\rm mono} \approx 0.06 \text{ fm} , \tag{46.3}$$

defined in terms of the full non-Abelian action associated with the monopole and not in terms of the projected action. It is relatively small compared with the temperature of the confinement–deconfinement phase transition:

$$T_c \approx 300 \text{ MeV} , \tag{46.4}$$

corresponding to a distance $d_{\rm mono} \sim 1/T_c \sim 0.5$ fm. An attempt to understand the origin of this scale hierarchy has been investigated in [617] using monopole cluster assuming that monopole condensation occurs when the monopole action is UV divergent. However, one expects that the onset of condensation in the standard field theoretical language corresponds to the zero mass of the magnetically charged field ϕ. This apparent contradiction can be understood from the kinematical relation between the physical mass $m_{\rm phys}$ entering in the propagator of the scalar field and the mass $M \equiv S/L$ defined in terms of the Euclidian action, where L is the length of the trajectory and S the corresponding action on a cubic lattice with spacing a. To leading order in ma:

$$m^2_{\rm phys} \cdot a = M - \frac{\ln 7}{a} , \tag{46.5}$$

where $\ln 7$ originates from the fact that a trajectory of length L can be realized on a cubic lattice in $N_L = 7^{L/a}$ various ways. At each step, the trajectory can be continued on an adjacent cube, where in four dimensions one has eight such cubes. Zakharov [617] argues that the data on monopole action imply a fine tuning:

$$M_{\rm mono}(a) - \frac{\ln 7}{a} \ll M_{\rm mono}(a) \sim \Lambda_{\rm QCD} , \tag{46.6}$$

where $\ln 7$ is of pure geometrical origin and $M_{\rm mono}$ is the monopole energy defined on a compact $U(1)$ group as:

$$M_{\rm mono}(a) = \frac{1}{8\pi} \int \vec{B}^2 d^3r \sim \left(\frac{c}{e^2}\right)\left(\frac{1}{a}\right) , \tag{46.7}$$

where c is a constant, e is the electric charge and $g_m = 1/2e$ is the magnetic charge. Analysis of lattice data [618] suggests that the actual physical size $R_{\rm phys}$ of the monopole can be much smaller than that in Eq. (46.3). By $R_{\rm phys}$ one means the distance where the excess of the monopole action is parametrically smaller than the action associated with the zero-point fluctuations. Using the running of the QCD coupling and the condition due to the $U(1)$ critical coupling $e_c^2 \approx 1$ at which the monopole condenses, one obtains the scale:

$$M_{\rm phys} \approx 1 \text{ TeV} \tag{46.8}$$

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giving the electroweak scale rather than the QCD one of the order of $\Lambda_{\rm QCD}$, therefore indicating that QCD projected onto the scalar-filed theory via monopoles corresponds to a fine-tuned theory. This result suggests a $SU(2)$ lattice measurements at $\beta = 4$ rather than the present results at $\beta = 2.6$, which is too low to see the dissolution of monopoles at short distance. This is a subject that deserves further investigations.

It is also often stated that the symmetry responsible for confinement is different in pure gauge theory and in the presence of quarks. In pure gauge theory, the order parameter is the vacuum expectation value of the Polyakov line, and the symmetry is Z_N, the centre of the group. Since in the presence of quarks, Z_N is explicitly broken, one might expect that the order parameter is the chiral quark $\langle\bar\psi\psi\rangle$ condensate, which is responsible for spontaneous breaking of the chiral symmetry, although it is also known that the quark masses explicitly break chiral symmetry. However, the relation between confinement and chiral symmetry is not clear at all. Lattice simulations indicate that the two transitions take place at the same temperature, but there is no explanation of this numerical observation.

Another point is that if dual superconductivity in all Abelian projections is the symmetry behind confinement, then it should also work in full QCD.

Finally, there are the recent attempts [622,623] to tackle the confinement problem using QCD perturbation theory. The approach is based on a gluon chain model in the large N_C QCD, which gives a *string-like* picture of hadrons although confinement is not built in. One can modify the Born approximation by introducing a non-local counterterm for an *IR renormalization* of the Coulomb potential, which now possesses a linear term proportional to the QCD string tension. The arbitrary IR subtraction point can be optimized by using a variational method. It reaches its optimal value at that of the string tension. The procedure induces a mass to the gluon which, in some sense, is similar to the tachyonic gluon mass introduced by [161] at short distance. Some further examples of the applications of the approach to confinement are discussed in [623].

From this short summary, we conclude that though there has been progress towards an understanding of confinement via monopole condensation, but there remain some unclarified points that still need further investigation. The perturbative approach to confinement looks promising.