





The impact of magnetism on tidal dynamics in the convective envelope of low-mass stars

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Abstract. For the shortest period exoplanets, star-planet tidal interactions are likely to have played a major role in the ultimate orbital evolution of the planets and on the spin evolution of the host stars. Although low-mass stars are magnetically active objects, the question of how the star's magnetic field impacts the excitation, propagation and dissipation of tidal waves remains open. We have derived the magnetic contribution to the tidal interaction and estimated its amplitude throughout the structural and rotational evolution of low-mass stars (from K to F-type). We find that the star's magnetic field has little influence on the excitation of tidal waves in nearly circular and coplanar Hot-Jupiter systems, but that it has a major impact on the way waves are dissipated.

Keywords. MHD, waves, stars: evolution, stars: rotation, stars: magnetic fields, planetary systems

1. Introduction

Over the last two decades, about 4000 exoplanets have been discovered around low-mass stars (Perryman 2018). In close-in star-planet systems, tidal dissipation in the host star is known to affect the semi-major axis (and thus the orbital period) of the companion as well as the spin of the star over secular timescales (see e.g. Ogilvie 2014 for a review on this topic). In particular, the dissipation of the stellar dynamical and equilibrium tides (Zahn 1977) can vary significantly along the evolution of the star. It is highly dependent on stellar parameters like the angular velocity or the metallicity (Mathis 2015, Gallet *et al.* 2017 and Bolmont *et al.* 2017). Therefore, it is very important to identify and quantify in the most realistic way the dissipation processes that come into play. In this respect, we have examined the effect of stellar magnetism on the excitation and dissipation of dynamical tides inside the convective envelope of low-mass stars throughout their evolution. For this purpose, we have used detailed grids of rotating stellar models computed with the stellar evolution code STAREVOL, as well as databases of observed star-planet systems. We first examine (in Sect. 3) the impact of the star's magnetic field on the effective tidal forcing exciting magneto-inertial waves. The amplitude of a relatively large scale magnetic field is estimated via physical scaling laws at the base and the top of the convective envelope (Sect. 3). We then assess the ratio of the magnetic and

Table 1. Dynamo-like magnetic field derived from simple energy or force balances.

Regime	Balance	Estimation of B_{dyn}
Equipartition	ME = KE	$\sqrt{\mu_0 2\text{KE}}$
Buoyancy dynamo	ME/KE = $\text{Ro}^{-1/2}$	$\sqrt{\mu_0 2\text{KE}/\text{Ro}^{1/2}}$
Magnetostrophy	$\mathbf{F}_L = 2\rho_0\boldsymbol{\Omega} \times \mathbf{u}$	$\sqrt{\mu_0 2\text{KE}/\text{Ro}}$

Notes. KE and ME are the kinetic and magnetic energy densities of the convective flow, respectively. Ro is the fluid Rossby number.

hydrodynamic tidal forcings for several short-period exoplanets (Sect. 4) before analysing the relative importance of viscous over Ohmic dissipation of kinetic and magnetic energies (Sect. 5).

2. Influence of magnetism on the effective tidal forcing

In the presence of stellar magnetic fields, both the excitation and dissipation of tidal waves are theoretically modified when compared to the hydrodynamical case because of the Lorentz force and the magnetic diffusion. The linearised momentum equation for tidal waves in a convective region can be written as (Lin & Ogilvie 2018):

$$\rho_0(\partial_t \mathbf{u} + 2\boldsymbol{\Omega} \times \mathbf{u}) + \nabla p - \mathbf{F}_\nu - \mathbf{F}_L = \mathbf{f}_{\text{hydro}} + \mathbf{f}_{\text{mag}}, \tag{2.1}$$

where we have introduced ρ_0 the mean density, $\boldsymbol{\Omega}$ the spin of the star, \mathbf{u} , p the perturbed flow and pressure, and \mathbf{F}_ν , \mathbf{F}_L the effective viscous and Lorentz forces, respectively. Magneto-inertial waves (left-hand side of Eq. (2.1)) are forced by an effective tidal forcing (right-hand side), resulting mainly from the action of the Coriolis pseudo-force and the Lorentz force on the equilibrium tide ($\mathbf{f}_{\text{hydro}}$ and \mathbf{f}_{mag} , respectively). Since \mathbf{f}_{mag} is often neglected in studies of tidal interactions (see, e.g., Lin & Ogilvie 2018 and Wei 2016, 2018), we propose to examine its amplitude (f_{mag}) relative to f_{hydro} (the amplitude of $\mathbf{f}_{\text{hydro}}$) when varying the mass and age of low-mass stars. Using typical scales of a star-planet system such as R the radius of the star and σ_t the tidal frequency, the magneto-to-hydrodynamical forcing ratio can be recast as:

$$\frac{f_{\text{mag}}}{f_{\text{hydro}}} \sim \text{Le}^2 / \text{Ro}_t / \hat{\sigma}_{\text{max}}, \tag{2.2}$$

with $\text{Le} = B / (\sqrt{\rho\mu_0} 2\boldsymbol{\Omega}R)$ the Lehnert number (Lehnert 1954), $\text{Ro}_t = \sigma_t / (2\boldsymbol{\Omega})$ the tidal Rossby number, and $\hat{\sigma}_{\text{max}} = \max\{\sigma_t / (2\boldsymbol{\Omega}), 1\}$ a dimensionless factor close to unity. We refer to Astoul *et al.* (2019) for a detailed derivation of Eq. (2.2).

3. Scaling laws to estimate stellar magnetic fields

To evaluate the ratio of the effective tidal forces (Eq. 2.2), we made use of simple energy and force balances to give a rough estimate of the dynamo-generated magnetic field strength inside the convective zone of low-mass stars (see, e.g., Brun *et al.* 2015 and Augustson *et al.* 2019). These scaling laws are listed in Table 1. From this dynamo-like magnetic field (B_{dyn}), we also estimate a large-scale dipolar magnetic field at the top of the convective envelope:

$$B_{\text{dip}} = \gamma(r/R)^3 B_{\text{dyn}}, \tag{3.1}$$

where γ can be understood as the ratio of the large-scale to small-scale magnetic fields, or as the fraction of the total energy stored in the dipolar component of the magnetic field (see Astoul *et al.* 2019 for more details). Unless otherwise stated, B_{dyn} is computed at the radial interface (r) between the radiative and the convective zones, which is the expected location for the development of a large-scale dynamo (Brun & Browning 2017).

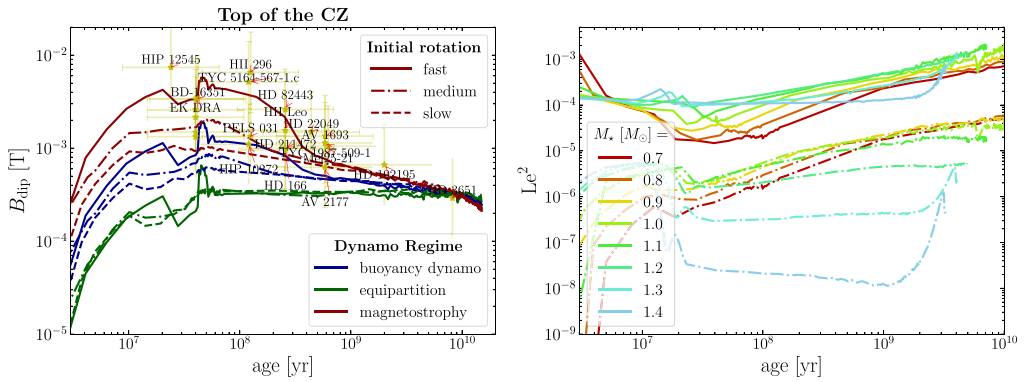


Figure 1. *Left:* surface dipolar magnetic field versus time for a $0.9M_\odot$ star. The curves are computed with a grid of STAREVOL models, for different scaling laws and initial stellar rotation rates (see legend). The symbols \star depict the mean dipolar magnetic field observed at the surface of $0.9M_\odot$ stars (See *et al.* 2019). *Right:* evolution of the Lehnert number squared over time, at the base (*solid curves*) and the top (*dashed dotted curves*) of the convective envelope for various low-mass stars, using the magnetostrophic dynamo regime and the median initial rotation.

In the left panel of Fig. 1, we display the time evolution of the surface dipolar magnetic field (B_{dip} , Eq. 3.1) for a $0.9M_\odot$ star and for the three scaling laws in Table 1. A few observational data are overplotted for comparison (See *et al.* 2019). To compute B_{dip} , parameters like the convective turnover time and velocity were obtained from grids of models computed with the 1D stellar evolution code STAREVOL (Amard *et al.* 2019). These grids were calculated for low-mass stars between $0.7M_\odot$ and $1.4M_\odot$, from the early pre-main sequence until the end of the main sequence. Three initial rotation rates (fast, median and slow) have been chosen (see Amard *et al.* 2019 for more details). We see that the observed and estimated surface dipolar magnetic fields are in good agreement when using the magnetostrophic regime, in particular when assuming fast initial rotation. Note that the large-scale to small-scale ratio γ has been kept constant in this analysis. The right panel in Fig. 1 shows the Lehnert number squared against time for stars of various masses for the magnetostrophic regime and median initial rotation. The panel reveals that Le^2 is higher at the base than at the top of the convective zone, similar to what can be expected from the magnetic field amplitude inside the convective envelope of a low-mass star. Moreover, Le^2 increases overall with time and with mass (except for the most massive stars).

4. Magnetic tidal forcing in observed star-planet systems

The tidal frequency σ_t in Eq. (2.2) depends on the orbital frequency of the planet Ω_o and the spin frequency Ω of the star. When the orbit is quasi-circular and coplanar, the quadrupolar component of the tidal potential dominates (Ogilvie 2014) and the tidal frequency writes $\sigma_t = 2(\Omega_o - \Omega)$. In the left panel of Fig. 2, the ratio of magnetic and hydrodynamical effective forcings has been calculated for observed quasi-circular and coplanar star-planet systems in the main-sequence. The masses, radius, age, and the orbital and/or spin frequencies of the planets and their host star have been extracted from the [Extrasolar Planets Encyclopaedia](#). For each system, Le^2 is calculated via the STAREVOL grid models. The ratio $f_{\text{mag}}/f_{\text{hydro}}$ is higher at the base of the convective zone than near the surface, and grows with stellar mass, in line with the trends identified in the previous section. We emphasize that the forcing ratio is far from unity for all considered star-planet systems, meaning that the contribution of the Lorentz force to the effective tidal forcing is weak.

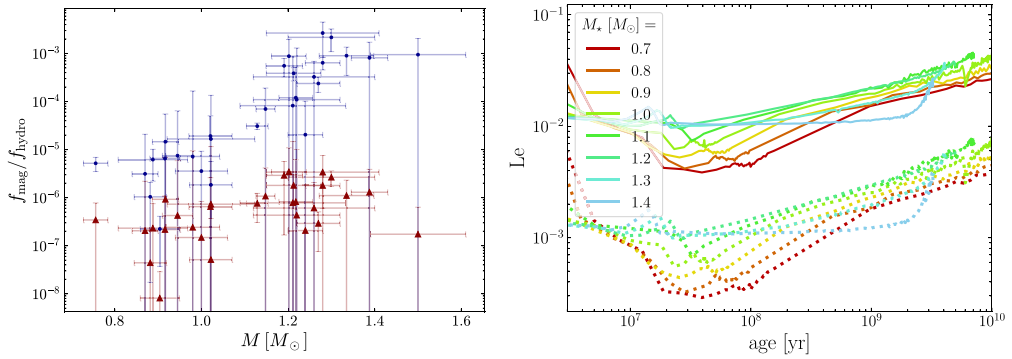


Figure 2. *Left:* ratio of the Lorentz and Coriolis tidal forcing against the mass of the star for various observed short-period exoplanetary systems. The ratio $f_{\text{mag}}/f_{\text{hydro}}$ (Eq. 2.2) is estimated at the base (*blue bullet*) and the top (*red triangle*) of the convective zone, and the magnetostrophic regime is used to calculate B_{dyn} . *Right:* Lehnert number at the base of the convective zone (*solid lines*) versus age for different masses of the star. The typical Lehnert number above which Ohmic dissipation of magneto-inertial waves dominates over viscous dissipation (Lin & Ogilvie 2018) is shown by dotted curves using a turbulent magnetic Ekman number.

5. Dissipation of kinetic and magnetic energies

In the previous section, we have highlighted the negligible impact of the star’s magnetic field in the effective tidal forcing. However, the star’s magnetic field may still play an important role in the propagation and dissipation of magneto-inertial waves. In that regard, Lin & Ogilvie (2018) have shown that the Ohmic dissipation of magneto-inertial waves becomes comparable to viscous dissipation when the Lehnert number becomes of order $\text{Em}^{2/3}$, where Em is the magnetic Ekman number. The right panel of Fig. 2 displays the time evolution of the Lehnert number at the base of the convective zone for various low-mass stars, as well as the threshold derived by Lin & Ogilvie (2018). We observe that Le is always an order of magnitude larger than this threshold. Therefore, Ohmic dissipation prevails over viscous dissipation for all low-mass stars at the base of the convective zone. At the top of the convective zone (not shown here), this statement is less clear-cut, especially for stars with $M \gtrsim 1.2M_{\odot}$, for which both Ohmic and viscous dissipations are comparable (Astoul *et al.* 2019).

6. Conclusion

We have shown that the large-scale dynamo-generated magnetic field of a star has a limited impact on the forcing of tidal waves in the convective envelope of K, G, F-type stars all along their evolution from the pre-main sequence until the terminal age main-sequence. Nevertheless, stellar magnetism is found to have a strong influence on the dissipation mechanism of dynamical tides inside the convective envelope of these stars. Our results therefore indicate that a full magneto-hydrodynamic treatment of the propagation and dissipation of tidal waves is needed to assess the impact of star-planet tidal interactions for all low-mass stars along their evolution.

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