*The Zeldovich Universe:*
*Genesis and Growth of the Cosmic Web*
*Proceedings IAU Symposium No. 308, 2014*
*R. van de Weygaert, S. Shandarin, E. Saar & J. Einasto, eds.*
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doi:10.1017/S1743921316009649

# Large-scale structure non-Gaussianities with modal methods

**Marcel Schmittfull**

Berkeley Center for Cosmological Physics,
Department of Physics and Lawrence Berkeley National Laboratory,
University of California, Berkeley, CA 94720, USA
email: mschmittfull@lbl.gov

**Abstract.** Relying on a separable modal expansion of the bispectrum, the implementation of a fast estimator for the full bispectrum of a 3d particle distribution is presented. The computational cost of accurate bispectrum estimation is negligible relative to simulation evolution, so the bispectrum can be used as a standard diagnostic whenever the power spectrum is evaluated. As an application, the time evolution of gravitational and primordial dark matter bispectra was measured in a large suite of N-body simulations. The bispectrum shape changes characteristically when the cosmic web becomes dominated by filaments and halos, therefore providing a quantitative probe of 3d structure formation. Our measured bispectra are determined by $\sim 50$ coefficients, which can be used as fitting formulae in the nonlinear regime and for non-Gaussian initial conditions. We also compare the measured bispectra with predictions from the Effective Field Theory of Large Scale Structures (EFTofLSS).

**Keywords.** cosmology: large-scale structure of universe, methods: n-body simulations, methods: data analysis, methods: statistical

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Due to non-linear gravitational collapse, the probability distribution function (pdf) of large-scale structure (LSS) dark matter (DM) density fluctuations deviates from a Gaussian pdf even if the initial conditions are Gaussian. This non-Gaussianity is characterized by non-vanishing higher-order $n$-point functions, e.g. the bispectrum, which is the Fourier transform of the 3−point function and corresponds to the probability of finding a third overdensity mode given two other overdensity modes. An additional source of non-Gaussianity is primordial non-Gaussianity of the initial conditions that can be generated by certain inflation models. Measuring these non-Gaussianities in observations can help to break degeneracies present at the level of 2-point statistics, e.g. between linear bias $b_1$ and the normalization of fluctuations $\sigma_8$, or to constrain inflation models.

While it is numerically straightforward to sample initial conditions from a Gaussian pdf and to estimate the power spectrum of a given density, sampling from a non-Gaussian pdf and estimating the bispectrum of a given density perturbation are numerically rather challenging tasks. We address these issues by expanding the bispectrum in separable basis functions which are particularly suited for efficient numerical evaluation.

## 1. Non-Gaussian initial conditions for $N$-body simulations

To draw a realization from a pdf with power $P_\Phi$ and bispectrum $f_{\rm NL}B_\Phi$, we add $\Phi^B = \int_{\mathbf{k}'} W(k,k',|\mathbf{k}-\mathbf{k}'|)\Phi^G_{\mathbf{k}'}\Phi^G_{\mathbf{k}-\mathbf{k}'}$ to a Gaussian field $\Phi^G_{\mathbf{k}}$, where (Wagner *et al.* (2012))

$$W(k,k',|\mathbf{k}-\mathbf{k}'|) \equiv \frac{f_{\rm NL}}{2}\frac{B_\Phi(k,k',|\mathbf{k}-\mathbf{k}'|)}{P_\Phi(k)P_\Phi(k') + 2\,\mathrm{perms}}. \tag{1.1}$$



https://doi.org/10.1017/S1743921316009649 Published online by Cambridge University Press

In practice, $\Phi^B$ can only be computed efficiently if $W$ is product-separable, i.e. consisting of terms of the form $f_1(k)f_2(k')f_3(k'')$, because then $\Phi^B$ reduces to a convolution of filtered Gaussian fields. Unfortunately, the symmetrized denominator in (1.1) often destroys separability. As a general solution to this problem, the kernel $W$ can be expanded in product-separable basis functions following Fergusson *et al.* (2012), which we show in Regan *et al.* (2012) to perform well in simulations.

## 2. Separable bispectrum estimation

Estimating the bispectrum of a density perturbation is computationally expensive due to the large number of possible triangle configurations and integrals similar to the expression for $\Phi^B$ above. Rather than estimating the bispectrum individually for every triangle, Fergusson *et al.* (2012) propose to estimate the amplitude of many independent separable basis bispectrum templates and reconstruct the full bispectrum by summing up the contributions. We restrict ourselves to a finite separable monomial basis that covers all theoretically motivated bispectra and implicitly uses all triangles. This reduces the computational cost of bispectrum estimation dramatically, e.g. the full bispectrum of a $1024^3$ grid is estimated in only one hour on six cores.

We implemented this method and tested it on a large suite of $N$-body simulations with Gaussian and different types of non-Gaussian initial conditions in Schmittfull *et al.* (2013). All bispectrum measurements agree with perturbation theory on large scales, validating our framework and implementation. In the non-linear regime, we find that the bispectrum shape characterizes the 3d dark matter structures, e.g. pancake-like structures correspond to flattened bispectra, while filaments and clusters enhance equilateral contributions to the bispectrum. The bispectrum characterizes these structures in a quantitative way which can both be modeled and extracted from simulations.

In our approach, the full bispectrum information is compressed to $n_{\max} = \mathcal{O}(50)$ amplitudes of (orthonormalized) basis bispectrum shapes. The first ten modes already contain 99% of the information, which we exploit to construct fitting formulae for the dark matter bispectrum with ten parameters at every redshift and $k_{\max}$. The effect of primordial non-Gaussianity can be modeled by a time shift compared to gravitational evolution in a universe with Gaussian initial conditions, which leads to simple fitting formulae for the excess bispectrum due to primordial non-Gaussianity of various types (see Schmittfull *et al.* (2013) for details).

In another application of the separable bispectrum estimation method, we compare the measured bispectra against predictions of Effective Field Theory of Large-Scale Structure (EFTofLSS) in Angulo *et al.* (2014) (see also Baldauf *et al.* (2014)), finding that EFTofLSS significantly improves upon standard perturbation theory.

**Acknowledgements** I thank my collaborators for the work summarized here, R. Angulo, J. Fergusson, S. Foreman, D. Regan, L. Senatore and P. Shellard.

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https://doi.org/10.1017/S1743921316009649 Published online by Cambridge University Press