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EXISTENCE, UNIQUENESS AND APPROXIMATION OF SOLUTIONS TO THE STOCHASTIC LANDAU–LIFSHITZ–GILBERT EQUATION ON THE REAL LINE

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The Landau–Lifshitz–Gilbert (LLG) equation is a partial differential equation describing the motion of magnetic moments in a ferromagnetic material. In the theory of ferromagnetism, an important problem is to study noise-induced transitions between different equilibrium states. Hence, the LLG equation needs to be modified in order to incorporate random fluctuations into the dynamics of the magnetisation. Including the noise effects in the theory of evolution of magnetic moments requires a proper study of the stochastic version of the LLG equation. The aim of this thesis is to lay the foundation of the theory of the stochastic LLG equation for a magnetic nanowire of infinite length that is widely used in physics to study the dynamics of the domain walls. The deterministic version of this equation has been intensely studied in recent years due its importance for the fabrication of magnetic devices. It is customary to study the nanowire of infinite length. This approach allows for a relatively simple mathematical description and, at the same time, provides a useful approximation of the wires of finite length.

First, we propose a semi-discrete finite difference method to find approximate solutions to the stochastic LLG equation on the real line. Then, we transform the discretised equation into a partial differential equation with random coefficients, without the Itô term, to prove the convergence of approximate solutions. We deduce the existence and uniqueness of a global unique strong solution to the stochastic problem on the whole real line. The main novelty of our approach is that we prove the existence of pathwise solutions, unique for each trajectory of the noise which is given in advance.

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Second, in order to solve the stochastic LLG equation numerically on the real line, we truncate the infinite line into a bounded interval. We consider the stochastic problem on a bounded interval $[-L, L]$ with physically relevant homogeneous Neumann boundary conditions and we show that when L tends to infinity, the solution of the problem on a bounded interval converges to the solution of the original stochastic problem on the real line. We also provide pathwise error estimates depending on L.

Finally, to solve the stochastic LLG equation numerically, we propose a fully discrete finite difference scheme based on the midpoint rule for the stochastic LLG equation on a bounded interval. We perform first numerical experiments which shows that the fully discrete solutions converge to the solution of the stochastic problem on a bounded interval $[-L, L]$ for vanishing discretisation parameters. Next, we implement a numerical experiment which validates the convergence of the solution on a bounded interval $[-L, L]$ to the solution on the real line when L is large enough.

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