SENSITIVITY OF INTERNAL STRUCTURE TO THE SURFACE BOUNDARY CONDITION

Pierre Demarque
Yale University Observatory

It is now nearly fifty years since Eddington and Milne had a lively controversy on the importance of the surface boundary condition on the internal structure of stars [see Eddington (1930) and Milne (1930)]. We remember that Eddington believed that the internal structure of stars is basically determined by the physical processes occurring in the deep interior and that what happens in the surface layers has little effect on the total stellar luminosity. On the other hand, Milne emphasized the importance of the properties of the outer layers and the effect these could have on the run of pressure and temperature in the deep interior of the stars. We know now that both Eddington and Milne were correct. Eddington's considerations apply to the hot stars, the early-type stars which have surface layers in radiative equilibrium. Milne's arguments are relevant to the cool stars, the late-type stars which have deep convective envelopes. In the former case, one can safely assume in calculations of stellar structure that the density and the temperature both approach zero simultaneously at the surface (the so-called "zero" surface boundary conditions). For late-type stars, most of the convective envelope is adiabatic and its structure is determined by the adiabatic equation:

$$P = KT^{\gamma/\gamma-1} \qquad (1)$$

which requires the parameter K to be determined by the run of P and T near the stellar surface (Schwarzschild 1958).

Now let us consider the properties of these convective zones. One can show that for a star with a deep convective envelope in a diabatic equilibrium, the radius is chiefly determined by the specific entropy in the adiabatic region [see Larson (1973)]. For a perfect gas, the specific entropy s is given by:

$$s \propto c_v \ \ln \,(P/_{\rho\gamma}) \propto c_v \ \ln(T/_{\rho}\gamma-1) \qquad (2)$$

where $\gamma = c_p/c_v$, the ratio of specific heats.

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For a star in hydrostatic equilibrium, of given mass M and radius R, one finds that dimensionally:

$$\rho \propto \frac{M}{R^3} \quad \text{and} \quad P \propto \frac{GM^2}{R^4}$$

so that:

$$R \propto \left[\frac{\exp(s/c_V)}{GM^{2-\gamma}} \right]^{\frac{1}{(3\gamma-4)}} \qquad (3)$$

For a polytrope of index n = 1.5 (i.e. γ = 5/3), using the tables of Chandrasekhar (1939), one can then write:

$$R = 2.36 \; \frac{\exp(s/c_V)}{GM^{1/3}} \qquad (4)$$

This expression gives the radius of configurations with a deep adiabatic envelope with accuracy of 10 to 15 percent.

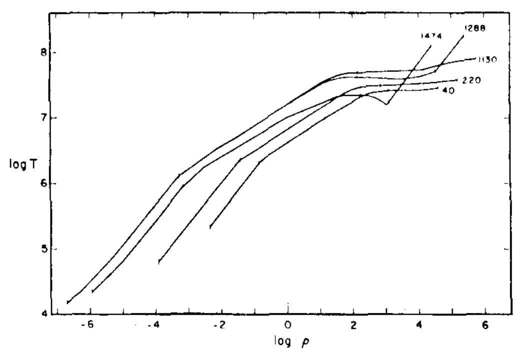

Figure 1. Interior characteristics of models for red giant stars with 0.85 $M_\odot$. The labels refer to the model number on the evolutionary sequence. The onset of helium burning occurs between models 1130 and 1288. The surface convection zone is for each model bounded by the first two tick-marks (starting at the low-temperature end). Note that in model 1474, which has a smaller radius than 1288, the specific entropy in the convection zone has also begun to decrease [from P. Demarque and J.G. Mengel (1971). Courtesy of the University of Chicago Press].

Figure 1 illustrates this result in terms of models for red giants with increasing luminosities along an evolutionary sequence which reaches the onset of helium burning between models 1130 and 1288. At

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model 1474, the star begins to move down the giant branch toward the horizontal branch. The radii of each of the configurations of Figure 1, which have deep convective envelopes, is determined by the specific entropy in the convective region. This is illustrated by Figure 1 which shows that as the luminosity and therefore the radius increase, the specific entropy increases also, since one moves toward higher temperatures and lower densities in the (log T-log ρ)-plane. And once the core flash has begun and the radiative layers below the convection zone are cooling down (as in model 1474), the radius decreases in turn.

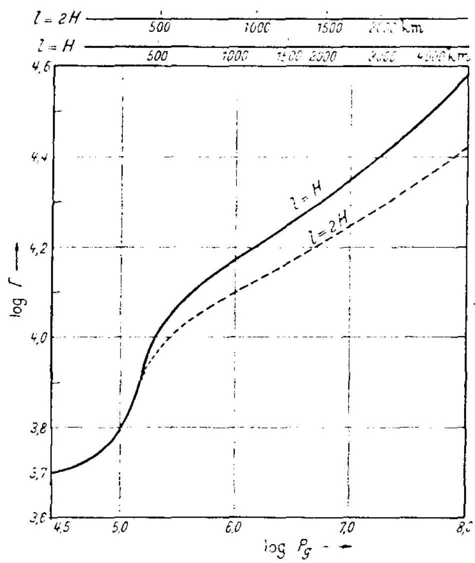

Figure 2. The run of gas pressure P_g and temperature T in the solar convection zone for two values of the mixing length ℓ in terms of the local pressure scale height H [from E. Vitense (1953). Courtesy of Springer-Verlag].

The calculation of the radius of the stellar model would thus seem to reduce to the problem of determining the specific entropy in the adiabatic envelope. One might think that a radiative model for the atmospheric layers which gives the quantities P and T at the point of the onset of convection, could by substituting into equation (1), provide all the information that one requires to fix the stellar radius. The problem is, however, complicated by the presence of a superadiabatic layer at the top of the convection zone and is best illustrated in the case of the sun. Although the sun is believed to have a relatively shallow superadiabatic zone compared to red giants, it has the advantage of readily observable surface features and a measurable limb darkening. Karl Schwarzchild (1906) was aware of this problem a long time ago, and it has not yet been fully resolved: al-

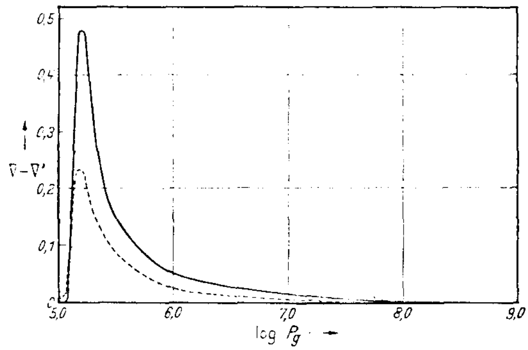

Figure 3. The "degree of superadiabaticity", plotted in ordinate, in the solar convection zone for two values of ℓ. Note how thin this superadiabatic layer is. [from E. Vitense (1953). Courtesy of Springer-Verlag].

though there is evidence for convection in the solar photosphere, the law of limb darkening observed on the solar disk is that which is characteristic of radiative equilibrium. In other words, we are seeing a layer which is unstable with respect to convection, but which undergoes a very inefficient kind of convection because of the low densities and the large radiative losses, and in which the temperature gradient is as a result intermediate between the local radiative and adiabatic gradients. For lack of a better theory, it is customary in studies of stellar interiors to use the mixing length formalism to describe this layer, after the work of Vitense (1953). Figure 2 and 3 show the models of Vitense for the solar convective zone. Note how sensitive the run of temperature and pressure are on the choice of the mixing length in Figure 2. Note also that it is in the thin superadiabatic region shown in Figure 3 on the outer part of the convective zone that the specific entropy of the whole adiabatic envelope is determined, i.e. that the boundary condition which fixes the radius is set.

Figure 4 illustrates the effect of the choice of the mixing length on main sequence position in the H-R diagram, Figure 5 shows the well known result that the structure of red giant envelopes are even more sensitive to the choice of the mixing length.

The problem that we face in the determination of what one might call an effective mixing length for late-type stars is compounded by other major gaps in our understanding of the relevant physical processes. The structure of the superadiabatic region is sensitive not only to the choice of the mixing length, but also to the opacity. This situation can be particularly complicated since various molecular species can be found in the atmospheres of cool stars which can affect the opacities in an important way.

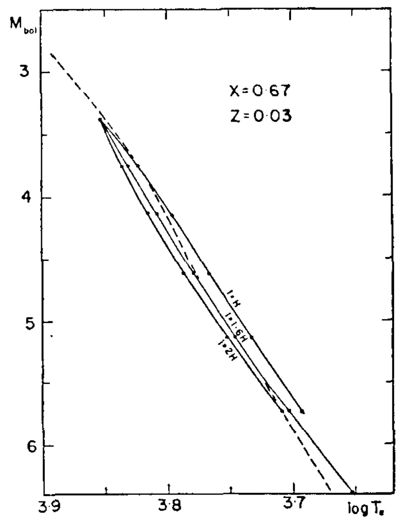

Figure 4. Effect of the choice of ℓ on the position of the main-sequence in the theoretical H-R diagram for stars in the mass range 1.3-0.8 $M_\odot$.[from P. Demarque and R.B.Larson (1964). Courtesy to the University of Chicago Press].

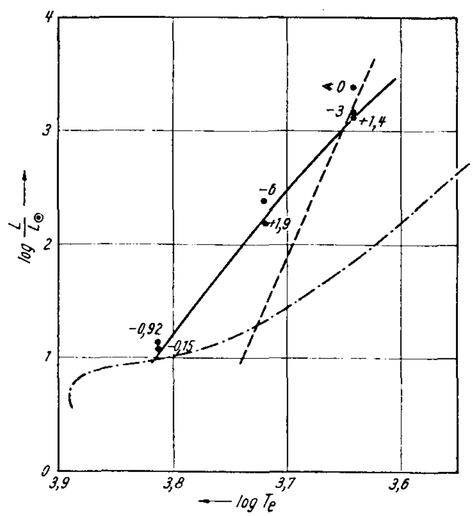

Figure 5. The large effect of the choice of the mixing length ℓ on the position of the giant branch is illustrated in this figure for a star of 1.3 $M_\odot$. The continuous line was obtained with ℓ=H, the dot-dashed line with ℓ=2H. [from R. Kippenhahn, St. Temesvary and L. Biermann. Courtesy of Springer-Verlag].

The details of the outer radiative layers can also affect the structure of the surface convection zone and in turn modify the radius. For example, a recent experiment by Prather (1975) in which he varied the function $q(\tau)$ of the grey solution of the equation of transfer from Milne's $q=2/3$ to the Krishna Swamy empirical fit to the sun meant a shift of 0.01 in log T_{eff} on main sequence interior models. This shift corresponds to a change in metallicity from Z=0.01 to 0.02. For red giants the sensitivity is greater still. And it is quite possible that non-LTE effects are important in this context.

In summary, one can say that proper surface boundary conditions for interior models of late-type stars require a detailed understanding of the structure of the stellar atmosphere. Much progress still remains to be made on several problems which are separated here for convenience, but which are obviously closely interrelated: 1) the problems of the treatment of convection and of the uncertainties in convective efficiency, and the related problem of convective overshoot; 2) the problem of the opacities and of the molecular equilibrium in late-type stellar atmospheres; 3) the problem of the radiative transfer itself and its implications for the construction of atmospheric models of great spectral complexity.

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