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LINEAR MAPS ON FACTORS WHICH PRESERVE THE EXTREME POINTS OF THE UNIT BALL

Dedicated to Zsuzsa Ágnes Molnár

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ABSTRACT. The aim of this paper is to characterize those linear maps from a von Neumann factor A into itself which preserve the extreme points of the unit ball of A. For example, we show that if A is infinite, then every such linear preserver can be written as a fixed unitary operator times either a unital $*$-homomorphism or a unital $*$-antihomomorphism.

Introduction and statements of the results. Linear preserver problems deal with the question of characterizing those linear maps on matrix algebras which leave a certain subset, function or relation invariant. For example, in the first mentioned case this means that the problem is to describe those linear maps Φ on a matrix algebra M for which $\Phi(S) \subset S$ holds true where S is a given subset of M. In fact, these problems represent one of the most active research areas in matrix theory (see the survey paper [LiTs]). In the last decade considerable attention has been paid to the infinite dimensional case as well, *i.e.* to linear preserver problems concerning linear maps acting on operator algebras rather than matrix algebras (see the survey paper [BrSe]). The linear preserver problem we intend to investigate below is in an intimate connection with the problem of unitary group preservers which are the linear maps leaving the set of unitaries in M invariant. The finite dimensional case of this problem was treated in [Mar], while the one on $B(H)$ (the algebra of all bounded linear operators acting on the Hilbert space H) and on a general C^*-algebra were solved in [Rai] and in [RuDy], respectively. In [Rai, Section 4] the problem of characterizing those linear maps on $B(H)$ which preserve the extreme points of the unit ball of $B(H)$ was implicitly raised and concerning bijective linear selfmaps of $B(H)$ which preserve the extreme points in question in both directions (*i.e.* the maps as well as their inverses are supposed to preserve the set of those extreme points) the author obtained a complete description. The connection between unitary group preservers on $B(H)$ and linear maps preserving the extreme points of the unit ball of $B(H)$ is that in the first case our maps preserve the set of all bijective partial isometries while in the

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second case they preserve the set of all injective or surjective partial isometries (see [Hal, Sections 98, 99] and Lemma 1, Lemma 2 below).

The other motivation to our present investigations is the following. In the paper [LaMa] linear maps between C^*-algebras whose adjoint preserve the extreme points of the dual ball were studied and they turned out to give a valuable clue as to what objects may be regarded as "non-commutative composition operators". Now, it seems to be a natural problem to consider linear maps in general, *i.e.* without the assumption of being the adjoints of linear maps on a C^*-algebra, which preserve the extreme points of the unit ball. For example, since $B(H)$ is the dual space of the Banach algebra of all trace-class operators on H (which is a highly non-C^*-algebra), in this particular but undoubtedly very important case the problem has nothing to do with the one treated in [LaMa]. In fact, as one can see below, we follow a completely different approach to attack this problem.

As indicated in the abstract, we solve the problem of determining all linear maps which preserve the extreme points of the unit ball in the case when the underlying algebra is a von Neumann factor. One might have the opinion that we should consider the problem for example for general C^*-algebras but an easy example shows that in that generality we cannot expect almost anything. In fact, if the underlying algebra is the C^*-algebra $C(H)$ of all compact operators acting on an infinite dimensional Hilbert space H, then, since the unit ball of this Banach space has no extreme points at all, our preservers are the linear maps on $C(H)$ without any further properties. So, to obtain a more satisfactory result we have to suppose something more. In what follows we solve the problem in the case when our preservers act on von Neumann factors emphasizing the particular case of $B(H)$.

In view of the results as well as their proofs we have to remind the definition of Jordan $*$-homomorphisms. A linear map J between $*$-algebras $\mathcal{A}$ and $\mathcal{B}$ is called a *Jordan $*$-homomorphism* if

$$J(x)^2 = J(x^2)$$
$$J(x)^* = J(x^*)$$

hold true for every $x \in \mathcal{A}$. Observe that by linearization, *i.e.* replacing x by $x+y$, the first equation above is equivalent to $J(x)J(y) + J(y)J(x) = J(xy + yx)$ $(x, y \in \mathcal{A})$.

Let us now summarize the results of the paper.

THEOREM 1. *Let $\mathcal{A}$ be an infinite factor. The linear map $\Phi: \mathcal{A} \longrightarrow \mathcal{A}$ preserves the extreme points of the unit ball of $\mathcal{A}$ if and only if either there are a unitary operator $U \in \mathcal{A}$ and a unital $*$-homomorphism $\Psi: \mathcal{A} \longrightarrow \mathcal{A}$ such that Φ is of the form*

$$\Phi(A) = U\Psi(A) \quad (A \in \mathcal{A})$$

or there are a unitary operator $U' \in \mathcal{A}$ and a unital $*$-antihomomorphism $\Psi': \mathcal{A} \longrightarrow \mathcal{A}$

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such that Φ *is of the form*

$$\Phi(A) = U'\Psi'(A) \quad (A \in \mathcal{A}).$$

As a consequence of this result we immediately have the structure of surjective linear selfmaps of $B(H)$ which preserve the extreme points of the unit ball. In fact, Corollary 1 below is a significant generalization of a result of Rais [Rai, Lemma 3 and Corollary 1] who obtained a similar result but worked under the quite restrictive assumption that the maps under consideration are bijective and preserve the extreme points of the unit ball in both directions.

COROLLARY 1. *Let* H *be an infinite dimensional Hilbert space. Then the surjective linear map* $\Phi: B(H) \to B(H)$ *preserves the extreme points of the unit ball of* $B(H)$ *if and only if either there are unitaries* $U, V \in B(H)$ *such that* Φ *is of the form*

$$\Phi(A) = UAV \quad \big(A \in B(H)\big)$$

or there are antiunitaries $U', V' \in B(H)$ *such that* Φ *is of the form*

$$\Phi(A) = U'A^*V' \quad \big(A \in B(H)\big).$$

REMARK. We note that, as one can see from the proof of Corollary 1, it would have been sufficient to assume that the range of Φ contains a rank-one operator instead of supposing that Φ is surjective.

If the underlying Hilbert space is separable, then we can write our linear preservers on $B(H)$ in a more detailed form than it was obtained in the statement of Theorem 1.

COROLLARY 2. *Let* H *be a separable infinite dimensional Hilbert space. The linear map* $\Phi: B(H) \to B(H)$ *preserves the extreme points of the unit ball of* $B(H)$ *if and only if either there are a unitary operator* V *and a collection* $\{U_\alpha\}$ *of isometries with pairwise orthogonal ranges which generate* H *such that* Φ *is of the form*

$$\Phi(A) = V\Big(\sum_\alpha U_\alpha A U_\alpha^*\Big) \quad \big(A \in B(H)\big)$$

or there are a unitary operator V *and a family of anti-isometries* $\{V_\alpha\}$ *with pairwise orthogonal ranges which generate* H *such that* Φ *is of the form*

$$\Phi(A) = V\Big(\sum_\beta V_\beta A^* V_\beta^*\Big) \quad \big(A \in B(H)\big).$$

Our second main result describes our linear preservers in the case of any finite von Neumann algebras.

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THEOREM 2. *Let $\mathcal{A}$ be a finite von Neumann algebra. The linear map $\Phi: \mathcal{A} \longrightarrow \mathcal{A}$ preserves the extreme points of the unit ball of $\mathcal{A}$ if and only if there exist a unitary operator $U \in \mathcal{A}$ and a unital Jordan $*$-homomorphism Ψ such that*

$$\Phi(A) = U\Psi(A) \quad (A \in \mathcal{A}).$$

Concerning matrix algebras, we immediately have the last assertion of the paper (*cf.* [Mar]).

COROLLARY 3. *Let $\mathcal{M}$ be the algebra of all complex $n \times n$ matrices. The linear map $\Phi: \mathcal{M} \longrightarrow \mathcal{M}$ preserves the extreme points of the unit ball of $\mathcal{M}$ if and only if there are unitary matrices $U, V \in \mathcal{M}$ such that Φ is either of the form*

$$\Phi(A) = UAV \quad (A \in \mathcal{M})$$

or of the form

$$\Phi(A) = UA^{\mathrm{tr}}V \quad (A \in \mathcal{M})$$

where $^{\mathrm{tr}}$ denotes the transpose.

Proofs. The statements of Lemma 1 and Lemma 2 are guessed to be well-known. However, since we have not found any trace of them in the bibliography of Kadison and Ringrose [KaRi1-2], for the sake of completeness we present them with proofs.

LEMMA 1. *Let $\mathcal{A}$ be a factor. The operator $A \in \mathcal{A}$ is an extreme point of the unit ball of $\mathcal{A}$ if and only if A is either an isometry or a coisometry.*

PROOF. It is well-known that in an arbitrary C^*-algebra $\mathcal{B}$, the extreme points of the unit ball are exactly those partial isometries $W \in \mathcal{B}$ for which $(I - W^*W)\mathcal{B}(I - WW^*) = \{0\}$ [KaRi2, 7.3.1. Theorem]. In a factor every two projections are comparable [KaRi2, 6.2.6. Proposition]. Let, for example, $V \in \mathcal{A}$ be a partial isometry such that $I - W^*W = V^*V$ and VV^* is a subprojection of $I - WW^*$. Therefore, we have $(V^*V)(V^*)(VV^*) = 0$. But V is a partial isometry and hence $VV^*V = V$. Consequently, we obtain that $0 = (V^*V)(V^*VV^*) = V^*VV^* = V^*$ which implies $V = 0$. This gives us that W is an isometry. ∎

PROOF OF THEOREM 1. The sufficiency is trivial to check. Let us assume that Φ preserves the extreme points of the unit ball of $\mathcal{A}$. First observe that Φ is necessarily norm-continuous. Indeed, since in an arbitrary C^*-algebra every self-adjoint operator of norm ≤ 1 is the arithmetic mean of two unitaries, we easily obtain that $\|\Phi\| \leq 2$.

Consider now the operator $V = \Phi(I)$. Since V is either an isometry or a coisometry, without loss of generality we may and do suppose that $V^*V = I$. Since the unitary group is arcwise connected in $\mathcal{A}$, we infer that $\Phi(U)$ is an isometry for every unitary $U \in \mathcal{A}$.

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Let us define a linear map $\Psi\colon \mathcal{A} \to \mathcal{A}$ by $\Psi(A) = V^*\Phi(A)$ $(A \in \mathcal{A})$. In what follows we prove that Ψ is a Jordan $*$-homomorphism. To verify this, first observe that, by the fact that Φ sends unitaries to isometries, we have

$$\Phi(e^{itS})^*\Phi(e^{itS}) = I \quad (t \in \mathbb{R})$$

for every self-adjoint operator $S \in \mathcal{A}$. Using the power series expansion of the exponential function as well as its uniqueness, it is easy to conclude that

$$\Phi(I)^*\Phi(S) - \Phi(S)^*\Phi(I) = 0$$
$$-\frac{1}{2}\Phi(I)^*\Phi(S^2) + \Phi(S)^*\Phi(S) - \frac{1}{2}\Phi(S^2)^*\Phi(I) = 0.$$

These identities imply that

$$\Phi(S)^*\Phi(S) = \Phi(I)^*\Phi(S^2). \tag{1}$$

Since $\Psi(S^2) = \Phi(I)^*\Phi(S^2)$, this shows that Ψ preserves the positive as well as the self-adjoint operators. If we replace S by $S + T$ in (1) where $S, T \in \mathcal{A}$ are self-adjoint, then we obtain

$$\Phi(S)^*\Phi(T) + \Phi(T)^*\Phi(S) = \Phi(I)^*\Phi(ST + TS)$$

and one can check that this results in

$$\Phi(A^*)^*\Phi(A) = \Phi(I)^*\Phi(A^2) \quad (A \in \mathcal{A}).$$

If we linearize this equation, *i.e.* replace A by $A + B$, we get

$$\Phi(A^*)^*\Phi(B) + \Phi(B^*)^*\Phi(A) = \Phi(I)^*\Phi(AB + BA) \quad (A, B \in \mathcal{A}). \tag{2}$$

Let $P \in \mathcal{A}$ be an arbitrary projection. Since $\mathcal{A}$ is an infinite factor, by [StZs, E.4.11, p. 105] it follows that either $P \sim I$ or $I - P \sim I$. Suppose that this latter possibility is the case. Then we have an isometry $U \in \mathcal{A}$ for which $UU^* = I - P$. By (2) we can compute

$$\begin{aligned} \Phi(U^*)^*\Phi(U^*) + \Phi(U)^*\Phi(U) &= \Phi(I)^*\Phi(UU^* + U^*U) \\ &= \Phi(I)^*\Phi(2I - P) = 2 - \Psi(P). \end{aligned} \tag{3}$$

Since, by our assumption on Φ, $\Phi(U^*)^*\Phi(U^*)$ and $\Phi(U)^*\Phi(U)$ are projections, we infer that there are projections $Q_1, Q_2 \in \mathcal{A}$ such that $\Psi(P) = Q_1 + Q_2$. On the other hand, by the positivity preserving property of Ψ and (1) we have

$$\Psi(P)^2 = \Psi(P)^*\Psi(P) = \Phi(P)^*VV^*\Phi(P) \leq \Phi(P)^*\Phi(P) = \Psi(P)$$

which results in $Q_1Q_2 + Q_2Q_1 \leq 0$. Since Q_1, Q_2 are projections, we easily obtain $Q_1Q_2 = Q_2Q_1 = 0$ and this gives us that $\Psi(P)$ is a projection. It is now a standard argument to show that Ψ is a Jordan $*$-homomorphism. Indeed, if $P_1, P_2 \in \mathcal{A}$ are mutually orthogonal projections, then we know that $\Psi(P_1 + P_2) = \Psi(P_1) + \Psi(P_2)$ is a

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projection, too. Therefore, $\Psi(P_1)$ and $\Psi(P_2)$ are also orthogonal, *i.e.* Ψ preserves the orthogonality between projections. If $P_1,\dots,P_n \in \mathcal{A}$ are mutually orthogonal projections and $\lambda_1,\dots,\lambda_n \in \mathbb{R}$, then we have

$$\Big[\Psi\big(\sum_{k=1}^{n} \lambda_k P_k\big)\Big]^2 = \Big[\sum_{k=1}^{n} \lambda_k \Psi(P_k)\Big]^2 = \sum_{k=1}^{n} \lambda_k^2 \Psi(P_k) = \Psi\Big(\Big(\sum_{k=1}^{n} \lambda_k P_k\Big)^2\Big).$$

Since Ψ is norm-continuous, by the spectral theorem we have

$$\Psi(S)^2 = \Psi(S^2)$$

for every self-adjoint operator $S \in \mathcal{A}$. Using the trick of linearization we readily obtain

$$\Psi(A)^2 = \Psi(A^2) \quad (A \in \mathcal{A}).$$

Since Ψ is linear and positivity preserving, we obviously have

$$\Psi(A)^* = \Psi(A^*) \quad (A \in \mathcal{A}).$$

Consequently, we infer that Ψ is a Jordan $*$-homomorphism.

Our next claim is that $\Phi(A) = V\Psi(A)$ $(A \in \mathcal{A})$. To this end, let $U \in \mathcal{A}$ be an arbitrary unitary operator. Then $W = \Phi(U)$ is an isometry and we have

$$\Psi(U)^*\Psi(U) + \Psi(U)\Psi(U)^* = \Psi(U^*U + UU^*) = \Psi(2I) = 2I,$$

i.e.

$$W^*VV^*W + V^*WW^*V = 2I. \tag{4}$$

Since $W^*VV^*W \leq I$ and $V^*WW^*V \leq I$, by (4) it follows that $W^*VV^*W = I$ and $V^*WW^*V = I$. Since VV^*, WW^* are projections, these relations imply $\operatorname{rng} W \subset \operatorname{rng} V$ and $\operatorname{rng} V \subset \operatorname{rng} W$. Consequently, $\operatorname{rng} \Phi(U) = \operatorname{rng} V$ for every unitary operator $U \in \mathcal{A}$. Since the linear span of the unitaries in $\mathcal{A}$ is $\mathcal{A}$, it follows that $\operatorname{rng} \Phi(A) \subset \operatorname{rng} V$ for every $A \in \mathcal{A}$, and this gives us that $V\Psi(A) = VV^*\Phi(A) = \Phi(A)$ $(A \in \mathcal{A})$.

We now prove that V is unitary. Since Ψ is a unital Jordan $*$-homomorphism, by [Sto, Theorem 3.3] there is a projection E in the centre of the von Neumann algebra generated by the image of Ψ, such that the maps Ψ_1 and Ψ_2 on $\mathcal{A}$ defined by $\Psi_1(A) = \Psi(A)E$ and $\Psi_2(A) = \Psi(A)(I - E)$ are a $*$-homomorphism and a $*$-antihomomorphism, respectively. Now, if we suppose on the contrary that $\operatorname{rng} V \neq \mathcal{H}$, then by the extreme point preserving property of Φ and the equality $\Phi(A) = V\Psi(A)$ it follows that Φ sends every isometry and coisometry to a proper isometry. Let $W^*W = I$ or $WW^* = I$. In both cases we have $I = \Psi(W)^*V^*V\Psi(W) = \Psi(W)^*\Psi(W) = \Psi_1(W^*W) + \Psi_2(WW^*)$. This readily implies that

$$\Psi_1(W^*W) = E \quad \text{and} \quad \Psi_2(WW^*) = I - E. \tag{5}$$

Let $P \in \mathcal{A}$ be a projection such that $P \sim I$ and $I - P \sim I$ [KaRi2, 6.3.3. Lemma (Halving)]. Using (5) we have $\Psi_1(P) = \Psi_1(I - P) = E$, $\Psi_2(P) = \Psi_2(I - P) = I - E$.

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Since the operators appearing here are all projections, we deduce $\Psi_1(P) = \Psi_1(I) = 0$ and $\Psi_2(P) = \Psi_2(I) = 0$. It now follows that $\Psi_1 = 0$ and $\Psi_2 = 0$ which is an obvious contradiction. Therefore, we obtain that V is unitary.

It only remains to prove that either $\Psi_1 = 0$ or $\Psi_2 = 0$. Just as before, let $P \in \mathcal{A}$ be a projection for which $P \sim I$ and $I - P \sim I$. Let $W \in \mathcal{A}$ be such that $W^*W = I$ and $WW^* = P$. Then $\Psi(W)$ is either an isometry or a coisometry. Suppose that this latter one is the case. Then we have

$$I = \Psi(W)\Psi(W)^* = \Psi_1(WW^*) + \Psi_2(W^*W)$$

which gives us that $\Psi_1(P) = \Psi_1(I)$. If W' is an isometry for which $I - P = W'W'^*$, then we have $\Psi_1(W')\Psi_1(W')^* = 0$. Therefore, $\Psi_1(W') = 0$ and hence $0 = \Psi_1(W')^*\Psi_1(W') = \Psi_1(I)$. It follows that $\Psi_1 = 0$ and thus we obtain that Ψ is a $*$-antihomomorphism. In case $\Psi(W)$ is an isometry, one can argue in a very similar way. ∎

PROOF OF COROLLARY 1. Using Theorem 1, without serious loss of generality we may and do suppose that our map Φ is a surjective unital $*$-homomorphism of $\mathcal{B}(\mathcal{H})$. We assert that Φ is injective. Let P be a rank-one projection. Then there is an operator $A \in \mathcal{B}(\mathcal{H})$ such that $\Phi(A) = P$. Since $\Phi(A^*A) = \Phi(A)^*\Phi(A) = P$, we can assume that our operator A is positive. Let us consider its spectral resolution. By the monotonicity and continuity of Φ it follows that there is a Borel subset B of the spectrum of A having a positive distance from 0 for which $E(B)$ (E is the spectral measure corresponding to A) has nonzero image under Φ. Since there is a positive constant c for which $cE(B) \leq A$, we obtain $0 \neq c\Phi\big(E(B)\big) \leq P$. By the minimality of P we have $\Phi\big(E(B)\big) = P$. Now, if Φ is not injective, then its kernel, being a nontrivial ideal of $\mathcal{B}(\mathcal{H})$, contains the ideal of all finite rank operators. Hence, the projection $E(B)$ is infinite dimensional. But in this case $E(B)$ is the sum of two orthogonal projections Q_1, Q_2 which are both equivalent to $E(B)$. Using the minimality of P once again, we see that either $\Phi(Q_1) = 0$ or $\Phi(Q_2) = 0$. Now, Q_1 and Q_2 are equivalent to each other and Φ is a $*$-homomorphism, thus if any of Q_1, Q_2 is in the kernel of Φ, then so is the other one. Hence, we deduce $0 = \Phi(Q_1) + \Phi(Q_2) = P$ which is a contradiction. Consequently, we obtain the injectivity of Φ and so Φ is a $*$-automorphism of $\mathcal{B}(\mathcal{H})$. To conclude, it is a folk result that in this case there is a unitary operator $U \in \mathcal{B}(\mathcal{H})$ such that $\Phi(A) = UAU^*$ $\big(A \in \mathcal{B}(\mathcal{H})\big)$. This completes the proof. ∎

PROOF OF COROLLARY 2. Using the notation of Theorem 1 we can suppose without loss of generality that $\Phi: \mathcal{B}(\mathcal{H}) \longrightarrow \mathcal{B}(\mathcal{H})$ is a unital $*$-homomorphism. Now, using the separability of $\mathcal{H}$ and the classical result [KaRi2, 10.4.14. Corollary] on the form of the representations of $\mathcal{B}(\mathcal{H})$ on $\mathcal{B}(\mathcal{H})$, we obtain the assertion. ∎

LEMMA 2. *Let $\mathcal{A}$ be a finite von Neumann algebra. The extreme points of the unit ball of $\mathcal{A}$ are exactly the unitaries in $\mathcal{A}$.*

PROOF. By [KaRi2, 7.3.1. Theorem] every unitary operator in $\mathcal{A}$ is an extreme point of the unit ball of $\mathcal{A}$. Referring to that theorem again, suppose that $V \in \mathcal{A}$ is a partial isometry for which $(I - V^*V)\mathcal{A}(I - VV^*) = \{0\}$. Let $V^*V = E, VV^* = F$. Since $\mathcal{A}$ is

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finite, by [KaRi2, 6.9.6.] we have $I - E \sim I - F$. Let $W \in \mathcal{A}$ be a partial isometry for which $I - E = W^*W$ and $I - F = WW^*$. Since $W^*W\mathcal{A}WW^* = \{0\}$, it follows that $0 = W^*WW^*WW^* = W^*WW^* = W^*$. Consequently, we have $E = F = I$ which means that V is unitary. ∎

PROOF OF THEOREM 2. If Ψ is a unital Jordan $*$-homomorphism on $\mathcal{A}$, then taking the fact that Ψ is necessarily a contraction into consideration, by the equality

$$\Psi(U)^*\Psi(U) + \Psi(U)\Psi(U)^* = \Psi(2I) = 2I$$

we easily obtain that $\Psi(U)$ is unitary for every unitary operator $U \in \mathcal{A}$. On the other hand, if Φ preserves the extreme points of $\mathcal{A}$, *i.e.* the unitary group of the C^*-algebra $\mathcal{A}$, then by [RuDy, Corollary 2] we obtain the other part of the assertion. ∎

PROOF OF COROLLARY 3. The sufficiency is obvious. To the necessity we may suppose that Φ is a unital Jordan $*$-homomorphism. Since the algebra under consideration is finite dimensional, we obtain that Φ is an injective and hence surjective Jordan $*$-homomorphism, *i.e.* a Jordan $*$-automorphism. By a well-known theorem of Herstein [Her] Φ is either a $*$-automorphism or a $*$-antiautomorphism and, just as in the proof of Corollary 1, we are done. ∎

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