THE EARTH'S CONSTITUTION AND THE NUTATIONS

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The theoretical researches on this subject were first developed in a consistent theory, in 1765, by Euler, who studied the theory of the motion of a rotating rigid body. Bradley shortly before, in 1737, had discovered the first forced nutation, with a period of 18.6 years, which was explained by d'Alembert.

The beginnings of the history of this subject show already a constant feature of science, namely, the interaction between theory and observation. The astronomical observations of the principal nutation preceded the theory, but Euler's theory predicted some nutations that were not yet known at the time of the formulation of the theory. The knowledge of the internal constitution of the Earth was lacking for many centuries, and the theory remained based on rigid models.

The dynamical equations of the motion of a rigid body can be expressed by the vector equation

$$\frac{d\vec{H}}{dt} = \vec{G}$$

showing that the time derivative of the angular momentum $\vec{H}$ around the centre of mass is equal to the vector $\vec{G}$ of the external torques. The projection of this equation, on a suitable system of axes, corresponds to the Euler equations. The motion of the Earth, relative to its centre of mass, is represented, at every instant, by the instantaneous rotation $\vec{\omega}$ around an axis through the centre of mass.

The case $\vec{G} = 0$ corresponds to the free motion of the Earth, and the most important is the free eulerian nutation; recently, the nearly diurnal nutation has been the subject of several researches.

The case $\vec{G} \neq 0$ corresponds to the forced motions of the Earth, and the main external forces are due to the Sun and the Moon, giving rise to the luni-solar motion; there are several dozens of forced nutations, classified by their periods.

E. P. Fedorov, M. L. Smith and P. L. Bender (eds.), Nutation and the Earth's Rotation, 139–151.

This is a simple and correct way of defining these motions. It is unfortunate that the names "wobble" and "sway" have been employed to describe some of these motions.

The motion of the Earth can be described by a simple and elegant representation, due to Poinsot (1852), which is based on the conical motions that the axes of figure, rotation and angular momentum have relative to one another.

The eulerian motion is represented in Fig. 1 showing the positions of the polhode cone, fixed within the Earth, that rolls on the herpolhode cone, fixed in space.

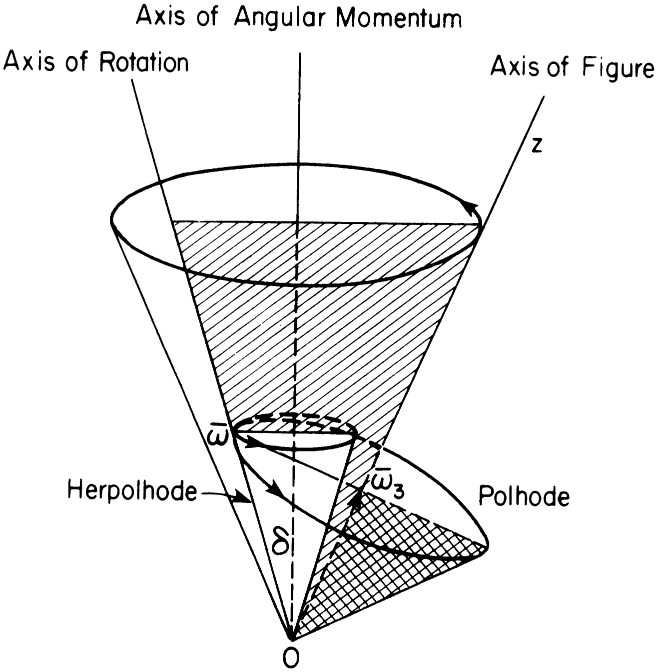

Figure 1. The Eulerian motion.

In the free Eulerian motion, the angle between the axes of rotation and figure is always small, about 0".3 or 10 m at the surface of the Earth, that is, the geographic poles are 10 m away from the poles of figure. The angle between the axes of rotation and angular momentum is even smaller, about 0".001 or 3 cm at the Earth's surface.

The luni-solar motion is represented in Fig. 2, where OZ is the axis of the system of coordinates, considered as fixed for a certain epoch. The luni-solar motion of the axes of figure and rotation in space can be considered as the resultant of the motions of these axes around $\vec{H}$ and, then, the motion of $\vec{H}$ in space.

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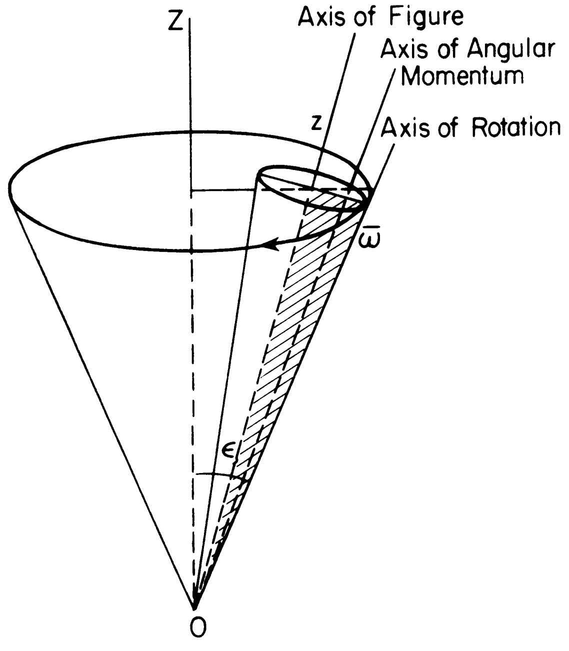

Figure 2. The luni-solar motion.

The variation of the position of the axis of rotation in space, and the corresponding motion of the geographic poles on the Earth's surface, depends on the free eulerian and luni-solar motions, but the free eulerian motion is more important than the luni-solar motion.

It is a pity that this representation is not employed more often because it would avoid many confusions and misrepresentations that have appeared lately, since geophysicists have become interested in these phenomena.

The steady improvement in the precision of astronomical instruments led to the possibility of observing the free nutation, already forecast by Euler's rigid body theory. Astronomers were skeptical about the possibilities of determining variations of latitude of the order of magnitude of 0ʺ.3 at the beginning of the 19th century, but before the end of the century these small variations of latitude were really observed, and we must give full credit to the patience and persistence of the observational astronomers who dedicated themselves to this task.

The researches of Chandler (1891-2) determined a value for the period of the free nutation of the order of 14 months. This observed value was in disagreement with the value of about 10 months forecast by Euler's theory based on a rigid Earth model. This disagreement remained without explanation for nearly a year, and there were many discussions between observers and theoreticians, each side supporting the values they found. It is interesting to think how long it took to solve this disagreement, when everybody knew that the theoretical value was determined from a very simple Earth model, while the observed value corresponded to the behaviour of the Earth.

This period is called the Chandler period and corresponds to the free nutation observed for the Earth, a complex planet constituted by a lithosphere, a hydrosphere, an atmosphere and a magnetosphere. If we consider a simple theoretical model of a rigid solid Earth, and employ Euler's dynamical equations, we have the so-called free Eulerian nutation with a period of 305 days. It was Newcomb (1892) who gave a qualitative explanation of the disagreement, saying that it was due to the elasticity of the Earth and the existence of the oceans.

This was an important explanation giving the first proof that the Earth behaved as an elastic body. It is interesting to notice that such an important geophysical result was obtained by astronomical techniques. This fact led to the setting up of the International Latitude Service (ILS) in 1899 that, I think, is one of the most remarkable programmes of international cooperation. Some of the features, adopted by our colleagues at the time, are worth noting, like the same observing programmes and instruments and the computations organized in a Central Bureau. They gave us a good example that should be followed nowadays, when we think about the modern techniques of observing the same phenomena.

Another important discovery, resulting from the observations of the ILS at the beginning of our century, was the annual component of the polar motion that could not be forecast by any theoretical model of the constitution of the solid Earth because it is due to meteorological causes. This is another good example of the care we have to take when we set up our theoretical models of the constitution of the Earth, because we must not forget that they are always approximations to the complex structure of our planet.

If we read the first papers published on the operation of the ILS, we can see the optimism of our colleagues, saying that the problems derived from the study of the variations of latitude would be solved in a few years, but we are still trying hard to understand the problems of polar motion.

We should not forget these two lessons from the history of our subject, one about the limitations of our theoretical models and the other about the optimism of solving quickly the difficulties arising in the study of this problem, because they can be applied to the problems we are encountering with the new techniques, nearly three quarters of a century later.

While astronomers were busy with these problems, and the first important result in geophysics about the elasticity of the Earth was obtained, there were some interesting developments in mathematical physics referring to the study of the behaviour of liquids contained in envelopes. Some of the more interesting results were obtained by Hough (1895) who considered elaborate models, some of them ellipsoidal, but the envelopes were always considered as rigid. Sluodsky (1895) also dealt with this subject. An obvious application was to the case of the Earth, considered as a liquid core surrounded by a rigid envelope, but the values found for the free nutation were shorter than the values obtained by Euler's theory, that is, the consideration of a possible fluid core for the Earth gave values smaller than the Chandler period corresponding to the behaviour of the Earth.

It is an interesting fact to think that no one, at the time, had the idea to employ a theoretical model composed of a liquid core surrounded by an elastic envelope, even though the elasticity theory was already well developed and Newcomb had already given a qualitative elastic explanation. If any theoretician had done that, the existence of a liquid core in the Earth could have been discovered at the beginning of the century, and not have to wait till the observational seismologists detected it by analysing their seismograms. This fact, in the history of our subject, teaches us another lesson that we must work together and consider the inter-disciplinary nature of the problem where theory and observation have to work closely.

The theory of the forced nutations was developed during the last century (Oppolzer 1886, Tisserand 1891) and the main terms of the theory were applied in the calculations of the stars' positions. The number of terms employed has increased with the improvement in the precision of the observations.

I should like to remark that, at the beginning of the operation of the ILS, only the main terms were employed in the computations but, later on, it was decided to include more terms. It is evident that this change of computation procedure introduces a source of systematic error in the coordinates of the pole. In order to avoid this and other inconsistencies that have appeared in the computations of the coordinates of the pole, some of them unavoidable at the time, it was proposed in 1970 (Vicente 1971) to set up a working group of the IAU with the task to determine the coordinates of the pole from the original observations and for the whole period of this service; it is the "Working Group on Pole Coordinates."

The theory of the forced nutations was based on a rigid Earth model, but we have to introduce the value obtained from the observations in order to compute the several coefficients of the different terms, that is, although we have a rigid body Earth model we employ an observed value corresponding, therefore, to the behaviour of the real Earth. The value so far employed for the main term in obliquity is 9".210, the so-called constant of nutation. This value was proposed by Newcomb (1895); the period is 18.6 years. One difficulty with the determination of this observed value is its long period, and, unfortunately, very few instruments have been kept observing for at least 20 years.

It was pointed out by Jackson (1930) that there was a discrepancy between the theoretical and observed values of the constant of nutation. This discrepancy led to inconsistencies among the other astronomical constants (H. S. Jones 1941). We see that astronomers were aware, in the decade 1930-40, of the difficulties encountered with the values of the constant of nutation, but no one tried to explain this discrepancy by the consideration of more complicated models for the structure of the Earth.

The end of the 19th century saw also the identification of the seismic waves P and S from seismographic records. The types of waves had been forecast by Poisson, studying the equations of motion of a disturbance in a perfectly elastic substance. The theoretical results were obtained nearly 70 years before the experimental confirmation, showing that the theory was more advanced than the observations of seismic waves. It was already in 1913 that Gutenberg found seismological evidence for the existence of a central core, the so-called liquid core of the Earth. As is well known, the expression "liquid" is applied in the strict seismological sense that the secondary (S) waves do not propagate inside this part of the core, because we do not yet know the real physical state of the matter in this region of the Earth. The first evidence of the existence of the Earth's inner core was obtained in 1936 by Lehmann.

The values of the other forced nutations, with smaller amplitudes, were more difficult to analyse in order to discover the inconsistencies between theoretical and observed values. The increase in the number of years of observation of the ILS stations, and better methods of computation, led to the confirmation of the difficulties already found for the constant of nutation (Fedorov 1958).

The studies in mathematical physics about the behaviour of liquids contained in envelopes did not concern themselves very much with the possible applications to the Earth, after the initial unsuccessful efforts at the end of last century. There was a paper by Poincaré (1910) where he studied the oscillations of a rotating ellipsoidal mass of liquid contained in a rigid envelope, analysing the effects of elasticity on the values found for the precession and nutations. This paper employs an elegant method for solving the difficulties of considering the ellipticities of the envelope and core, that have such an importance in the values of the nutations.

We are not here concerned with the bodily tide of the Earth, but we should notice that the external forces, due to the Sun and Moon, that give rise to the forced nutations, also originate tidal attractions, which deform the Earth, giving rise to the tides of the solid Earth. Different components of the Earth tide produce the nutations that we have considered, and the forced nutations correspond to diurnal tides.

The problem of the Earth tides was first considered by Kelvin (1863), and many researches have been made since then, considering several

Earth models with different values for the elastic parameters and the distribution of density. The Earth models adopted were not realistic, in the sense that they were not based on any observational results obtained on the density and elastic parameters in the interior of the Earth. The seismological studies were not yet sufficiently developed to give us more accurate results about the structure of the Earth.

It is fortunate that all the studies concerned with the Earth tides express their results in terms of the bodily tide numbers $\underline{h}$, $\underline{k}$ and $\underline{\ell}$, always defined in the same manner. The definition of these numbers is given, for instance, by Jeffreys (1976).

It is interesting to notice that in spite of the obvious connection between nutations and Earth tides, the earlier investigators did not try to solve the problems together. I think a possible explanation is that the subject was studied independently by astronomers and by people who, later on, were called geophysicists.

A great aid to the improvement of our knowledge about the internal constitution of the Earth was provided by the publication, in 1935, of the Jeffreys-Bullen tables with the travel times of bodily waves. The so-called J.B. tables relate to a model Earth in which each surface of equal P (or S) velocity in the interior is spherical and encloses the same volume as the corresponding surface of the actual Earth. An essential feature of this work has been the consistent use of statistical procedures, including significance tests.

It was then possible, by 1940, to consider the Earth's interior divided into a number of regions occupying ranges of depth from the surface to the centre. The next development was the construction of Earth models based on the seismological knowledge acquired, and Bullen (1963) published his Earth Models A (1940-2) and B (1950). Most of the researchers of that time who needed to employ Earth models considered Bullen's models because computers were not yet easily available. It was an advantage, because it was easier to compare the results obtained by different scientists, due to the fact they always employed the same Earth model.

There is an interesting series of papers by Jeffreys (1948,1949,1950) dealing with the problem of the nutations and the structure of the Earth, specially the consideration of the liquid core. It was proved that the application of a statical theory to the core is still valid for the semi-diurnal, fortnightly and semi-annual tides, but the application of statical theory, that is, the neglect of rigidity and inertia in the core, is not any more valid for the diurnal tides.

The free and forced nutations tend to alter the position of the axis of rotation of the Earth, and this fact means that the boundary conditions at the core cannot be satisfied by a statical theory with the approximation necessary for the solution of the problem. These researches show that the study of the nutations needs the application of a dynamical theory to the motion of the core.

The integration of the elastic equations of the Earth, considering for the first time an Earth model based on the observations (Bullen's model A), was done by Takeuchi (1950) with the purpose of determining the bodily tide numbers. This work was still done without electronic computers. Molodensky (1953) computes the values of the bodily tide numbers for different density distributions of the Earth models, that are arranged in agreement with the observed velocity of seismic waves.

The theory of the motion of the Earth, considered as a rigid body, around its centre of mass was very well investigated in a paper by Woolard (1953). This was an improvement on the theory previously adopted, and the number of terms considered in the astronomical ephemerides for the nutation in longitude and obliquity was nearly three times larger after 1960. While before 1960 the values were tabulated only to 0".01, after that date the series included all terms with coefficients greater than 0".0002.

The values of the luni-solar nutations in obliquity and longitude were determined considering the equations of motion of the instantaneous axis of rotation. Oppolzer (1886) was the first to show the advantages of this procedure, and we must remember that Poisson's equations give a better approximation to the motion of the axis of rotation than to the motion of the axis of figure. As is well known, the axis of rotation is very near to the axis of angular momentum and, therefore, Poisson's equations give a very good approximation to the motion of the axis of angular momentum.

It is easy to adapt the theory of the oscillations of an ellipsoid, composed of a rotating fluid contained in a rigid envelope, to the case of the rotating Earth formed by a liquid core contained in a rigid shell. Poincaré (1910) analyzes this problem employing a very elegant principle, called the simple motion of a liquid, that is, the velocity components of the liquid are linear functions of the coordinates. Transforming the coordinates and the corresponding velocities by homogeneous strain, it is possible to reduce the problem to the case of a liquid within a spherical boundary, without affecting the boundary conditions, and facilitating the solution of the problem.

The problem of the nutations of a spheroidal Earth model, composed of a shell and a liquid core, can be treated in two ways: 1) in terms of angular velocities that show already the importance of the ellipticity and fluidity of the core; 2) in terms of the displacements, revealing one of the essential features of the theory of nutation, referring to the fact that the roots of the equation for the free periods are grouped in pairs near certain values. In the case of the free Eulerian nutation one pair of roots is zero and near zero, and the other pair is $-\omega$ and near $-\omega$.

This feature is related with the general problem of resonance that appears in all problems of a dynamical system in equilibrium, where of equilibrium. A good example of this problem is the study of the nutations that appear in the motion of the Earth around its centre of mass. Considering the disturbing forces, due to the Sun and Moon, we have a pair of roots with periods very near to the period of one of the free oscillations of the system, and, therefore, we have double resonance; the system does not behave as a solid body and the influence of the liquid core is important. This is the case corresponding to the periods of the 18.6 year, semi-annual and fortnightly nutations.

The consideration of elasticity does not alter the conclusions referring to double resonance, in an Earth model composed of a shell and a core. The existence of a root very near to the zero root, and then the effects due to double resonance, are associated with the value of the core ellipticity. If the ellipticity of the core were not very small, we could not have double resonance, and the amplitudes of the nutations considered would be similar to the values found for a solid body.

The theory of nutation proposed by Jeffreys and Vicente (1957) considers an Earth model composed of a shell and a core, taking into consideration the researches of Bullen on Earth models, and Takeuchi's statical theory of the shell. At the time, the results of Bullen (1955) referring to the inner core had greater uncertainty than the values determined for the outer core, and, for this reason, simplified models for the core were adopted. It was believed that the two models considered bounded the actual behaviour of the core.

Molodensky (1961), in a paper published later than the investigations above mentioned, also solves the problem of the nutations and the bodily tides of the Earth. He employs two different models and finds, by numerical methods, a certain type of solution for a system of partial differential equations.

Several astronomical observatories have tried to observe and compute the nearly diurnal nutation in spite of the well-known difficulties in the detection of nutations with this periodicity. The observations are difficult to make because of several astronomical and geophysical causes associated with diurnal phenomena. The above mentioned theories forecast some likely periods for this nutation (Vicente and Jeffreys 1964).

The differential equations for the bodily tide and free oscillations have been derived in different ways, and expressed in different forms. We can mention spherical polar coordinates (Alterman, Jarosch and Pekeris 1959), rectangular coordinates with an Eulerian specification (Jeffreys 1929, Takeuchi 1950) or with a Lagrangian specification (Jeffreys and Vicente 1957). The difference between the Eulerian specification (actual position x_i, original position x_i-u_i) and the Lagrangian one (actual position x_i+u_i, original position x_i) makes a slight change in the boundary conditions. It was demonstrated that the equations of motion derived in these several researches are equivalent (Jeffreys and Vicente 1966).

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Some of the published researches do not take account of the different specifications employed, without due care to the boundary conditions, specially the outer and inner cores. The result is that the values obtained for the bodily tide numbers are not in agreement.

The number of papers concerned with these problems increased steadily in the last two decades, and the availability of computers resulted in the adoption of different Earth models. Some researches even include features of one model and features of another model, forgetting that the models are not, sometimes, consistent with one another.

There are so many Earth models, thanks to the easy access to computers and the rapid advances made in the physics of the Earth interior, that it is difficult to choose a convenient model. Another difficulty, resulting from the proliferation of models, is the fact that it makes difficult, if not to say impossible, the comparison of results obtained by the adoption of different Earth models.

A paper (Vicente 1973), presented at the Earth Tides Symposium during the IUGG General Assembly in Moscow, advocated the constitution of a working group, formed by members of the IAG and IASPEI, with the task to propose a reference model for the internal constitution of the Earth. This working group was formed at once and called the "Standard Earth Model Committee" (see Garland 1973).

This working group published a report of its sub-committees [Physics of the Earth and Planetary Interiors 9 (1974) 1-44], and the work to set up a reference model has been made more difficult because of the rapid advances of seismology and physics of the Earth interior during the last few years. We have to consider the possibility of adopting more detailed models including regional differences, taking account of the existence of the hydrosphere, and including the important effects of damping which Jeffreys has been calling to our attention for a number of years (see Jeffreys 1975).

We hope that in the next few years we can have a reference model for any studies about the structure of the Earth, and this model will be in agreement with the reference ellipsoid adopted in geodesy. The possibility of adopting one or a few reference models will be an improvement on the present situation, facilitating the comparison of results obtained by different scientists.

The existence of a reference model will not in any way restrict the researches on the subject, but if any paper will include, as one of their models, the international adopted reference model it will improve the possibilities of comparison. Another advantage will be the fact that scientists, who are not specialists in this field, will not have the temptation of adopting features of one model and features of another model, forgetting that the properties of the models they have adopted are not consistent.

We have described some of the researches dealing with the solid Earth (lithosphere) but the influences of the hydrosphere, atmosphere and magnetosphere are important for the determination of the values of the nutations.

The core has special relevance for this problem and a number of researches have tried to solve the equations of magnetohydrodynamics for the core, but the problem is a very difficult one.

Smith (1974) considered the theoretical problem of the infinitesimal normal modes of a rotating, slightly elliptical Earth with an interior in hydrostatic equilibrium and an isotropic perfectly elastic constitutive relation. His theory assumes ellipticity to be small but does not require rotation to be slow in any sense, and he shows how the symmetries of the problem constrain the most general form of the displacement eigenfunctions.

Shen and Mansinha (1976) have considered several types of oscillations in the outer core generalizing and confirming previous results, referring to the nutations, like, for instance, Poincaré's simple motion of a liquid.

M. S. Molodensky has studied the influence of the ellipticity (1974) and of the Coriolis' forces (1976) on the values of the nutations and bodily tides. S. M. Molodensky (1976) investigated the variation of the bodily tide numbers with different models of the Earth's structure.

Kinoshita (1976) developed a theory of the rotation of the rigid triaxial Earth considering Andoyer's variables and a moving reference plane. It has the following advantages: 1) it is easier to treat separately the motions of the axes of figure, rotation and angular momentum; 2) the development of the disturbing function is greatly simplified, because the theories of the Sun and Moon are referred to the moving ecliptic of date. A comparison with Woolard's theory, referring to the angular momentum axis, shows a fairly good agreement but there is a small discrepancy in the Oppolzer terms.

Jeffreys (1965) has criticized average Earth models based on free oscillations periods that ignored the effect of attenuation on dispersion of seismic waves, saying that it is a first order effect and based on the empirical creep law of Lomnitz. The damping of seismic waves, free oscillations and nutations is an important and difficult problem.

The dissipation in the Earth, represented by the dimensionless number Q, and the viscosity in the core are some of the difficult problems that have been considered.

Akopyan, Zharkov and Lyumibov (1975) called attention to the importance of dissipation on the values of the dynamical shear modulus of the Earth's interior. Anderson and Hart (1978) consider an Earth model taking account of the values of Q for the Earth, saying that the average Q of the mantle, and the variation of Q with depth, is likely to be quite different at tidal and free nutation periods than at seismic periods. It is stated that the absorption correction is also important in calculating bodily tide numbers from seismic Earth models that are appropriate at tidal, annual and free Eulerian periods.

There are different values obtained for Q from the Chandler period (Currie 1974), the oceanic pole tide (Currie 1975) and the 30-year period of polar motion (Vicente and Currie 1976).

One of the difficult problems we have to face when studying the nutations is the fact that they vary in period from a day to decades. Therefore, the behaviour of the materials of the Earth, that is, the rheology of our planet, is difficult to forecast for such different intervals of time.

This brief review of the influence of the Earth's constitution on the nutations presents several cases where the theory was sometimes more advanced than the observations, and at other times it was the opposite. It also shows some of the shortcomings of theoretical models and we have to keep in mind that our models, in spite of the advances made, have to be considered as asymptotic approximations to the physical reality of the Earth.

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