The Oscillations of a Mass-Spring System with Multi-Step Friction Damping

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Summary

An analysis is made of the free and forced oscillations of a single degree of freedom system damped by a multi-step friction damper It is shown that in free oscillation the decay is similar to that obtained with viscous damping, but in this case the frequency increases as the friction increases

In forced oscillation the exact solution is little different from that obtained using an equivalent viscous damper if the ratio (natural frequency/forcing frequency) is less than 1 37 For higher values the mass will remain at rest during some part of the cycle if the friction is large enough , when the above ratio is 3, 5, 7, etc , " stops " occur, however small the friction The variation of phase angle is unusual , under certain conditions the displacement leads the exciting force

On the basis of the results a criterion for the use of the equivalent viscous damper in ground resonance calculations is suggested

(1) Introduction

On some helicopters dampers are fitted to the drag hinges which rely upon sliding friction, rather than fluid viscosity, to provide for energy absorption The primary purpose of these dampers is to prevent ground resonance, but it has not so far been possible to represent their characteristics exactly in an analytical solution of the ground resonance problem Instead it is usual to assume that the friction damper can be replaced by an equivalent viscous damper, *i e* , a viscous damper which absorbs the same energy per cycle This assumption does make the analysis tractable, but since the equivalent damping coefficient is in general a function of the (unknown) frequency, rotational speed and amplitude, some process of iteration is necessary and the labour of calculation is long Also, since no exact solution is available for comparison, it is not possible to be sure that the answers obtained are correct either in detail or in principle It is, therefore, essential to know under what conditions the assumption of viscous damping is valid

As a preliminary approach, a study has been made of the free and forced oscillations of a mass-spring system restrained by a multi-step friction damper This is not, of course, a solution to the ground resonance problem, but it is sufficient to show when and where errors might arise from the use of the equivalent viscous damper It also applies directly to the problem of the blade motion in the drag plane which occurs in forward flight

The calculation of the free oscillations of a mass-spring system with Coulomb damping is well known and is discussed in several text-books on vibration[1,2] The calculation of the forced oscillations in that case is possible under certain conditions, but the method of solution is less well known This was first given by Den Hartog[3] , the results are summarised in his book on Mechanical Vibrations[4] The method is to look for those solutions which, when the steady state oscillation has been reached, have the same period as the exciting force It is necessary to introduce a phase difference between the peaks of the displacement and the exciting force and it is assumed that a transient oscillation occurs at the instants when the velocity changes sign The solution is valid so long as there are no " stops " , these occur whenever, at the instant of coming to rest, the friction force exceeds the resultant of the spring and exciting forces

An alternative approach has been described by Davidson[5], but this would seem to be in error This assumption is that the (discontinuous) variation of friction force with velocity can be represented by a Fourier series having the same period as the exciting force The analysis is then simple, but it leads to the curious result that friction has no influence on the amplitude of motion unless viscous damping is also present The fault arises from the Fourier representation of the friction , on this basis the friction is zero when the velocity is zero whereas in fact a true discontinuity does occur and the friction may lie anywhere between zero and the limiting value Numerical methods can, of course, be used and a very ingenious semi-graphical technique, due to Meissner, has been used This method, which is not widely known in this country, has been described by Kamke[6]

In this paper Den Hartog's method is used In order to obtain an analytic solution it is necessary to assume that the friction force varies linearly with amplitude , in other words the damper is assumed to have a large number of small steps If the number of steps is small, the analytical approach is not convenient and either a graphical method or some type of automatic computer must be used

(2) Notation

M	mass
K	spring stiffness
P	exciting force
F	friction force
x	displacement
x_0	displacement at which last velocity sign change occurred
t	time
k	slope of friction force/displacement line
ω_n^2	K/M
f	k/K

ω^2	$(K + k)/M$
α	$f/(1 + f)$
A, B	arbitrary constants
X_0	initial displacement
X_1	displacement after first half-cycle of free oscillation
a	P_0/K
ϕ	phase angle
p	(circular) frequency of exciting force
β	$a\omega n^2/(\omega^2 - p^2)$
n	ω/p
B_e	equivalent viscous damping coefficient
[A]	amplitude of " equivalent " oscillation

(3) Theory

General Fig 1 shows a mass M restrained by a spring of stiffness K and acted upon by an exciting force P and a friction force F The differential equation of motion is then

$$Md^2x/dt^2 + Kx = P - F \tag{1}$$

The variation of F with displacement is shown schematically in Fig 2a and an approximate representation assuming that the number of steps is large is shown in Fig 2b This representation was first suggested by C H Jones[7] as a simple means of calculating the equivalent viscous damper

At the point A, where the displacement is x_0, the velocity changes sign and the friction drops abruptly to zero at B As the mass M returns towards its mean position (0) the friction increases linearly until the mass again comes to rest at C If the velocity changes sign here then the force drops again to zero (D) and increases once more as the mass moves towards O The direction of F is always such as to oppose the motion

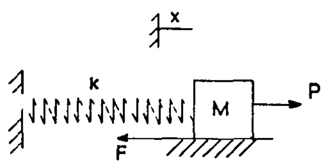
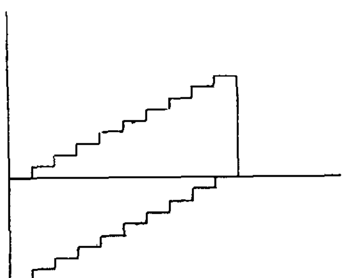
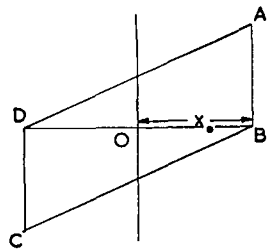


Fig 1



Actual variation of friction with displacement

Fig 2a



Assumed variation of friction with displacement

Fig 2b

Thus we may represent F analytically by a relation of the form

$$F = k(x \pm x_0) \quad (2)$$

where k is the slope of the force/displacement line and x_0 is the displacement at which the velocity last changed sign The positive sign is taken if the velocity is positive , the negative sign is taken if the velocity is negative With this choice of signs x_0 is always reckoned positive (Note that there is a difference between points (or instants) at which the mass comes to rest and points at which its velocity changes sign , it is possible for the mass to come to rest instantaneously and then to move on in the same direction)

Substituting (2) in (1) we get

$$Md^2x/dt^2 + (K + k)x = P \pm kx_0, \begin{pmatrix} + & x & - & ve \\ - & x & + & ve \end{pmatrix} \quad (3)$$

Free Oscillations In this case P = 0 and (3) becomes

$$Md^2x/dt^2 + (K + k)x = \pm kx_0, \begin{pmatrix} + & x & - & ve \\ - & x & + & ve \end{pmatrix} \quad (4)$$

It will be seen that one effect of this type of friction is to increase the effective stiffness Equation (4) is very similar to that for the free oscillations of a mass-spring system with Coulomb damping, but there is the difference that the " constant " term on the right hand side changes at the end of each half-cycle

If we put $K/M = \omega_n^2$, $k/K = f$ then (4) becomes

$$x + \omega^2 x = \pm f\omega_n^2 x_0, \begin{pmatrix} + & x & - & ve \\ - & x & + & ve \end{pmatrix} \quad (5)$$

where dots denote differentiation with respect to time

The solution of (5) is

$$x = A \cos \omega t + B \sin \omega t \pm \alpha x_0, \begin{pmatrix} + & x & - & ve \\ - & x & + & ve \end{pmatrix} \quad (6)$$

At t = 0 let the mass be displaced to X_0 and then be released , if the velocity is initially zero the displacement is given by

$$x = (1 - \alpha)X_0 \cos \omega t + \alpha X_0 \quad (7)$$

The positive sign is taken because the mass moves towards the centre of oscillation from a positive displacement

Equation (7) represents an oscillation of amplitude $[(1 - \alpha)X_0]$ about a point which is displaced αX_0 from the origin , this oscillation continues until x next changes sign at the end of the first half cycle, *i e* , when $t = \pi/\omega$

At this instant the displacement is

$$- X_0(1 - 2\alpha) = - X_1 \text{ (say)} \quad (8)$$

Then during the next half-cycle when the velocity is positive the displacement is given by

$$x = (1 - \alpha)X_1 \cos \omega t - \alpha X_1 \quad (9)$$

At the second stopping instant ($t = 2\pi/\omega$) the displacement is

$$X_1(1 - 2\alpha) = X_0(1 - 2\alpha)^2 \quad (10)$$

Thus at the end of one cycle the amplitude has decreased by a factor of $(1 - 2\alpha)^2$ But

$$\alpha = f/1 + f$$

and therefore at the end of the first cycle the displacement is

$$(1 - f)^2X_0/(1 + f)^2$$

From this point the motion follows the same pattern (see Fig 3) At the end of each half cycle the displacement is reduced by a factor $(1 - f)/(1 + f)$, *i e*, the successive peak displacements on any one side are in a geometric progression of common ratio $(1 - f)^2/(1 + f)^2$ The envelope of the peaks is similar to that obtained with viscous damping, but there the

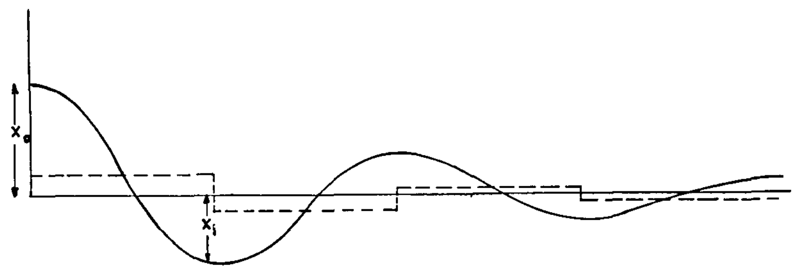


Fig 3

peaks become more and more widely spaced as the damping increases until, when the critical damping is reached, the oscillation degenerates into a subsidence On the other hand, with multi-step friction damping, the effective stiffness increases and the peaks move closer together as the friction increases It will be seen that the mass never actually comes to rest, but that the amplitude decays until it becomes imperceptible These results hold good provided that $f < 1\ 0$, *i e*, the " stiffness " of the damper must be less than the spring stiffness If the two stiffnesses are equal then the mass is in neutral equilibrium If the damper " stiffness " exceeds the spring stiffness no free oscillation is possible in one degree of freedom In the absence of mechanical springing, this state of affairs will always occur on a helicopter rotor at a sufficiently low rotational speed

Provided that $f < 1$, there is some similarity between the above results and those given by Bishop[8] for the case of "hysteretic" damping In fact it can be shown (see below, Section (4)—The Equivalent Viscous Damper) that the equivalent viscous damper for a multi-step damper is of the "hysteretic" type At one time it was thought that the type of damping represented by Fig 2b might in some cases be a useful alternative to "hysteretic" damping since the analysis is not complicated and there is no difficulty in defining the frequency of oscillation, but unfortunately any advantage which there may be does not extend to forced oscillations

Forced Oscillations As in the case of viscous damping there is a phase difference between the exciting force and the displacement It is convenient to include this phase difference in the expression for the exciting force rather than in that for the motion, *i e*, we put

$$P = P_0 \cos(pt + \phi) = a\omega_n^2 \cos(pt + \phi) \tag{11}$$

where $a = P_0/K$ (12)

and ϕ is the (as yet unknown) phase angle

Equation (1) then becomes

$$\ddot{x} + \omega^2 x = a\omega_n^2 \cos(pt + \phi) \pm f\omega_n^2 x_0, \quad \begin{pmatrix} + \dot{x} - ve \\ - \dot{x} + ve \end{pmatrix} \tag{13}$$

The solution of (13) is

$$x = A \cos \omega t + B \sin \omega t + \beta \cos(pt + \phi) \pm \alpha x_0, \quad \begin{pmatrix} + \dot{x} - ve \\ - \dot{x} + ve \end{pmatrix} \tag{14}$$

where $\beta = a\omega_n^2/(\omega^2 - p^2) \qquad \omega \neq p$ (15)

and A and B are arbitrary constants

Now when the forced motion first begins there will be a transient (free) oscillation which will ultimately decay, leaving a steady oscillation The form of this transient depends only on the initial conditions and it can be suppressed altogether , but this does not mean that the "free" oscillation terms in (14) are completely absent In fact they must be present since each time the velocity changes sign the friction force is abruptly removed and there is a discontinuity in the acceleration Therefore we must assume that the ultimate steady oscillation is made up partly from the free and partly from the forced oscillation terms in (14)

In addition to the four unknowns A, B, ϕ, x_0, (14) contains an "uncertainty," since either the positive or negative sign may be taken To eliminate this, it is necessary to assume that the period of oscillation is the same as that of the exciting force, *i e*, the overall period of (14) is $2\pi/p$ So far it has not been possible to prove that this is the only possible period, in fact when "stops" occur it cannot be so, but for motions without stops there is no reason to assume that it does not hold

The instant at which the starting transient may be assumed to have vanished is not known, but since the oscillation after that time is continuous and steady, we may measure time from any instant we choose This provides a means of determining ϕ We therefore assume that when $t = 0$, $x = x_0$, $\dot{x} = 0$, i e, the origin is taken at an instant when the amplitude is a maximum If the oscillation is steady and of constant amplitude, when $t = \pi/p$, $\dot{x} = 0$ and $x = -x_0$ By imposing these four conditions we are defining a steady oscillation as one in which the displacements and velocities at the starting instant are the exact reversals of those at the stopping instant With this definition we need consider the motion in one half-cycle only, i e, in the interval $0 < t < \pi/p$

To obtain A, B, ϕ and x_0, when $t = 0$ put $x = x_0$, $\dot{x} = 0$

$$i\ e,\ x_0 = A + \beta \cos \phi + \alpha x_0 \qquad (16)$$

$$0 = \omega B - p\,\beta \sin \phi \qquad (17)$$

and when $t = \pi/p$ put $x = -x_0$, $\dot{t} = \pi/p$

$$i\ e,\quad -x^0 = A \cos n\pi + B \sin n\pi - \beta \cos \phi + \alpha x_0 \qquad (18)$$

$$0 = -\omega A \sin n\pi + \omega B \cos n\pi + p\beta \sin \phi \qquad (19)$$

where $n = \omega/p$ (20)

From (16)—(19) we get

$$A = -x_0 \alpha \qquad (21)$$

$$B = -x_0 \alpha \tan (n\pi/2) \qquad (22)$$

$$x_0 = \beta \cos \phi = \beta/[1 + \alpha^2 n^2 \tan^2(n\pi/2)]^{\frac{1}{2}} \qquad (23)$$

$$\tan \phi = -\alpha\, n \tan (n\pi/2) \qquad (24)$$

These results are best shown in the form of a frequency response diagram, but before doing this it must first be established that the solution is in fact valid To obtain (21)—(24) it was necessary to impose certain conditions and if these should lead to the result that the velocity changes sign in the interval $0 < t < \pi/p$, then the solution is invalid Also (14) does not hold when the free oscillation and exciting frequencies are equal ($n = 1\ 0$) and this case requires special study

(4) Special Cases

Applicability of the Method On substituting (21)—(24) in (15) and differentiating, the expression for the velocity in the interval $0 < t < \pi/p$ becomes

$$\dot{x}/px_0 = \left\{ \frac{\alpha n[\sin n\theta - \sin \theta \sin(n\pi/2)]}{\cos (n\pi/2)} - \cos \theta \right\},\ n \neq 1 \qquad (25)$$

where $\theta = pt - \pi/2$ $\quad$ i e, $-\pi/2 < \theta < \pi/2$ (26)

The first term on the right-hand side of (25) represents the modification to the velocity due to friction If, for any value of θ within the prescribed range, this term is positive and exceeds cos θ then a " stop " will occur and the solution no longer applies Evidently when n = 3, 5, 7, etc , this term is infinite however small the value of α so that the first restriction is that a steady oscillation without " stops " is not possible if the frequency of free oscillation is an odd-integral multiple of the forcing frequency For intermediate values of n it can easily be shown that stops will occur if α exceeds some value which depends on n, *e g* , when n = 2, α must not exceed 0 35 for the solution to be valid As n decreases the value of α required to cause stops increases rapidly and, since α cannot exceed unity, it follows that there must be some value of n below which " stops " do not occur, however large the friction Calculation shows that this value of n is 1 37 approximately

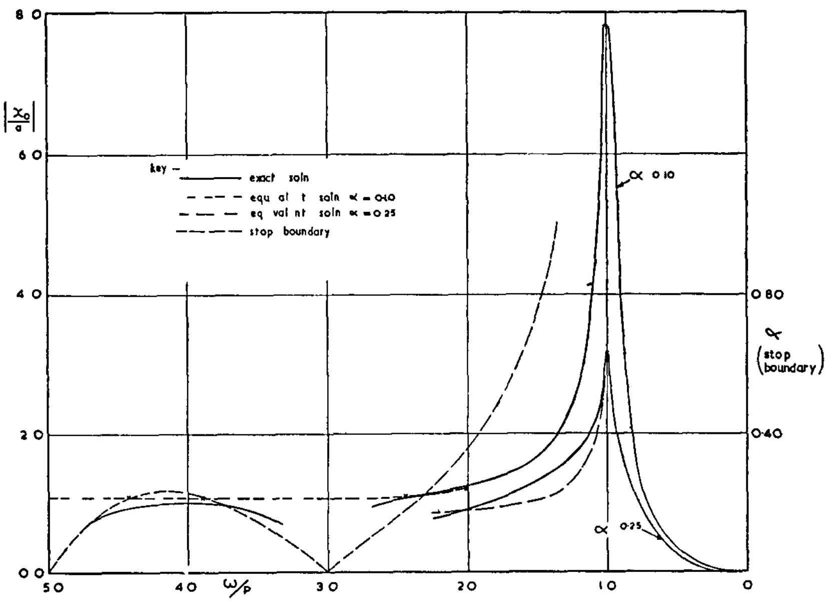


Fig 4

In other words the solution given by (21)—(24) applies without restriction provided that the frequency of free oscillation is less than 1 37 times the exciting frequency For larger values of n the solution given is valid provided that α lies below the boundary line shown in Fig 4

It is interesting to compare these results with those obtained by Den Hartog[3] for the case of Coulomb friction damping There too it was found that stops will occur for n = 3, 5, 7, etc , however small the friction, but it was also found that stops will occur for all n providing that the friction force is a sufficiently large fraction of the (peak) exciting force In the present case it will be seen from (25) that the ratio of the instantaneous velocity to the maximum velocity is independent of β so that the occurrence, or non-occurrence, of stops is independent of the magnitude of the exciting force With Coulomb damping, however, since the amplitude is not proportional to β and the friction is independent of amplitude, it follows that stops will always occur under some conditions

Resonance In the special case when the free and exciting frequencies are equal (*i e*, n = 1 0) the solution of (13) is —

$$x = (1 - \alpha)x_0 \cos pt - (a\omega_n^2/2p^2)\sin pt + (a\omega_n^2/2p)t \cos pt + \alpha x_0 \quad (27)$$

$$\text{where } x_0 = \frac{\pi a}{4f} \quad (28)$$

and the peaks of exciting force and displacement are 90° out of phase

The velocity is

$$x/px_0 = -\frac{\sin pt}{(1+f)}\left\{1 + \frac{2pt}{\pi}f\right\} \quad (29)$$

Since the term in brackets is never negative it follows that " stops " do not occur at resonance however large the friction

The result (28) for the amplitude at resonance can also be obtained from (23) provided that the limit as n tends to unity is taken in the correct manner

The Equivalent Viscous Damper The " equivalent viscous damper " is defined as the viscous damper which absorbs the same energy per cycle as the actual damper—assuming a sinusoidal oscillation

Energy absorbed per cycle by friction damper

$$= 2x_0\, 2kx_0 = 4kx_0^2 \quad (30)$$

Energy absorbed per cycle by equivalent viscous damper

$$= \pi p B_e x_0^2 \quad (31)$$

where B_e is the equivalent viscous damping coefficient

Then from (30) and (31)

$$B_e = \frac{4k}{\pi p} \quad (32)$$

i e, the equivalent viscous damping coefficient is independent of the amplitude and inversely proportional to the frequency But this is precisely the form of the " hysteretic " damper described by Bishop[8] In the present notation Bishop's results for the amplitude and phase are —

$$[A] = \frac{\beta}{(1+f)\left\{1 + \frac{16\alpha^2 n^2}{\pi^2(1-n^2)^2}\right\}^{\frac{1}{2}}} \tag{33}$$

$$\tan\phi = \frac{4\alpha}{\pi}\frac{n^2}{n^2-1} \tag{34}$$

We now have sufficient information to plot a frequency response diagram for the continuous oscillations with friction damping and to compare these results with those obtained for the equivalent viscous damper In Fig 4 the " magnification factor " $[x_0/a]$ is plotted against n and the variation of " phase " with n is shown in Fig 5 On both diagrams α has the values 0 1 and 0 25

(5) Discussion of the Results

Since $n = \omega/p$ it follows that resonance occurs when the frequency of the (damped) free oscillation is equal to the exciting frequency, *i e*, at a frequency which is greater than that of the undamped free oscillations in the ratio $(1+f)^{\frac{1}{2}}$ 1 For $n < 1\ 37$ the amplitude frequency curves are of conventional form but for larger n this is no longer the case Consider first the results for $\alpha = 0\ 10$

As n increases beyond 1 37 the magnification factor drops rapidly towards unity but, from (25), the solution ceases to be valid when $n \approx 2\ 65$ At this value of n the " stop " boundary—shown dotted in Fig 4—intersects the line $\alpha = 0\ 10$ and no continuous solution with the period of the exciting force is possible until $n = 3\ 35$ when the " stop " boundary and $\alpha = 0\ 10$ again intersect Continuous solutions are then possible until $n \approx 4\ 8$ where a third intersection occurs Further intersections will occur at higher n but ultimately a value of n is reached beyond which no continuous solution can occur at all This is illustrated by the results for $\alpha = 0\ 25$ The line $\alpha = 0\ 25$ intersects the " stop " boundary at $n \approx 2\ 2$ and beyond this no further intersections occur, *i e*, no continuous oscillations are possible if $n > 2\ 2$

In the vicinity of n = 2, 4, etc , the magnification factor is the same as for undamped oscillations but as n approaches 3, 5, etc , it decreases rapidly For smaller values of α it is possible to maintain continuous oscillations at values of n which are closer to 3, 5, etc , but the accompanying decrease in amplitude is then much more abrupt It is to be expected from these results that the amplitude of an oscillation with " stops " will be considerably less than that of the undamped vibration

Further, and perhaps more important, departures from conventional behaviour at large n are shown by the phase-frequency curves Instead of the phase angle remaining approximately constant at some positive value for all large n it does in fact become zero at n = 2 for all α—provided that the solution is valid For larger n the phase angle becomes negative, *i e*, the peaks of displacement then *lead* the peaks of the exciting force In the vicinity of n = 3 the solution is not valid but when n = 4 the lead angle is 180°, *i e* , there is a phase change of 2π between n = 0 and n = 4 Beyond n = 4 this cycle of phase change is repeated until continuous solution ceases to be possible

Also shown on Figs 4, 5, are the corresponding results using the equivalent viscous damper* Up to the value of n where " stops " first occur there is not much difference between the amplitudes of the " equivalent " and " exact " solutions At resonance they are exactly equal , at other values of n the exact amplitude is slightly larger, the difference increasing with increasing α The greatest differences occur when n $\approx$ 3, 5, etc , when the

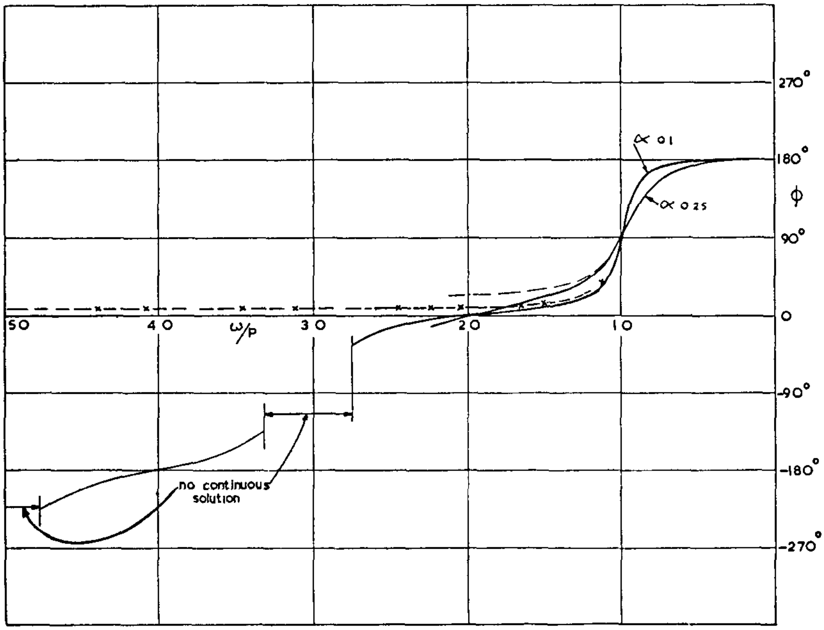


Fig 5

equivalent amplitude is about 50% greater than the exact Since the phase-frequency curve for the equivalent damper is of conventional form there is a considerable difference between the phase angles given by the two methods

(6) Conclusions

It has been shown that exact solutions can be obtained for the free and forced oscillations of a mass-spring system with a (simplified) multi-step friction damper Unlike Coulomb damping the multi-step damper does not cause complete stops in the free oscillations, nor do stops occur at all frequencies in forced oscillations Stops will occur in forced oscillations if the friction and/or the ratio between the free oscillation and exciting frequencies is large enough

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* It is quite legitimate to plot the results for the equivalent viscous damper against ω/p and not against ω_n/p since the additional *stiffness* contributed by the friction damper must still be taken into account

The equivalent viscous damper gives a very good approximation to the exact solution provided that the frequency of the (damped) free oscillations is less than about 1 37 times the exciting frequency At resonance the amplitude and phase given by the two methods are exactly equal For larger values of n the phase angles are completely different and errors of 50% or more in amplitude are possible

It is evident from these results that the equivalent viscous damper is sufficiently accurate for ground resonance calculations only if the true frequency of the blade oscillations is more than about 0 70 of the natural frequency Therefore since the important blade oscillation frequency is the difference between the chassis oscillation frequency and the rotational speed and as the blade natural frequency is about 1/3 of the rotational speed it follows that the chassis frequency must be less than about 3/4 of the rotational speed If this condition is not satisfied stops will occur if the friction is large enough But even if stops are avoided the use of the equivalent viscous damper may still lead to a large error since the occurrence of coupled self-excited oscillations depends very much on achieving the correct phase relationship The model test results described by C H Jones[7] show that the chassis frequency is only close to the rotational speed if the blade and chassis motion, or the chassis motion alone, is undamped Otherwise the chassis frequency is about 3/4 of the rotor speed and stops seem unlikely to occur This cannot be regarded as a final answer however, since a motion with stops may have very different characteristics (see Chap XI of Ref 9)

The method of Section (3) on Forced Oscillations, could be extended to deal with free, coupled, continuous oscillations but the calculation would probably be very elaborate because of the several frequencies of the undamped free oscillations There is no doubt that the best way to solve this problem is by means of an analogue computer (On this type of machine the multi-step damper is in fact easier to simulate than the equivalent viscous damper) Some work to this end has already begun, but the problem is complicated by the fact that the periodic terms in the differential equations of motion cannot now be eliminated by a transformation of the type used by Coleman[10]

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