

SUTHERLAND, W. A., *Introduction to Metric and Topological Spaces* (Clarendon Press: Oxford University Press, 1975), xiii+181 pp., £2.50 (paper covers), £5.00 (boards).

This book provides an excellent introduction to the theory of metric and topological spaces suitable for a student who has already met rigorous analysis at the level of a single real variable. It concentrates almost exclusively on the analytic aspects of the theory, the aim being to show how the notions of continuity and convergence can be studied to advantage in the setting of metric or, more generally, topological spaces.

The contents of the book are for the most part standard, centring round the four C's: continuity, compactness, connectedness and completeness. After an initial chapter reviewing the necessary background material, the basic theory of metric spaces is discussed and then topological spaces are introduced as a more appropriate framework in which to study continuity. Once topological spaces have been defined and their elementary properties discussed, there follow two chapters on compactness and connectedness. The remainder of the book specializes again to metric spaces, covering topics such as sequential compactness and its equivalence to compactness for metric spaces; uniform convergence, mainly as an example of convergence in a suitable metric space of functions; completeness, including Banach's contraction mapping theorem and Baire's category theorem; and, finally, total boundedness and the Ascoli–Arzelà theorem. There is also an appendix giving constructions of, firstly, the reals from the rationals via Cauchy sequences and, secondly, the completion of an arbitrary metric space.

The emphasis is very much on introduction, this being particularly the case in the chapters on general topological spaces. On the whole, I felt that the contents and level of presentation were about right for a first course. Wisely, the author stops short of anything involving the full axiom of choice. Thus, for instance, product topologies are only defined for the case of a finite number of factors and Tychonoff's compactness theorem consequently gets just a passing reference. (Strictly speaking, a countable version of the axiom of choice is in fact used at various places without any explicit mention; but this, sensibly, seems to be the customary practice at this level.)

The author has a pleasantly informal, yet precise, style and has made a great effort to motivate new concepts and abstractions by relating them to ideas which have already been discussed. There are plenty of examples to illustrate the theory, several useful diagrams, and good sets of exercises. All in all, this is a most welcome book which will be of value to many students.

T. A. GILLESPIE

COLTON, D. L., *Partial differential equations in the complex domain* (Pitman Research Notes in Mathematics No. 4, 1976), 96 pp., Paperback £5.00.

This book consists of lectures given at Indiana and Glasgow by the author in 1971–72. In it he develops an analytic theory for certain improperly posed initial value problems based on the theory of functions of a complex variable. The following example, taken from the introduction, illustrates the scope of the book:

“The uniqueness of a solution to the improperly posed elliptic Cauchy problem is equivalent to the Runge approximation property. In order to exploit such a property one is led to construct an integral operator which maps analytic functions onto solutions of the elliptic equation under investigation, thus giving a practical method of constructing both a complete family of solutions and analytic approximations to the elliptic Cauchy problem. However, in order to construct this integral operator it is necessary (in the case of three independent variables) to again examine an improperly posed problem, this time an exterior characteristic initial value problem for a hyperbolic equation. Having now constructed the desired integral operator it can in turn be used not only to construct a complete family of solutions but also to analytically continue solutions of the elliptic equation from a knowledge of the domain of regularity of their Cauchy or (complex) Goursat data along prescribed analytic surfaces.”

The book is clearly written and should serve as a useful introduction to the use of function theoretic methods in improperly posed problems.

J. CARR