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TEST OF A NEW THEORY FOR STELLAR CONVECTION USING HELIOSEISMOLOGY

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ABSTRACT Two evolutionary models of the Sun have been tested using helioseismological data. The two models use the same input microphysics (nuclear reaction rates, opacity, equation of state) and the same numerical evolutionary code, but differ in the treatment of turbulent convection. The first model employs the standard mixing - length theory of convection, while the second one employs a new turbulent convection model which overcomes some basic inconsistencies of the standard theory of convection.

The test rests on the calculation of p-mode eigenfrequencies and on the comparison with the helioseismological data.

The comparison shows an overall improvement of the eigenfrequencies calculated with the new model with respect to those calculated with the standard model, although it appears that both models still suffer from inaccuracies especially in the treatment of the surface layers.

INTRODUCTION

The problem of stellar convection has been treated, for decades, by means of the mixing-length theory (Böhm - Vitense 1958; henceforth cited as MLT) which, in spite of the practical advantage of simplicity and the apparent ability to fit observation by adjusting a free parameter, makes use of drastic assumptions, physically unsuitable to be applied to the almost inviscid stellar interior.

A new approach to the problem of stellar turbulent convection has been recently proposed by Canuto and Mazzitelli (1991; henceforth cited as CM). The CM model overcomes some basic limitations of the standard mixing-length theory and can be employed in stellar codes without major modifications.

In order to test to what extent the CM model has improved the description of turbulent convection vis à vis the standard MLT model, two models of the Sun have been adopted. The first of them is based on the standard mixing-length theory, the second one on the new CM model in its more recent version (Canuto and Mazzitelli 1992).

A comparison between the helioseismological data (Libbrecht et al. 1990) and the p-mode eigenfrequencies calculated for the two models has been carried out.

MODELS

The convective turbulence in the interior of the stars is compressible and is characterized by a wide spectrum of eddies of all sizes.

The mixing-length theory treats the energy spectrum of turbulence as if it consisted of only one large eddy and provides an expression for the mixing length Λ which is assumed to be proportional to the local scale height through a free parameter adjusted to reproduce the observed position of a given star in the H-R diagram.

The CM treatment employs a new turbulent convection model which accounts for the full spectrum of turbulent eddies and derives a new expression for the turbulent flux as well as for the turbulent pressure, usually ignored in the MLT approach. While retaining the assumption of incompressibility , the CM model also provides an expression for Λ, given as a function of local and non-local variables, substantially independent of free parameters.

The two MLT and CM approaches have been then applied to construct two evolutionary models of the Sun adopting a numerical code described in detail elsewhere (Mazzitelli 1989). The two models have the same $L_\odot$ and $R_\odot$, use the same microphysics (equation of state, opacity, nuclear reaction rates) and the same numerical code. The difference lies only in the treatment of convective transport. Thus differences in the predicted p-mode eigenfrequencies should be ascribed to the different treatment of convection since it reflects on differences in physics variables of helioseismological interest.

In view of a helioseismological analysis, radial and non-radial p-mode adiabatic eigenfrequencies have been computed for the two models.

RESULTS AND DISCUSSION

A first comparison has been based on the analysis of modes with $0 \leq \ell \leq 11$, following the procedure described in Dziembowski et. al. (1988). For $\ell < 20$ deviations in frequencies $\delta\nu$ of theoretical eigenfrequencies either from those of another model or from the observed ones, can be expressed as:

$$\delta\nu(\nu,\ell) = [f(\nu,\ell)]_{0,r_e} + [a\nu + b]_{r_e,r_i} + [g(\nu)]_{r_i,R_\odot} \quad (1)$$

where the different functional forms describe the dependence of $\delta\nu$ on ν and ℓ in different regions located at different depths inside the Sun. In the present

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analysis $r_e = 0.735 R_\odot$ and $r_i = 0.95 R_\odot$.

In so doing the source of inaccuracies in the models can be identified by looking at the behavior of $\delta\nu$ as frequency function for the modes considered.

The dependence of the frequency differencies between CM and MLT models and observations on the observed frequencies, for modes with $0 \leq \ell \leq 11$, is dominant in both the models, thus indicating inaccuracies in the treatment of surface layers, especially above r_i=0.95 $R_\odot$ where $\delta\nu$ is a non-linear function of ν. However, the ν-dependence appears less pronounced in the CM model than in the MLT model. This result indicates that the CM model represents an improved approach to the problem of stellar convection with respect to the classical one. Moreover, since all the curves are not parallel, a small ℓ-dependence cannot be ruled out, thus indicating that also the physics of the interior needs revisions.

When the MLT and CM eigenfrequencies are compared with the observed values for some high degree modes with ℓ ranging from 40 to 1300, the CM eigenfrequencies appear, in general, closer to the observations than the MLT eigenfrequencies for all the ℓ values examined. The deviations of the theory from observations for $\ell > 400$ are in any case positive. This indicates that the theoretical sound speed in the convection zone is overestimated by both the CM and the MLT, although in the CM model the deviations are sensibly smaller. The largest deviations occur in the $1000 < \ell < 1300$ range, which concerns modes trapped in a very thin surface layer (0.006 $R_\odot$).

CONCLUSION

The present analysis has shown that both the CM and the MLT models still suffer from inaccuracies especially in the treatment of the surface layers, although some problems concerning the interior cannot be ruled out.

As a final result we infer that, compared with the MLT model, the new CM model predicts eigenfrequencies closer to the observed values. In any case some improvements to the model are required.

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