RESTORATION OF TURBULENCE DEGRADED IMAGES - A REVIEW

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ABSTRACT

The success of speckle interferometry in recovering high resolution information from atmospheric turbulence degraded images has renewed interest in the restoration of images recorded through a turbulent medium. There are a number of different approaches to image restoration which have been proposed. The effectiveness of each of these techniques is strongly dependent on the brightness and the angular extent of the object being observed. This paper will attempt to summarize the areas of application of such techniques, the current state-of-the-art in this field, and the expected performance range of the various techniques.

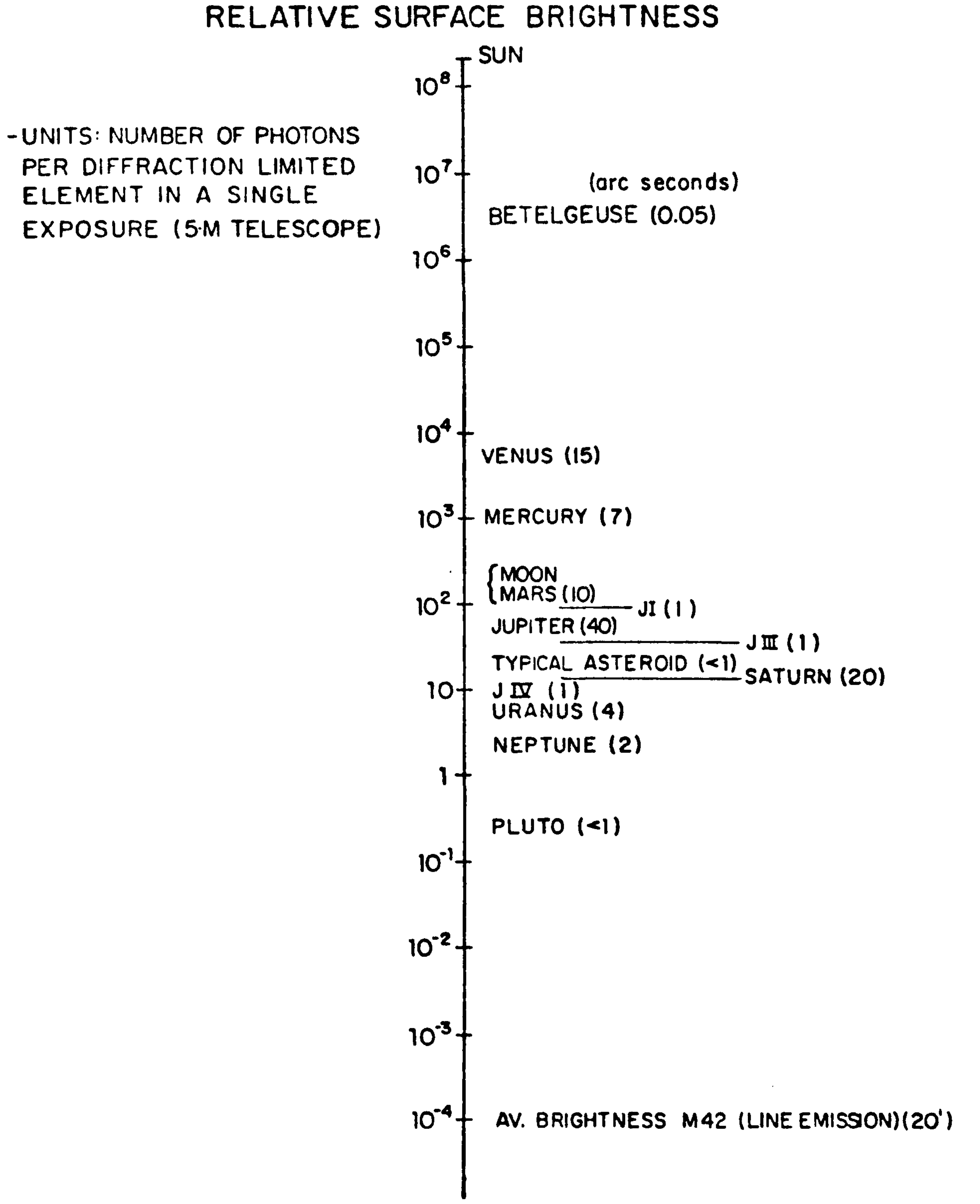

Fig. 1. Surface Brightness of Solar System Objects

https://doi.org/10.1017/S0252921100118937 Published online by Cambridge University Press

1. INTRODUCTION

The recent successes of speckle interferometry in obtaining high angular resolution measurements of objects much smaller than the seeing limited point spread function has led to the proposal and development of a number of techniques which could restore images of extended objects from speckle data. While most of the interferometry experiments are dedicated to measuring objects whose angular extent is close to the resolution limit of the interferometer, there are several classes of objects for which full image reconstruction is required, including the solar surface, planets, satellites and asteroids, as well as small extra-solar system objects with non-centrosymmetric detail.

While the earliest attempts at image restoration did not meet with success,[1] some more recent approaches, stimulated by recent work in speckle and active optics, appear extremely promising. There are considerable differences between proposed techniques, particularly in the range of object brightness and the angular extent of objects to which they could be applied.

2. HIGH RESOLUTION IMAGING PROBLEMS

Before discussing the techniques in detail, we will consider the kinds of problems one could address with high resolution imaging. Imaging small features on the solar surface is potentially one of the more physically significant problems one could consider as well as being amenable to techniques which must work at high light levels. The relative ease of image recovery, proportional to object brightness, is graphically shown in Fig. 1. There is considerable evidence for a great deal of small scale features on the sun, obtained from balloon images, images obtained under extraordinary seeing conditions with a vacuum telescope and from interferometry. A detailed program of solar surface imaging with high resolution could yield answers to what Leo Goldberg described[2] as the most important questions facing solar physics (Table 1).

There are a large number of interesting problems associated with planetary imaging, with the difficulty of restoring images approximately proportional to the square of the distance of the objects from the sun (along with the angular resolution required and the surface albedo) Such a program could include:

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Table 1

(1) Photosphere

- Size and velocity distribution of granules
- Source and structure of weak magnetic fields

(2) Chromosphere:

- Velocity distribution, horizontal and vertical, and its time dependence
- Detailed structure of chromospheric network cells
- Origin and microstructure of spicules

(3) Flares:

- Origin and physical nature
- Degree of localization
- Relation to magnetic field and coronal condensations
- Origin and physical nature of active regions

(4) Sunspots:

- Physical structure, including fine structure of umbra and penumbra
- Relation of magnetic field to fine structure
- Motions

(5) Plages and Faculae:

- Microstructure of plages and faculae
- Oscillations in faculae

(6) Prominences:

- Smallest sizes of filamentary structures
- Magnetic fields

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- Martian dust storm monitoring
- Asteroid imaging - > 250 are resolvable with a 5 m telescope
- Jupiter, surface features and satellites - $\sim$ 50 resolution elements across Galilean satellites for 5 m telescopes
- Saturn - 35 resolution elements across Titan
- Uranus - rotation rate by tracking spots, polar flattening (rings??)
- Neptune - polar flattening and rotation
- Pluto - image disk

With the proposed very large telescopes or telescope arrays, imaging stellar disks could start to yield interesting results and small extragalactic objects such as quasars or galatic nuclei could start to show some asymmetric detail if the imaging processes can perform at those light levels.

3. IMAGE RESTORATION TECHNIQUES

In recent years there has been a rapid development of image restoration techniques for general image restoration problems. A detailed discussion of various inverse filtering techniques such as constrained least squares, maximum entropy and pseudo-inverse filtering using single valued decomposition, is available in such texts as Andrews and Hunt.[3]

The limitations in the application of inverse filtering to restoration of atmospherically degraded images was explored by Goodman and Belcher[4] for long exposure (with recentering) images. They concluded that at high light levels with good seeing, near diffraction limited resolution was obtainable. However, extremely rapid degradation of the restoration was found as photon noise or seeing increased. It is worth considering that such techniques as psuedo-inverse restoration could be used for wide field solar imaging, since it has been demonstrated to work well on space-variant point spread functions, and since light levels are sufficient so that photon noise is not a problem.

4. SPECKLE HOLOGRAPHY AND BETELGENCE IMAGING

There are two techniques which have been proposed and successfully applied to image restoration from speckle data, but which appear to have a very limited set of observations to which they can be applied. The first technique, speckle holography, was originally proposed by Liu and Lohmann.[5] The approach is applied to speckle data which has been recorded under conditions where an unresolved star is in the same recorded isoplanatic field as the object one wishes to image. Standard speckle interferometry processing is then applied to the data, yielding the ensemble averaged auto-correlation image as its result. This image has the following form: on-axis is the auto-correlation of the object added to the auto-correlation of the reference star; shifted off-axis by the separation of the reference star from the object on either side are the cross-correlations of the object and reference (an identical result as is obtained in the laboratory with conventional lensless fourier transform holography) Since the reference is a single point, the off-axis terms are just deblurred images of the object. Weigelt[6] has demonstrated the imaging of a triple star system using this technique, but since most recent measurements of isoplanatism indicate that the uplooking isoplanatic angle is under 10 arc-seconds,[7] it is extremely rare that extended objects of interest will have an unresolved star in their field (see Table 2).

Table 2

Mean Seperation of Stars

Magnitude	Separation(b=0)
6 or brighter	2.5°
10 " "	20'
14 " "	2.5'
18 " "	30"
22 " "	8"

The technique applied by Worden, et al.[8] to reconstructing an image of the disk of Alpha Orionis (Betelgeuse) is also relatively simple in concept. It is based on two observations about speckle images: 1) some speckles are brighter than others; and 2) very small objects, very close to the diffraction limit will have separable images corresponding to the positions of these bright speckles. In the Worden technique, the centroids of bright speckles were identified for 40 different speckle frames taken on the KPNO 4 meter telescope. The individual Betelgeuse frames were then extracted, recentered and averaged. The resulting images had approximately the right angular size (twice 4 m airy disk size) and reportedly showed some evidence of nonuniformity across the disks which could be due to a surface feature. Obviously, this technique is only useful for bright objects close to the diffraction limit, an extremely restrictive class of potential targets.

5. NON-REDUNDANT APERTURE IMAGING (NRAI)

A much more generally applicable technique has been suggested and developed by T. Brown,[9,10] of the Sacramento Peak Observatory. Figure 2 is helpful in describing the recording and processing operations for NRAI.

The image from a large telescope is reimaged onto a mask having a set of holes each less the the r_o in diameter (r_o is the atmospheric correlation length) which are spaced so that the distance between every pair of holes in each direction is unique. This is equivalent to the positioning of antennae in a radio telescope array or of detectors in an amplitude interferometer. The image is then reformed and recorded with short exposure in narrow band light. The recorded speckle pattern differs from the usual full aperture image, since each pair of apertures has yielded an interference pattern which is unique in either orientation or direction, but with random relative shifts of the patterns corresponding to the random phase errors introduced by the atmosphere across each hole. Fourier transformation of the image (either digitally or, as in the diagram, optically) allows phase corrections to be applied to each equivalent subaperture position in the transform while observing the intensity in an image plane and optimizing for a parameter such as the sharpness parameters developed by Muller, et al.[10] Due to the non-redundancy of the subaperture positions, there is a unique relationship between a position in the image transform and an aperture pair in the pupil. Therefore, phase corrections in the fourier transform affect only the phase errors at a given aperture position. The sharpness parameters used for serially improving the image (such criterion as $\int_A I^3$) have been demonstrated to work by Muller and Buffington.[10]

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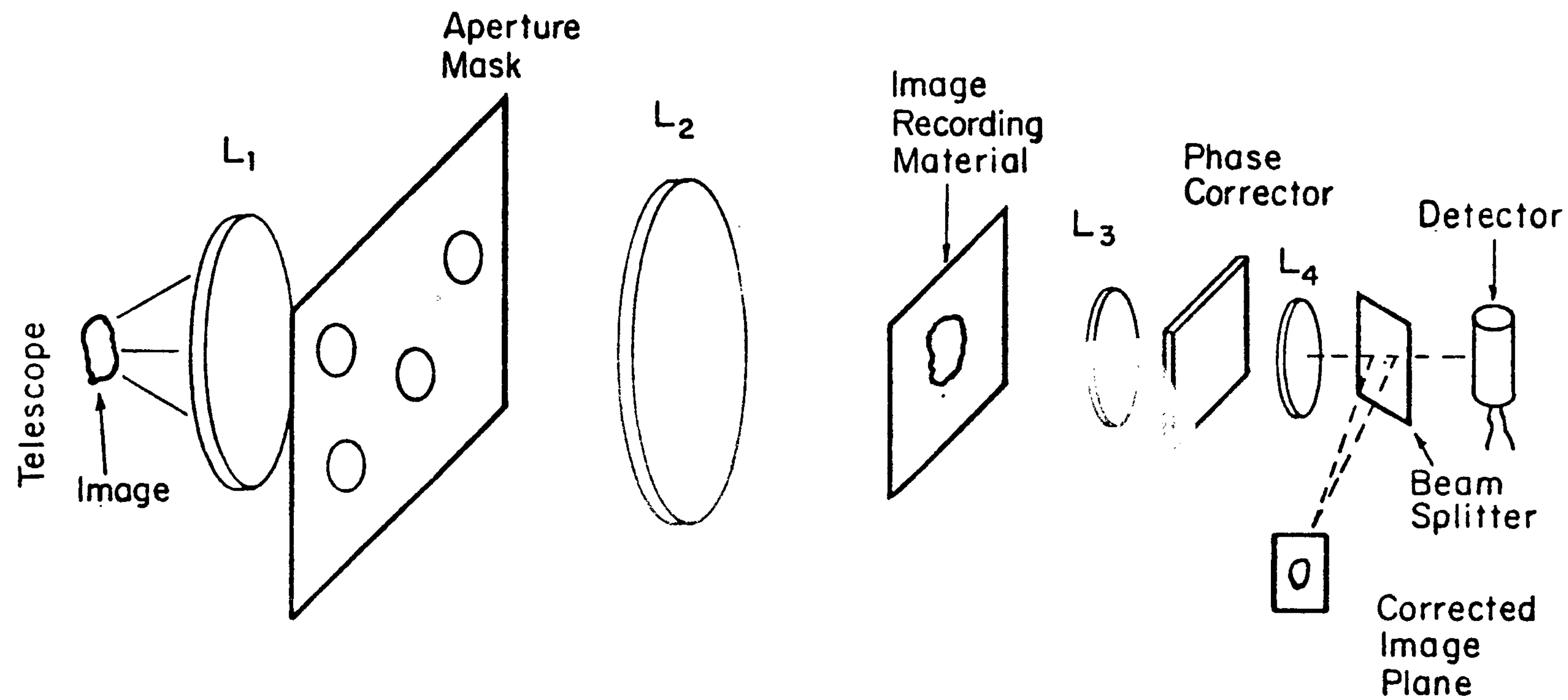

Fig. 2. NON REDUNDANT APERTURE IMAGING
T. BROWN (JOSA, JULY 1978)

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Brown reports successful laboratory tests and digital simulations of the techniques, and he is currently testing it with real atmospherically degraded astronomical data.

NRAI has a number of major advantages over the Muller and Buffington real time system, since real time operation requires correctors which work at speeds faster than the atmospheric change time (10 milliseconds) and since the Muller approach corrects one element at a time to converge on the optimum setting, the process is limited to only a small number of corrector elements. Since no such time restriction occurs with the post-processing of NRAI, there is no limit to the number of subapertures used. Brown estimates that good correction can be obtained with as few as 10 detected photons/subaperture and it is the integrated brightness of the object that is important for determining the phase at each subaperture. This differs from speckle techniques whose brightness per diffraction limited resel is the governing light level. Finally, while NRAI has not been tested at these light levels, the theoretical expectations calculated for some of the real time corrections systems suggest that technique should work on faint enough objects to yield many worthwhile scientific results.

6. PHASE RETRIEVAL

Another technique which holds great promise for the image restoration problem is the one suggested for electron microscopy by Gerchberg and Saxton,[11] and recently developed for image restoration by Fienup.[12] This technique is conceptually very simple. If one knows the modulus of the fourier transform of a two-dimensional object (the result of speckle interferometry) then one can determine the corresponding object distribution (or equivalently, the phase of the transform) by successive fourier transform and inversion, at each stage forcing the image or transform to fit the known constraints of the object. For example, one knows that object is real and non-negative and one also knows the modulus of the transfrom. In addition, an accurate estimate of the size and general shape of the object is available from the auto-correlation image (transform of the modulus). Figure 3 contains block diagrams of two alternative iterative schemes, the first from Gerchberg and Saxton, and the second from Fienup. In the first, one modifies, alternatively, the transform, then the image, then the transform, etc. to meet the preset constraints. In the second, called the input-output approach, convergence is speeded up by modifying the previous input to form a new input based on results from the previous cycle. In one such operation, the new input is

$$g_{x+1}(x) = \begin{cases} g_{k'}(x) & x \in \gamma \\ g_k(x) - \beta\, g_{k'}(x) & x \notin \gamma \end{cases}$$

where β is a constant and γ is the region that includes all points that comply with the object-domain constraints. Fienup points out that this approach is similar to that of negative feedback: compensate the input for violation of the constraints of the output

Fienup shows some dramatic results from computer simulations, even in the presence of as much as 11% rms noise. Since this technique works on the output of standard speckle interferometry, its implementation could be limited by only the expected limitation of interferometry, though the quality of restoration for a given signal-to-noise is as yet not fully determined.

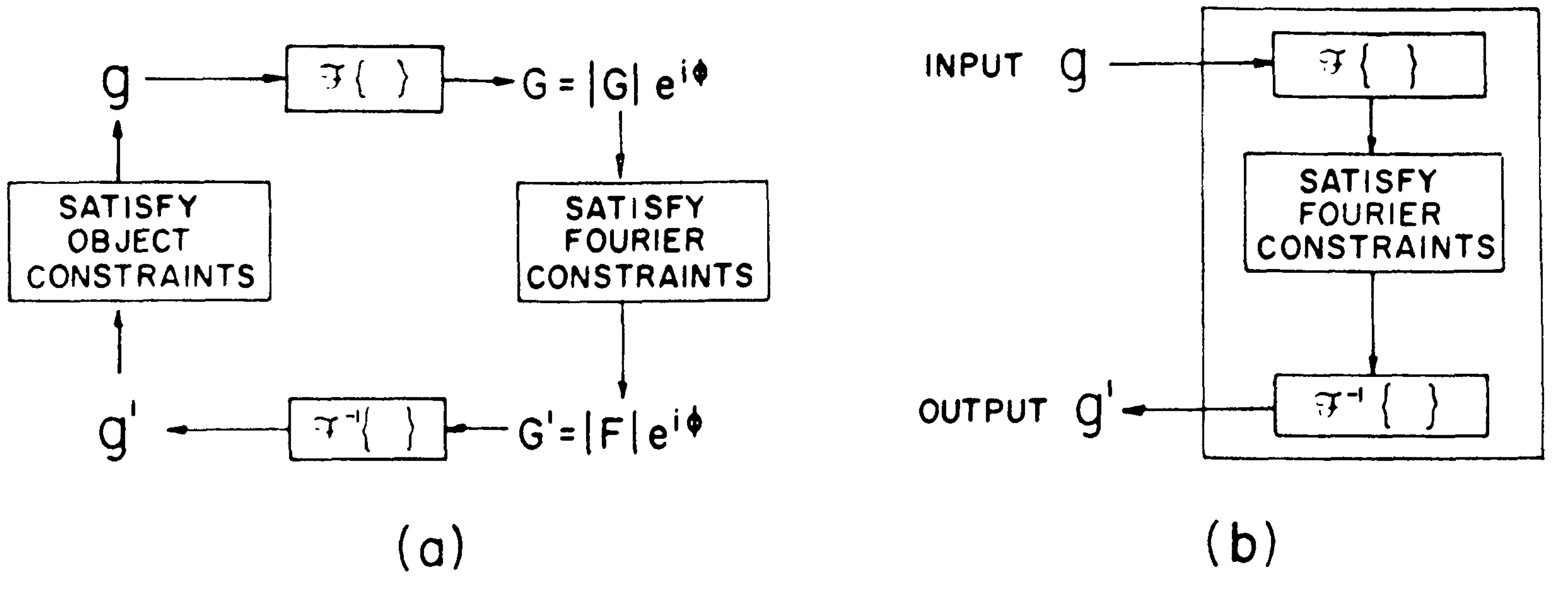

Fig. 3. (a) Block diagram of the error-reduction approach; (b) block diagram of the system for the input-output concept. (Reprinted from Optics Letters, 3, 27, 1978. Copyright 1978 by the Optical Society of American and reprinted by permission of the copyright owner.)

7. SPECKLE IMAGING

In speckle imaging, speckle data are recorded with identical requirements to speckle interferometry. Each image is then fourier transformed and, unlike interferometry, the phase in the transform is encoded as point-to-point phase differences, a technique first suggested by Knox and Thompson.[14] This is performed in two dimensions by calculating two complex products for each frame and then summing the results. The resulting averaged complex amplitudes have the form:

$$\langle Au(u,v)\rangle = \langle I(u,v)I^*(u + \Delta u,v)\rangle$$

$$\langle Av(u,v)\rangle = \langle I(u,v)I^*(u,v + \Delta v)\rangle \qquad (1)$$

where I(u,v) is the image transform.

Expanding each of the components in Eq. (1) yields (only the "u" term is shown for the sake of brevity):

$$\langle A(u,v)\rangle \simeq |O(u,v)|^2 \exp\{i[\phi(u,v) - \phi(u + \Delta u,v)]\}$$
$$\times\langle |T(u,v)|^2 \exp\{i[\psi(u,v) - \psi(u + \Delta u,v)]\}\rangle$$

where $|O(u,v)|$ is the modulus of the object fourier transform and $\phi(u,v)$ is its phase, and T(u,v) is the atmospheric transfer function and $\psi(u,v)$ is its phase.

When Δu is small compared to the atmospheric correlation length, r_o, the phase differences are small and the average transfer function approaches

$$\langle |T(u,v)|^2\rangle \exp\{i\langle\psi(u,v) - \psi(u + \Delta u,v)\rangle\} .$$

This transfer function has an expected value whose phase approaches zero (since the atmospheric phase fluctuations are a stochastic, mean zero process) and whose amplitude is the conventional speckle interferometry transfer function. In order to obtain a reconstructed image, the adjacent point phase differences in the object transform must be summed to obtain the object phase which is then combined with the modulus and transformed to form the image.

Since for any real data set, due to photon, recording and residual atmospheric noise, there will be an error associated with each phase difference, a simple addition of adjacent values will lead to a cumulative error in the calculated phase. In principle, the phase should have no greater errors than the phase differences. This can be accomplished by calculating a least squares fit of the phases to the phase difference data either by performing the indicated matrix inversion or by using an iterative method, such as the one described by Hardy.[14] In addition, the phase should be calculated radially out from the center (since the amplitude is largest at its center) and each phase path should be weighted by the product of the amplitudes along the path, thus weighting the data with the best signal-to-noise and avoiding paths with amplitude zeroes. Finally, the problem of π phase shifts must be handled (if the amplitude weighting does not avoid them all) by including a decision-making algorithm to determine the sign of the phase shift.

The following is a list of the steps in the speckle imaging process for a film recording system:

- Recording requirements identical to speckle interferometry;
- Scan and digitize each image at two sample/resolution element over large enough field for two samples/r_o in the F.T.;
- Correct for film sensitometry (linearization);
- Screen each image for defects;
- Digital FFT of each image
- Calculate
  - $A(u,v) \cdot A^*(u + \Delta u,v)$
  - $A(u,v) \cdot A^*(u,v + \Delta v)$
  - $<|A(u,v)|^2>$
  - $|<A(u,v)>|^2$
- Average each quantity for all frames;
- Set phase at center of transform to zero;
- Calculate phase from averaged complex products using amplitude weighted least squares algorithm;
- Adjust amplitude by division by reference star transform, subtraction of $|<A>|^2$, or low frequency scaling;
- Subtract bias level due to accumulated averaged photon noise;
- Invert combined amplitude and phase to an image and display

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This technique has had a recently demonstrated success in both digital simulation[15] and reconstruction of solar surface features.[16] A straightforward analysis of the convergence of the imaging process for various photon levels can be carried out with results similar to those of Roddier,[17] but with a requirement set the error in the phase differences (or phase) estimate instead of the amplitude (as for interferometry) As in the Roddier calculation, the number of frames required to obtain a given signal-to-noise (or phase error) in the reconstruction is inversely proportional to the square of the number of detected photons.

An example of the results of such a calculation project that a reconstruction of Europa with 1/8 wave phase accuracy at the 5 meter telescope requires approximately 5000 frames and that a similar result at a 2 meter telescope requires 300 frames (due to the wider bandwidth needed at a 2 meter telescope). Due to the inverse square relationship between frame number and photons/resel, the number of frames for diffraction limited imaging of faint objects such as Pluto gets prohibitively large unless an on-telescope integrating system is available.

8. SUMMARY

In this paper, we have attempted to show that not only are there a large number of worthwhile scientific projects for high resolution imaging, but also that there are several techniques potentially capable of restoring images at the required performance levels. Since these techniques are new and relatively untested, it is too early to say whether they can be practically applied at their predicted limits. However, the payoff from ground based telescopes with proposed apertures up to 25 meters yielding diffraction limited images of extended objects makes it well worth the effort.

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We gratefully acknowledge many useful discussions with Dr. Richard Goady of Harvard University and Dr. Robert Noyes of the Harvard College Observatory. This work was supported by the Air Force Geophysics Laboratory and the National Aeronautics and Space Administration.