SELF-ORDERING OF PHOTOSPHERIC MAGNETIC FIELDS

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ABSTRACT. We model the evolution of photospheric field elements by treating them as mean field structures undergoing a nonlinear self-interaction mediated by much smaller-scale, convectively driven plasma turbulence. Distributed fields can gather into discrete, strong elements of a minimum permitted scale. Also studied are the transport of flux from dissolving elements and to growing elements via weak intermediate fields and the cancellation of adjacent emements of opposite polarity.

1. THE FIELD EVOLUTION EQUATION

The photospheric Reynolds number $Re \sim 10^{12}$ and kinematic viscosity $\nu \sim 1\ cm^2/s$ indicate that the convectively driven plasma motions are turbulent down to ~ 1 m. The magnetic Reynolds number $Rm \sim 10^6$ and plasma magnetic diffusivity $\eta \sim 10^7\ cm^2/s$ indicate a strong influence of the turbulence on the magnetic field. Because there are no rigorous derivations of interactions of the magnetic field with the turbulence under these solar conditions, we must combine derivations valid under other conditions, numerical simulations, and observational phenomenology.

Acknowledging that there are unresolved theoretical questions about this procedure, we follow Steenbeck, et al. (Krause and Rädler 1980) and split the magnetic field into a vertical, large-scale, slowly-varying "mean field" $\langle\vec{B}\rangle = \hat{z}B(x,y,t)$, identified with observed photospheric fields, and a "turbulent" part, carried about by the rapid, small-scale eddies (Stenflo 1988). We assume vanishing helicity and mean velocity, and that statistical properties of the turbulence vary on mean field scales.

We adopt a magnetic "turbulent diffusivity" $\beta \sim \langle u^2\rangle/T$, where u is the turbulent velocity and T the eddy correlation time. See discussions by Parker (1979), Krause and Rädler (1980), Moffatt (1983), and especially numerical simulations by Drummond and Horgan (1986). In the photosphere $\beta \sim 10^{13}\ cm^2/s$.

A turbulent, conducting fluid behaves diamagnetically with respect to the mean field, leading to an equation, valid for $Rm \gg 1$, which

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reduces to

$$\partial B/\partial t = \vec{\nabla}\cdot[\beta^{\frac{1}{2}}\ \vec{\nabla}(\beta^{\frac{1}{2}}\ B)] = \vec{\nabla}\cdot[\beta^{\frac{1}{2}}\ \vec{\nabla}C] \qquad (1)$$

(Vainshtein and Zel'dovich 1972). See other discussions by Zel'dovich (1956), Rädler (1968), and Moffatt (1983).

Nonlinearity arises from the inhibition of turbulence by strong mean fields. This idea forms the basis of thermal plug models of sunspot cooling. For Rm<<1, Krause and Rädler (1980) show that, assuming the energy source is unaffected, most turbulent modes are suppressed by a strong mean field as $1/B^4$. Peckover and Weiss (1978) found by numerical simulation that overturning convective eddies are unaffected by weak fields, but are suppressed as $1/B^4$ in strong fields, thus reducing the turbulent energy supply. Beckers's (1976) observations of microturbulent velocities in sunspots indicate a turbulent diffusivity about 0.01 that of the photospheric value. We adopt a phenomenological diffusivity to model this behavior: $\beta(B)= \beta_o/(1+|B/B_c|^n)$. B_c is the critical field separating the kinematic and dynamical regimes of eddy suppression, which we estimate at ~100 G.

2. SOLUTIONS AND COMPARISON TO OBSERVATIONS

The functional $C[B]=B[\beta_o/(1+|B/B_c|^n)]^{\frac{1}{2}}$ acts as a potential governing the two-dimensional flow of B. When n>2, B[C] is double valued, so B can vary discontinuously between $B_1<B_m$ and $B_2>B_m$, where $B_m=B_c[2/(n-2)]^{1/n}$, while C remains continuous. When n>2 and $B>B_m$ the functional derivative $\delta C/\delta B<0$, and local maxima and minima of B and C anticoincide. Then fluctuations of B will grow. When $B<B_m$ or n<2 fluctuations are damped.

The smallest-scale fluctuations grow or decay first. A scale cut-off is imposed by requiring B to vary only over distances larger than the turbulent eddies. For numerical solutionsthe minimum scale will be associated with the grid spacing, which we identify with the granular scale ~1 Mm. We present a one-dimensional case using three point spatial derivatives, an explicit, trapezoidal time integration, and modelled random fluctuations. We choose n=4 and closed boundary conditions. An initially uniform field $B_o<B_m=1$ will remain uniform; Figure 1 shows the evolution for the maximally unstable case $B_o=1.3$. $B=B_o$ is metastable for a few hours and then breaks up on the scale of the grid as expected. Separate peaks undergoing a negative fluctuation develop local maxima of C and decay by transporting flux through the intervening weak field to the stronger peaks which, being local minima of C, grow. A stable state is reached after about 200 hours with only one strong peak $B_1>B_m$ and $B_2=B_c^2/B_1$ between; C is uniform. For $B_o=3000$ G, the initial uniform state lasts about 10 days.

The present investigation differs from earlier ones by Kraichnan (1976) and Knobloch (1978). These involve a linear diffusion equation with negative diffusivity. Such a system has no stable static solution.

We find that, under certain conditions, the field gathers itself into isolated, strong elements with weak field between. It evolves to an ordered state not in a minimum field energy configuration, thus evoking the "dissipative structures" discussed by Nicolis and Prigogine (1977) in connection with nonlinear chemical reactions. This process also suggests the sudden breakup of sunspot umbrae after the appearance of bright umbral dots (Zwaan 1987). In addition, such solutions include the growth and decay in place of apparently isolated flux elements, as observed by Simon and Wilson (1985) and by Topka, et al. (1986).

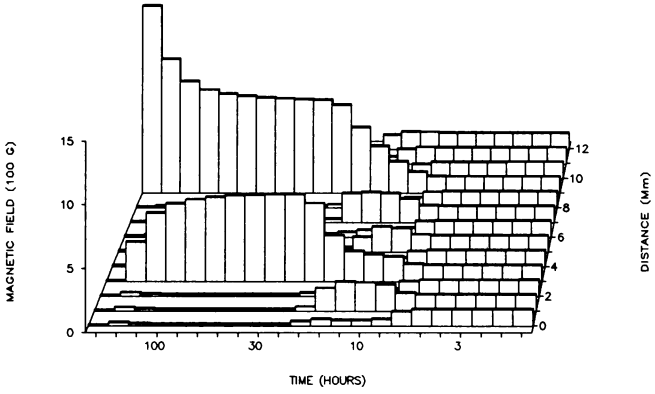

Figure 1. Magnetic field distribution versus time.

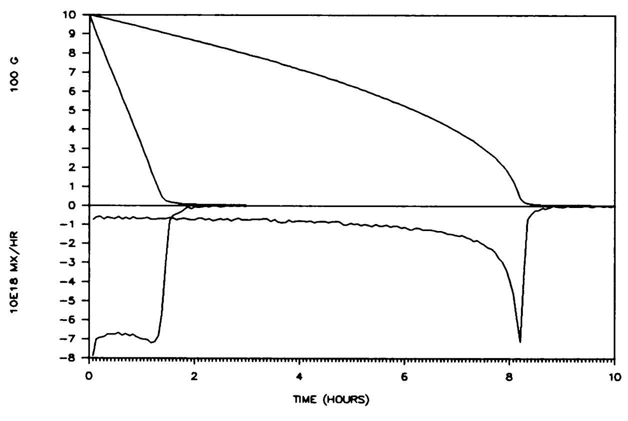

Figure 2. Peak field strength and rate of flux loss versus time for n=4 (long lived) and n=2 (short lived) cases.

Martin, et al. (1985) have observed the cancellation in active regions, over a few hours time, of adjacent but seemingly unconnected flux elements of opposite polarity. We have carried out a two-dimensional calculation for such elements of initial strength 1000 G and one intermediate grid point held at B=0. Figure 2 illustrates the element peak field value and the flux loss rate in one polarity as a function of

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time for n=4 and n=2. The n=4 case extends the lifetime of the peaks to ~8 hr compared to about 1.5 hr for the diffusive n=2 case. For purely atomic diffusion this would take ~10 years. The flux loss rate is a few times 10^{18} Mx/hr, in good agreement with observations. A single flux element with surrounding field held to zero will likewise survive for several hours; with surrounding field $B_c{}^2/B$ it will be stable.

3. CONCLUSIONS

Our treatment is far from rigorous. Further, it ignores much of the physics affecting photospheric fields, such as the balancing of magnetic and gas pressures which must limit the intensity of flux elements, and the large-scale velocity flows which can produce regions of $B>B_m$ leading to fragmentation. All the same, our solutions resemble some field phenomena which have proven difficult to explain satisfactorily. We believe we have made a plausible case that a field-turbulence interaction such as this is relevant to the behavior of photospheric fields. Such a viewpoint provides a different way to think about magnetic flux elements: rather than equilibrium structures representing a balance among various forces, they more resemble shock waves, maintained by a self-interaction of the mean field which is mediated by the small-scale turbulence.

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