PULSATION PROPERTIES OF MIRA LONG PERIOD VARIABLES

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INTRODUCTION.

A matter of great interest to variable star students concerns the mode of pulsation of Mira long period variables. In this report we first give observational evidence for the pulsation constant $Q = P(\mathcal{M}/\mathcal{R}^3)^{1/2}$ where P is the period and $\mathcal{M}$ and $\mathcal{R}$ are the mass and radius in solar units. We then compare the observations with calculations. Next, we review two interesting groups of papers dealing with hydrodynamic properties of long period variables. In the first, a fully dynamic nonlinear calculation maps out the Mira instability domain. In the second, special attention is paid to shock propagation beyond the photosphere which in large measure accounts for the complex spectra from this region. Finally, we review the paper by Wood-Zarro (1980) dealing with the pulsation constant Q.

Q VALUES: THEORY AND OBSERVATION

In order to estimate a value of Q for a Mira variable, estimates of mass and radius must be made. Mass is estimated from the period and mean spectral type at maximum as described by Cahn and Wyatt (1978). Recently we have revised Fig 2, the mass-luminosity region shown in Cahn and Wyatt (1978). The revision is shown here as Figure 1. The principle difference between these figures is that the current version 1) assumes that only stars of one solar mass or greater have thus far evolved onto the asymptotic giant branch; 2) that Q = 0.1 is an appropriate value of the pulsation constant; and 3) that the derived distances must be consistent with the proper motions. The zero point of the luminosity scale has been set by doing a statistical parallax of 123 Mira distances. The Osvalds and Risley (1961) proper motions and radial velocities were used. Assuming Q = 0.04 makes very little difference in Figure 1. The core mass-luminosity shown is from Iben (1980). The resulting Mira region shows a sharp cut-off near $\log \mathcal{L} = 3.82$. Whereas Figure 2 in Cahn and Wyatt (1978) included carbon stars, they have been effectively removed above $\log \mathcal{L} = 3.82$ where they may amount to 40% of the total.

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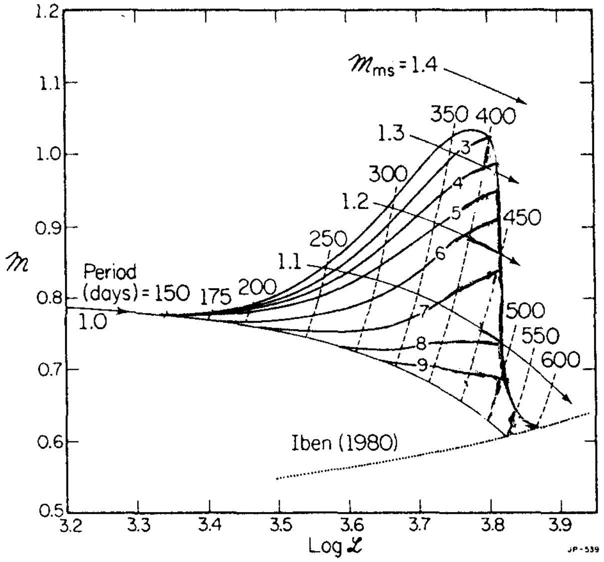

Fig. 1. Mira region in the mass-luminosity plane. Constant period, constant M spectral type lines and evolutionary lines are shown.

The radii may be determined from theoretical surface brightness and measured fluxes as described by Grey (1967, 1968). The approach to be followed here is to make use of the Wesselink (1969) idea

$$\begin{aligned} F_K &= 4.2207 - 0.1K - 0.5 \log \phi' & \text{1a} \\ &= 3.2629 + 0.25 \log S_K & \text{1b} \\ &= \log T_e + 0.1\ BC_K & \text{1c} \end{aligned}$$

as formualated by Barnes and Evans (1976a). Here K is the 2.2 μm magnitude, ϕ' the angular diameter in arc milliseconds, S_K is the surface brightness in Watts $cm^{-2} \mu m^{-1}$, and BC_K is the bolometric correction at K magnitude. There are two ways to proceed: we can, on the one hand, determine the function F_K empirically as a function of a color, V-R, as was done by Barnes et al. (1976a, 1976b, 1978). As will become clear, such a course is not possible for Miras. The other course of action is to make use of model atmosphere calculations of S_K and to generate the function F_K through Equation 1b. This is the method followed by Blackwell et al. (1977, 1979) and the one adopted here. In the present work, we adopt as color the unreddened color $(J-K)_o$ as suitable for such late type stars. The models of Johnson et al. (1980) were used to construct the function F_K for M2 to M9.5 at T_e = 3000 K as shown by the solid line in Figure 2. We used the Johnson et al. (1980) log g = 0.0 for the model curve in Figure 2, even though the value of log g probably lies between 0 and -1. As demonstrated by Piccirillo et al. (1980), the stellar gravity does not markedly affect the spectrum for variations of this amount. The effective temperature accompanying each spectral type are from Ridgeway et al. (1980). Using bolometric corrections BC_K obtained from Dyck et al. (1974), we find excellent agreement between Johnson et al. (1980) and Ridgeway et al. (1980) when the data are compared using Equations 1b and 1c. We have extended the functions F_K to spectral type K3 using the Ridgeway temperatures and the bolometric corrections in Eq. 1c along the dashed portion of the curve.

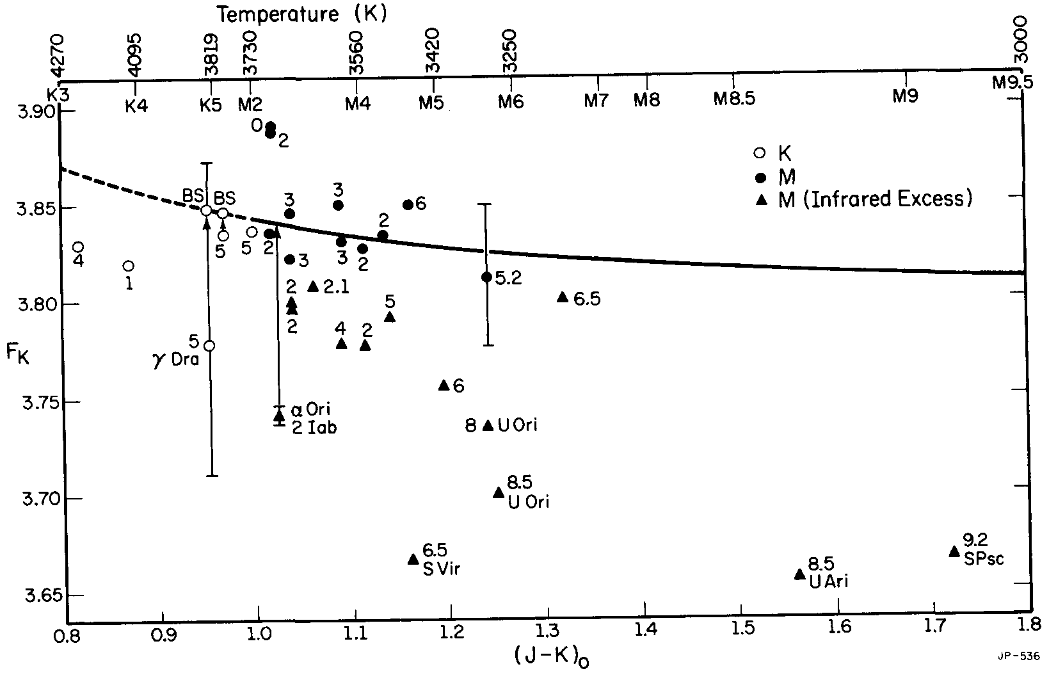

Fig. 2. Surface brightness parameter as a function of $(J-K)_0$. The solid curve from Johnson et al. (1980), dashed curve from Ridgeway et al. (1980). The numbers indicate spectral subtypes. The symbols indicate spectral type. Representative error bars shown.

From a totally different point of view, Iben (1980) has recently computed static models resembling Miras. He finds for a given carbon-oxygen core mass, a minimum in the effective temperature as a function of decreasing envelope mass. These models, unlike the model atmospheric calculations, are carefully fitted to the cores, but have the usual Cox-Stewart opacities. Yet the temperatures and radii are generally within 15% of those predicted by the Johnson et al. (1980) model atmospheres. It will be interesting to examine the Iben models with more appropriate opacities.

Using Equation 1a, several normal giants from spectral type K1 to M6 are shown in Figure 2. For each star, the angular diameter and K magnitude are known. The two error bars shown indicate that the curve F_K adequately represents the normal giants. The two stars marked BS show that the Blackwell et al. (1977) angular diameters are in excellent agreement with our adopted surface luminosities. A number of giants lying below the theoretical curve including Miras have been marked by the symbol ▲. Each of these stars, including the supergiant Betelgeuse, which has been included for historical reasons, is known to have an infrared excess [3.5] - [11.0] (Gillett et al. 1971, Gehrz et al. 1971, Dyck et al. 1971). The explanation of this effect is due to Tsuji (1978) who demonstrated that the presence of dust in a circumstellar shell having an optical depth τ_{vis} of a few tenths can easily cause

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a 50% enhancement of the angular diameter. The correction in the case of Betelgeuse puts the star squarely back on the theoretical F_K curve as shown by the arrow. Since all the Miras are known to have sizeable infrared excesses, the only way to estimate their photospheric radii is to make use of model atmosphere calculations.

Following a suggestion of Art Cox, we have computed the mean angular radii of a number of Miras, in order to estimate the pulsation constant Q. The distances were determined as in Cahn and Wyatt (1978). The resulting values of Q are shown in Figure 3. The abscissa

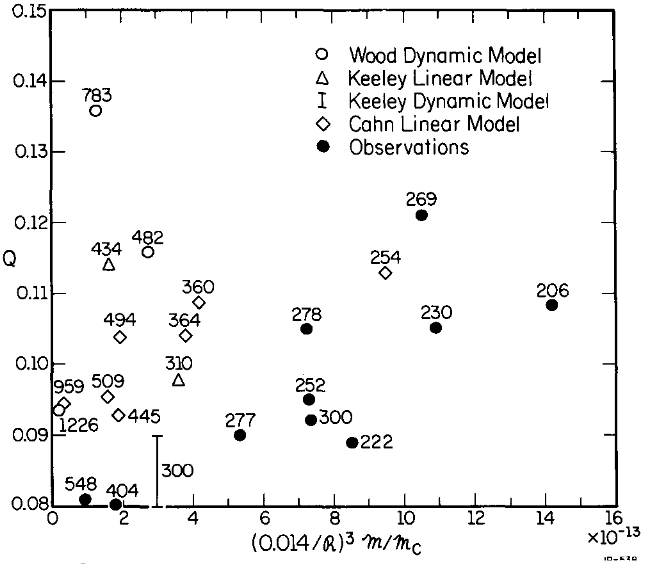

Fig. 3. Pulsation constant as a function of $\overline{\rho}/\rho_c$. Points labelled "observations" were derived from the theoretical surface brightness function in Figure 2.

in Figure 3 is suggested by Cox (1980) in which it is argued that Q should be an increasing function of $\overline{\rho}/\rho_c$. We have chosen to represent this ratio by $(.014/\mathcal{R})^3\, \mathcal{M}/\mathcal{M}_c$ where the core radius is assumed to be $.014R_\odot$ and $\mathcal{M}_c$ is the core mass in solar units. The empirical points seem to follow such a trend Also shown in Fig 3 are the Q values for the fundamental mode for the fully dynamical calculations of Wood (1974) as well as the fundamental mode linear nonadiabatic calculations of Keeley (1970 a,b). Of great interest is the dynamical calculation of Keeley which partakes of some of the characteristics of both the fundamental and first overtone radial modes. Such a result was first described by Wood (1976). The articles in this volume by Dave King and by John Cox and collaborators on RCrB stars may be related to Keeley's results. This "strange" mode only appears in highly nonadiabatic situations at high luminosity. Lastly, shown in Figure 3 are the results of recent Los Alamos linear nonadiabatic fundamental mode calculations using the Ross-Aller opacities and marked ◇. These opacities include all the appropriate molecules at the low temperatures of the Mira photosphere and emphasize the extreme central mass concentration in Miras. The trend of the Los Alamos calculations is quite similar to but displaced to larger Q relative to the observations. The periods of both the stars and the model calculations are shown next to each entry.

Contrary to earlier concerns (Wood, 1974), it now appears that model calculations can indeed simulate the stars both as to period and the ratio $\bar{\rho}/\rho_c$. Nonlinear studies of these models are planned. No mention of the many first overtone calculations are made, in view of the apparent discrepancy (Q ~ .04 days) with the results reported here.

THE MIRA INSTABILITY REGION

We next review the work of Tuchman, Sack and Barkat (1978, 1979) and of Barkat and Tuchman (1980) hereinafter referred to as BST. In this series of papers they attempted to derive the empirical period-number density relation of Wood and Cahn (1977) hereinafter WC. Following the hydro-dynamic code of Kutter and Sparks (1972), they determined from fully dynamic nonadiabatic calculations, (1), where the onset of the first overtone pulsational instability occurred and (2), where the occurrence of the fundamental pulsational dynamic instability led to rapid mass loss and the formation of planetary nebulae. We show in Figure 4 the results of their calculations in the mass-luminosity plane. Evolutionary paths of 1, 2, 4, and 6 $M_\odot$ asymptotic giant branch double shell burning stars are shown, (a) where they enter the Mira region (M) and (b) where they become unstable against fundamental mode oscillation upon intersecting the curve (P) and begin rapid mass loss.

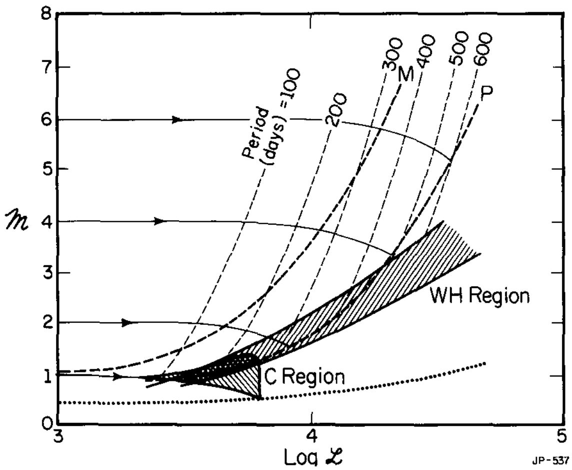

Fig. 4. Mass-luminosity plane from BST. Superimposed are the Mira regions of Fig. 1 (C region) and that of WH. The period lines are from BST. The WH region is made to agree with the BST period lines, as WH makes no assertion about luminosity.

The evolutionary curves are for Reimers' (1975) mass loss rates of $10^{-13}\mathcal{L}\mathcal{R}/\mathcal{M}\,M_\odot\,yr^{-1}$ where $\mathcal{L}$, the stellar luminosity, and $\mathcal{M}$, and $\mathcal{R}$ are in solar units. Region C below the BST Mira region is our current Mira region shown in Figure 1. The Mira region in Figure 1 extends from periods of 175 to 500 days and the corresponding luminosities are in accord with statistical parallax. The BST calculations are not quite in accord with the parallax requirements, say, of Clayton and Feast (1969). The overlap of the two Mira regions would be more complete were higher mass loss rates assumed.

In an effort to obtain agreement with the WC number density-period relation, BST found it necessary to invoke both mass-loss and helium shell pulsing. In so doing, as pointed out by Wood and Zarro (1980), they concluded that stellar masses below 2 $M_\odot$ must be excluded. This in their view (BST) was due to the rapid evolution time through the Mira region caused by the helium pulses.

Still another point of interest is that the BST calculations imply that the Mira region of Figure 1 is dominated by the fundamental mode. This accords with our conclusion that the observational evidence favors a Q very close to that of the fundamental. Much more research remains to be done before we know if such modes are stable.

What can be said of the BST Mira region in Figure 4 for masses clearly larger than the relatively large number of low mass Miras analyzed by WC of which 94% were oxygen rich? One can speculate that this constitutes a region populated by SRc variables, most of which are of M type, as well as those MS-S-CS-C spectral type semiregulars, some of which may prove to be high mass stars.

SHOCK HYDRODYNAMICS

We next summarize the papers by Willson (1976, 1979), Willson and Hill (1976, 1979), Hill and Willson (1979), and Pilachowski, Wallerstein and Willson (1980) hereinafter WH. These papers begin with an analysis of the spectra of long period variables in which the presence of shocks is shown to be an unavoidable conclusion of the complicated velocities associated with variable star spectra. In summary, it is shown that a shock emerges from the photosphere shortly after visual light maximum. This is a strong shock of the order of 50 km s^{-1}, and the excitation behind the shock accounts for ultraviolet lines of Mg II, Fe II, and Ti II. The ion lines do not appear until about halfway through the light cycle due to opacity effects. Ahead of the shock, and with an appropriate shock velocity jump velocity of 40 to 80 km s^{-1}, a large number of flourescent lines are excited. In this way, convincing proof of the shock is presented. The production of neutral Balmer lines as well as Si I, Fe I, and Mg I is either due to thermal excitation or recombination behind the shock or excitation ahead of the shock.

The success of the shock excitation of spectra prompted WH to construct dynamic models of the atmosphere of a pulsating red giant. The lower boundary was assumed to be periodic with a modest 12% amplitude, presumably the photosphere. Analysis of the atmosphere demonstrated a rising shock which lasted about one period. One period later, another shock commenced which meant that at certain times two shocks, one beginning and one high in the atmosphere, could be discerned. From the line velocity analysis, it was possible to determine pre- and post-shock velocities for both the upper and lower shock for a given star.

After a number of models were examined, it was found that the ratio of post-shock velocity of the lower shock, $v_o(0)$, to the escape velocity, v_e, was nearly a constant, 0.2. Also, the ratio of the shock velocity, v_s, to the post shock velocity, $v_o(0)$, was found to be nearly

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constant at 1.2. This made it possible to construct lines of constant $\mathcal{M} = (v_o(0)/0.2)^2 R(0)$ where R(0) is the radius of the lower shock. Further analysis showed that the post shock velocity for the upper shock $v_o(P)$ was given by $v_o(P) = v_o(0)[1 + 1.8v_o(0)P/R(0)]^{-1/3}$ where P is the period, so that lines of constant $v_o(P)$ can also be drawn on the mass-radius plane for each star. Observations then determine the region best satisfied by the data. Within the uncertainties of the data, reasonable values of mass and radius can be obtained as shown in particular for Mira in Figure 5. The bounding lines of constant $v_o(0)$ and $v_o(P)$ in this case are from the observations for Mira. Lines of constant Q are

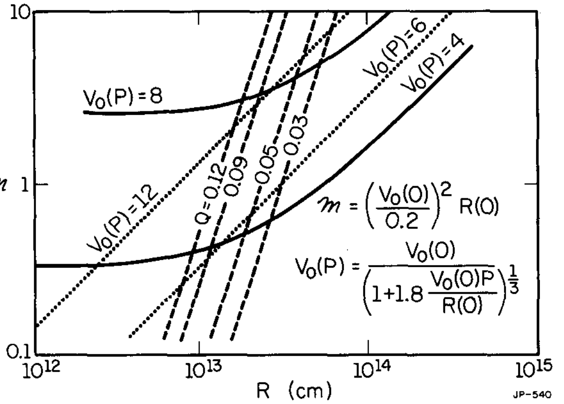

Fig. 5. Mass radius plane for Mira from WH Lines of constant $v_o(0)$ and $v_o(P)$ are shown.

also shown in Figure 5. It is clear that larger values of mass are required for smaller assumed values of Q. There does not seem to be a strong preference between a Q of 0.04 and 0.10. A stronger argument for mode selection comes from the analysis of the Payne-Gaposchkin and Whitney (1975) data giving shock position as a function of phase. Here the value of Q = 0.1 is preferrable to 0.04.

Both WH and Wood (1979) found that isothermal shocks, which would be expected deep within the stellar atmosphere, gave mass loss rates orders of magnitude lower than currently estimated mass loss rates of 10^{-8} to 10^{-6} $M_\odot$ yr^{-1}. On the other hand, both found that adiabatic shocks produced excessively high mass loss rates. Consequently, WH suggested that at a point in the atmosphere where the recombination length equals the static pressure scale height, the shock would cease to be isothermal and become adiabatic. The calculations in their paper were not correctly done; the mass loss rate should be written $\dot{\mathcal{M}} = - 4\pi R^2 \alpha n_e n_H kT/g(R)$ where α is the electron-ion recombination coefficient, n_e and n_H the electron and hydrogen number density, T the temperature and g(R) the acceleration of gravity at radius R. Such an expression, difficult though it is to evaluate, would make it possible to estimate mass loss rates from fundamental quantities. The mass loss expression yields acceptable rates. Were it written in the form $\dot{\mathcal{M}} = - L(n_e/n_+)/g(R)\Delta R$ where $r = 4\pi R^2 \Delta R \alpha n_e n_+ kT$ is a recombination luminosity between ions, n_+, and electrons, it is suggestive of Reimers' (1975) dimensional mass loss formula. The recombination luminosity is a measure of the shock strength.

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The WH and BST calculations may be compared as is done in Figure 4. The WH region definitely falls to the right hand side of the BST region, again favoring the higher Q value of 0.1.

HELIUM SHELL PULSES

In addition to the recent work affecting the choice of mode in long period variables, the work of Wood and Zarro (1980) should be mentioned, hereinafter WZ. WZ interpreted many years of data for two stars R Hya and R Aql for which the periods have been changing. The hypothesis was that each star was near maximum light due to a helium shell pulse; R Aql just before and R Hya just after. WZ show that change of luminosity is proportional to change of period. The argument is made that a Q of 0.04 is necessary to obtain the correct level of luminosity. We would have to agree with this result. Our current estimates of Mira temperatures are much higher than WZ, for our current Mira branch position on the HR diagram is $\log \mathcal{L} = 21.9 - 5.19 \log Te$ compared to $\log \mathcal{L} = 40.7 - 10.71 \log T_e$ used by WZ. A Q of 0.04 is still required to fit the helium shell pulse luminosities as depicted by WZ.

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DISCUSSION

J. COX: I was just asking myself whether or not this strange mode that you mentioned has anything to do with the Mira variable?

CAHN: I am not sure. You certainly have observed this in the linear calculations. It certainly is a challenge to see if that can be understood. We have found, using the new Ross-Aller opacities, with lots of molecules, that we have a difficult time getting down in mass into the star. We can get deep enough in radius but there is still too much mass remaining. We have to resolve that problem before we can hope to understand this.

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