Contents

Preface *page* xiii

1 Introduction 1
- 1.1 Petroleum Recovery 3
- 1.2 Reservoir Simulation 6
- 1.3 Outline of the Book 9
- 1.4 The First Encounter with MRST 14

Part I Geological Models and Grids 19

2 Modeling Reservoir Rocks 21
- 2.1 Formation of Sedimentary Rocks 21
- 2.2 Creation of Crude Oil and Natural Gas 26
- 2.3 Multiscale Modeling of Permeable Rocks 28
  - 2.3.1 Geological Characterization 29
  - 2.3.2 Representative Elementary Volumes 31
  - 2.3.3 Microscopic Models: The Pore Scale 33
  - 2.3.4 Mesoscopic Models 34
- 2.4 Modeling Rock Properties 34
  - 2.4.1 Porosity 35
  - 2.4.2 Permeability 36
  - 2.4.3 Other Parameters 38
- 2.5 Property Modeling in MRST 39
  - 2.5.1 Homogeneous Models 40
  - 2.5.2 Random and Lognormal Models 40
  - 2.5.3 The 10th SPE Comparative Solution Project: Model 2 41
  - 2.5.4 The Johansen Formation 44
  - 2.5.5 SAIGUP: Shallow-Marine Reservoirs 46

3 Grids in Subsurface Modeling 55
- 3.1 Structured Grids 57
- 3.2 Unstructured Grids 62

Published online by Cambridge University Press

3.2.1 Delaunay Tessellation 63
3.2.2 Voronoi Diagrams 66
3.2.3 General Tessellations 68
3.2.4 Using an External Mesh Generator 69
3.3 Stratigraphic Grids 72
3.3.1 Corner-Point Grids 73
3.3.2 2.5D Unstructured Grids 85
3.4 Grid Structure in MRST 88
3.5 Examples of More Complex Grids 96
3.5.1 SAIGUP: Model of a Shallow-Marine Reservoir 97
3.5.2 Composite Grids 101
3.5.3 Control-Point and Boundary Conformal Grids 103
3.5.4 Multiblock Grids 104

Part II Single-Phase Flow 111

4 Mathematical Models for Single-Phase Flow 113
4.1 Fundamental Concept: Darcy's Law 113
4.2 General Flow Equations for Single-Phase Flow 115
4.3 Auxiliary Conditions and Equations 120
4.3.1 Boundary and Initial Conditions 120
4.3.2 Injection and Production Wells 121
4.3.3 Field Lines and Time-of-Flight 128
4.3.4 Tracers and Volume Partitions 129
4.4 Basic Finite-Volume Discretizations 131
4.4.1 Two-Point Flux-Approximation 131
4.4.2 Discrete `div` and `grad` Operators 135
4.4.3 Time-of-Flight and Tracer 141

5 Incompressible Solvers for Single-Phase Flow 143
5.1 Basic Data Structures in a Simulation Model 144
5.1.1 Fluid Properties 144
5.1.2 Reservoir States 145
5.1.3 Fluid Sources 145
5.1.4 Boundary Conditions 146
5.1.5 Wells 147
5.2 Incompressible Two-Point Pressure Solver 149
5.3 Upwind Solver for Time-of-Flight and Tracer 153
5.4 Simulation Examples 156
5.4.1 Quarter Five-Spot 157
5.4.2 Boundary Conditions 160
5.4.3 Structured versus Unstructured Stencils 165
5.4.4 Using Peaceman Well Models 169

Published online by Cambridge University Press

6 Consistent Discretizations on Polyhedral Grids 174
6.1 The TPFA Method Is Not Consistent 174
6.2 The Mixed Finite-Element Method 177
6.2.1 Continuous Formulation 178
6.2.2 Discrete Formulation 180
6.2.3 Hybrid Formulation 182
6.3 Finite-Volume Methods on Mixed Hybrid Form 185
6.4 The Mimetic Method 188
6.5 Monotonicity 197
6.6 Discussion 199

7 Compressible Flow and Rapid Prototyping 202
7.1 Implicit Discretization 202
7.2 A Simulator Based on Automatic Differentiation 204
7.2.1 Model Setup and Initial State 205
7.2.2 Discrete Operators and Equations 206
7.2.3 Well Model 208
7.2.4 The Simulation Loop 209
7.3 Pressure-Dependent Viscosity 213
7.4 Non-Newtonian Fluid 215
7.5 Thermal Effects 220

Part III Multiphase Flow 229

8 Mathematical Models for Multiphase Flow 231
8.1 New Physical Properties and Phenomena 232
8.1.1 Saturation 232
8.1.2 Wettability 234
8.1.3 Capillary Pressure 235
8.1.4 Relative Permeability 239
8.2 Flow Equations for Multiphase Flow 243
8.2.1 Single-Component Phases 244
8.2.2 Multicomponent Phases 245
8.2.3 Black-Oil Models 246
8.3 Model Reformulations for Immiscible Two-Phase Flow 248
8.3.1 Pressure Formulation 248
8.3.2 Fractional-Flow Formulation in Phase Pressure 249
8.3.3 Fractional-Flow Formulation in Global Pressure 254
8.3.4 Fractional-Flow Formulation in Phase Potential 255
8.3.5 Richards' Equation 256
8.4 The Buckley–Leverett Theory of 1D Displacements 258
8.4.1 Horizontal Displacement 258
8.4.2 Gravity Segregation 264
8.4.3 Front Tracking: Semi-Analytical Solutions 266

Published online by Cambridge University Press

9 Discretizing Hyperbolic Transport Equations 272
9.1 A New Solution Concept: Entropy-Weak Solutions 272
9.2 Conservative Finite-Volume Methods 274
9.3 Centered versus Upwind Schemes 275
9.3.1 Centered Schemes 275
9.3.2 Upwind or Godunov Schemes 277
9.3.3 Comparison of Centered and Upwind Schemes 279
9.3.4 Implicit Schemes 283
9.4 Discretization on Unstructured Polyhedral Grids 286

10 Solvers for Incompressible Immiscible Flow 289
10.1 Fluid Objects for Multiphase Flow 290
10.2 Sequential Solution Procedures 292
10.2.1 Pressure Solvers 293
10.2.2 Saturation Solvers 294
10.3 Simulation Examples 297
10.3.1 Buckley–Leverett Displacement 298
10.3.2 Inverted Gravity Column 301
10.3.3 Homogeneous Quarter Five-Spot 303
10.3.4 Heterogeneous Quarter Five-Spot: Viscous Fingering 307
10.3.5 Buoyant Migration of CO_2 in a Sloping Sandbox 311
10.3.6 Water Coning and Gravity Override 313
10.3.7 The Effect of Capillary Forces – Capillary Fringe 319
10.3.8 Norne: Simplified Simulation of a Real-Field Model 323
10.4 Numerical Errors 326
10.4.1 Splitting Errors 326
10.4.2 Grid Orientation Errors 330

11 Compressible Multiphase Flow 337
11.1 Industry-Standard Simulation 337
11.2 Two-Phase Flow without Mass Transfer 342
11.3 Three-Phase Relative Permeabilities 348
11.3.1 Relative Permeability Models from ECLIPSE 100 349
11.3.2 Evaluating Relative Permeabilities in MRST 353
11.3.3 The SPE 1, SPE 3, and SPE 9 Benchmark Cases 355
11.3.4 A Simple Three-Phase Simulator 358
11.4 PVT Behavior of Petroleum Fluids 359
11.4.1 Phase Diagrams 360
11.4.2 Reservoir Types and Their Phase Behavior during Recovery 364
11.4.3 PVT and Fluid Properties in Black-Oil Models 370
11.5 Phase Behavior in ECLIPSE Input Decks 379
11.6 The Black-Oil Equations 388

Published online by Cambridge University Press

11.6.1 The Water Component 389
11.6.2 The Oil Component 390
11.6.3 The Gas Component 392
11.6.4 Appearance and Disappearance of Phases 392
11.7 Well Models 394
11.7.1 Inflow-Performance Relationships 394
11.7.2 Multisegment Wells 395
11.8 Black-Oil Simulation with MRST 399
11.8.1 Simulating the SPE 1 Benchmark Case 399
11.8.2 Comparison against a Commercial Simulator 405
11.8.3 Limitations and Potential Pitfalls 406

12 The AD-OO Framework for Reservoir Simulation 413
12.1 Overview of the Simulator Framework 414
12.2 Model Hierarchy 420
12.2.1 PhysicalModel – Generic Physical Models 422
12.2.2 ReservoirModel – Basic Reservoir Models 426
12.2.3 Black-Oil Models 430
12.2.4 Models of Wells and Production Facilities 433
12.3 Solving the Discrete Model Equations 434
12.3.1 Assembly of Linearized Systems 434
12.3.2 Nonlinear Solvers 436
12.3.3 Selection of Time-Steps 439
12.3.4 Linear Solvers 440
12.4 Simulation Examples 449
12.4.1 Depletion of a Closed/Open Compartment 449
12.4.2 An Undersaturated Sector Model 451
12.4.3 SPE 1 Instrumented with Inflow Valves 455
12.4.4 The SPE 9 Benchmark Case 460
12.5 Improving Convergence and Reducing Runtime 470

Part IV Reservoir Engineering Workflows 475

13 Flow Diagnostics 477
13.1 Flow Patterns and Volumetric Connections 478
13.1.1 Volumetric Partitions 479
13.1.2 Time-of-Flight Per Partition Region: Improved Accuracy 482
13.1.3 Well Allocation Factors 482
13.2 Measures of Dynamic Heterogeneity 483
13.2.1 Flow and Storage Capacity 483
13.2.2 Lorenz Coefficient and Sweep Efficiency 486
13.3 Residence-Time Distributions 489
13.4 Case Studies 495

Published online by Cambridge University Press

13.4.1 Tarbert Formation: Volumetric Connections 495
13.4.2 Heterogeneity and Optimized Well Placement 501
13.5 Interactive Flow Diagnostics Tools 505
13.5.1 Synthetic 2D Example: Improving Areal Sweep 510
13.5.2 SAIGUP: Flow Patterns and Volumetric Connections 514

14 Grid Coarsening 518
14.1 Grid Partitions 518
14.1.1 Uniform Partitions 519
14.1.2 Connected Partitions 520
14.1.3 Composite Partitions 522
14.2 Coarse Grid Representation in MRST 524
14.2.1 Subdivision of Coarse Faces 526
14.3 Partitioning Stratigraphic Grids 529
14.3.1 The Johansen Aquifer 529
14.3.2 The SAIGUP Model 532
14.3.3 Near Well Refinement for CaseB4 536
14.4 More Advanced Coarsening Methods 540
14.5 A General Framework for Agglomerating Cells 541
14.5.1 Creating Initial Partitions 541
14.5.2 Connectivity Checks and Repair Algorithms 542
14.5.3 Indicator Functions 544
14.5.4 Merge Blocks 545
14.5.5 Refine Blocks 547
14.5.6 Examples 550
14.6 Multilevel Hierarchical Coarsening 553
14.7 General Advice and Simple Guidelines 556

15 Upscaling Petrophysical Properties 558
15.1 Upscaling for Reservoir Simulation 559
15.2 Upscaling Additive Properties 561
15.3 Upscaling Absolute Permeability 563
15.3.1 Averaging Methods 564
15.3.2 Flow-Based Upscaling 569
15.4 Upscaling Transmissibility 575
15.5 Global and Local–Global Upscaling 578
15.6 Upscaling Examples 580
15.6.1 Flow Diagnostics Quality Measure 581
15.6.2 A Model with Two Facies 581
15.6.3 SPE 10 with Six Wells 585
15.6.4 Complete Workflow Example 589
15.6.5 General Advice and Simple Guidelines 595

Published online by Cambridge University Press

Appendix The MATLAB Reservoir Simulation Toolbox 597
A.1 Getting Started with the Software 598
A.1.1 Core Functionality and Add-on Modules 598
A.1.2 Downloading and Installing 601
A.1.3 Exploring Functionality and Getting Help 602
A.1.4 Release Policy and Version Numbers 606
A.1.5 Software Requirements and Backward Compatibility 606
A.1.6 Terms of Usage 608
A.2 Public Data Sets and Test Cases 609
A.3 More About Modules and Advanced Functionality 610
A.3.1 Operating the Module System 611
A.3.2 What Characterizes a Module? 612
A.3.3 List of Modules 613
A.4 Rapid Prototyping Using MATLAB and MRST 620
A.5 Automatic Differentiation in MRST 623

References 631
Index 650
Usage of MRST Functions 658

Published online by Cambridge University Press

Published online by Cambridge University Press