J. Austral. Math. Soc. (*Series A*) **48** (1990), 57–75

A NOTE ON RIESZ SETS AND LACUNARY SETS

R. G. M. BRUMMELHUIS

(Received 22 March 1988)

Communicated by J. F. Price

Abstract

W. Rudin has proved that the union of the Riesz set $\mathbf{N} \subseteq \mathbf{R}$ with a $\Lambda(1)$-subset of $\mathbf{Z}$ is again a Riesz set. In this note we generalize his result to compact groups whose center contains a circle group, thereby extending an earlier F. and M. Riesz theorem for such groups by the author. We also investigate the possibility of constructing $\Lambda(p)$-sets for these groups, departing from $\Lambda(p)$-sets for the circle group in the center.

1980 *Mathematics subject classification* (*Amer. Math. Soc.*) (1985 *Revision*): primary 43 A 05, 43 A 30, 43 A 46; secondary 22 E 30, 43 A 77.

1. Introduction

1.1 *Notations.* Throughout this note, let K be a metrizable compact group. Let $M(K)$ be the space of finite complex Borel measures on K and let dk denote the Haar measure on K, normalized to total mass 1. Also $L^p(K) = L^p(K, dk)$ and $\|f\|_p$ are defined as usual ($0 < p < \infty$).

Let $\hat{K}$, the dual of K, be a maximal set of pairwise inequivalent irreducible representations of K. For μ in $M(K)$ define the Fourier transform $\hat{\mu}$ of μ by

$$\hat{\mu}(\tau) := \int_K \tau(k^{-1})\, d\mu(k), \qquad \tau \in \hat{K}.$$

Let $\operatorname{spec}\mu := \operatorname{supp}\hat{\mu} = \{\tau \in \hat{K} : \hat{\mu}(\tau) \neq 0\}$.

The author was supported by the Netherlands Organization for the Advancement of Scientific Research (NWO).

https://doi.org/10.1017/S1446788700035187 Published online by Cambridge University Press

1.2 *Riesz sets.* A subset Δ of $\hat{K}$ is called a *Riesz set* if for any μ in $M(K)$, $\operatorname{spec}\mu \subset \Delta$ implies that μ is absolutely continuous with respect to dk: $\mu \ll dk$.

In [1] the author proved an F. and M. Riesz type theorem for compact groups K whose center contains an isomorphic copy of the circle group $\mathbf{T} = \{e^{i\theta}: \theta \in (-\pi, \pi]\}$. Let K be such a group and fix an injective homomorphism $\mathbf{T} \to Z(K)$; $e^{i\theta}$ will denote an element of $\mathbf{T}$ as well as of $Z(K)$. By Schur's lemma there exists for each τ in $\hat{K}$ a unique $n(\tau) \in \mathbb{Z}$ such that

$$\tau(e^{i\theta}) = e^{in(\tau)\theta} Id_{H(\tau)}, \quad \text{for all } e^{i\theta} \in \mathbf{T}. \tag{1.1}$$

Note that the map $\tau \to n(\tau)$ depends on the choice of the identification $\mathbf{T} \hookrightarrow Z(K)$.

One can prove the following extension of the F. and M. Riesz theorem (cf. [1, Theorem 3.2]):

1.3 THEOREM. *Let* $\Delta \subseteq \hat{K}$ *be such that* (i) *for all* $m \in \mathbb{Z}$ *the set* $\{\tau \in \Delta: n(\tau) = m\}$ *is finite and* (ii) *the set* $\{n(\tau): \tau \in \Delta\} \subseteq \mathbb{Z}$ *is bounded from above or from below. Then* Δ *is a Riesz subset of* $\hat{K}$

1.4 $\Lambda(p)$*-sets.* In [11] W. Rudin introduced the notion of a $\Lambda(p)$-subset of $\mathbb{Z}$. Of interest to us here is the following theorem from [11] which we will generalize in Section 2: the union of a $\Lambda(1)$-set of integers and the Riesz set $\mathbb{N}$ is again a Riesz subset of $\mathbb{Z} = \hat{\mathbf{T}}$.

The general theory of $\Lambda(p)$-sets was extended to arbitrary compact groups in Hewitt and Ross [6, Chapter IX, Section 37]. Here we will need little more than the definition.

Let $T(K)$ denote the space of trigonometric polynomials on K: $T(K) = \{f \in L^1(K): \operatorname{spec} f \text{ is a finite set}\}$. If $E \subseteq \hat{K}$ let $T_E(K) = \{f \in T(K): \operatorname{spec} f \subseteq E\}$, the space of E-spectral polynomials.

DEFINITION. Let $0 < p < \infty$. A subset E of $\hat{K}$ is called a $\Lambda(p)$*-set* if for some $q < p$ there exists a constant C such that

$$\|f\|_p \le C\|f\|_q, \qquad \text{for all } f \in T_E(K). \tag{1.2}$$

We call E *central* $\Lambda(p)$ if (1.2) holds for all f in $T_E(K)$ which are central, that is, for which $f(yxy^{-1}) = f(x)$ for all $x, y \in K$.

One can prove that "for some $q < p$" may be replaced by "for all $q < p$" (cf. [11], [6]).

For abelian groups $\Lambda(p)$-sets exist in great abundance, but for nonabelian groups the situation is rather disappointing: it is known that simple Lie groups do not possess infinite central $\Lambda(4)$ sets (Cecchini [2]), and Price [8] proved that $\widehat{SU}(2)$ does not contain an infinite $\Lambda(p)$ set for any $p > 0$. This result was extended to all $SU(n)$ in Rider [10]. On the other hand, the dual of each compact connected Lie group contains an infinite central $\Lambda(2)$-set (cf. Rider [9, Theorem 6]).

In Section 3 we try to construct $\Lambda(p)$-sets for groups K whose center contains the circle group. Roughly speaking there are, for nonabelian K, two obstructions to the existence of infinite $\Lambda(p)$-subsets of $\hat{K}$. The first, at least for $p \geq 4$, is the fact that the tensor product of two irreducible representations is not again itself irreducible but decomposes as a sum of irreducible ones, the number of which may tend to infinity (cf. [6, Example (37.21)], which in modified form also occurs in 3.4(a) below).

The second obstruction is the fact that there may not exist infinite subsets E of $\hat{K}$ for which the degrees d_τ, $\tau \in E$, remain bounded (cf. [8], [10] and Example 3.4(b) below).

This last obstacle can sometimes be circumvented by restricting oneself to central $\Lambda(p)$-sets. In case of Lie groups one then can use Weyl's integration and character formulae to show that the K's we are interested in do posses a large number of central $\Lambda(1)$-sets (cf. Proposition 3.3 below). This idea was taken from Dooley [4].

2. Riesz sets and $\Lambda(1)$ sets

In this section we prove the following result, which generalizes Rudin [11, Theorem 5.7], the latter being the case $K = \mathbf{T}$ of 2.1.

2.1 THEOREM. *Let K be a compact group whose center $Z(K)$ contains a copy of the circle group. Let Δ be a Riesz subset of $\hat{K}$ such that $\{n(\tau): \tau \in \Delta\}$ is bounded from above (where $n(\tau)$ is defined by (1.1)), $n(\tau) < N$ for $\tau \in \Delta$, say ($N \in \mathbf{N}$). If $E \subseteq \{\tau \in \hat{K}: n(\tau) \geq N\}$ is a $\Lambda(1)$-subset of $\hat{K}$, then $E \cup \Delta$ is a Riesz subset of $\hat{K}$.*

For the proof we need an auxiliary result. If f is a function on K and $k \in K$, define the "slice function" f_k on $\mathbf{T}$ by $f_k(e^{i\theta}) := f(e^{i\theta}k)$, $e^{i\theta} \in \mathbf{T} \to Z(K)$. Note that for f in $T(K)$, $f_k \in T(\mathbf{T})$ and

$$f_k(e^{i\theta}) = \sum_{m \in \mathbf{Z}} \Pi_m f(k) e^{\mathrm{i}m\theta}, \tag{2.1}$$

where the projections Π_m are defined by

$$\Pi_m f(k) = \sum_{\tau \in \hat{K}, n(\tau) = m} d_\tau \operatorname{Tr}[\hat{f}(\tau)\tau(k)]. \tag{2.2}$$

Define the projection P_N on $T(K)$ by $P_N[f] = \sum_{m > N} \Pi_m f$.

2.2 LEMMA. *For all p, $0 < p < 1$, there exists a constant $C = C(p)$ such that for all f in $T(K)$, $\|P_N f\|_p \leq C \cdot \|f\|_1$.*

PROOF. Let $\mathbb{S}$ denote the Cauchy-Szegő projection of $L^2(\mathbf{T})$ onto the Hardy space

$$H^2(\mathbf{T}) \colon \mathbb{S}(F)(e^{i\theta}) = \sum_{n \geq 0} \hat{F}(n) e^{in\theta}, \qquad F \in T(\mathbf{T}).$$

Note that $P_N f(e^{i\theta} k) = e^{iN\theta} \mathbb{S}(e^{-iN\theta} f_k(e^{i\theta}))$. Let $0 < p < 1$. By a theorem of Kolmogorov (cf., for example, Koosis [7, pages 137–138]) there exists a C such that for all F in $T(\mathbf{T})$: $\|\mathbb{S}(F)\|_p \leq C\|F\|_1$, the norms being taken in $L^p(\mathbf{T})$ and $L^1(\mathbf{T})$, respectively. Apply this inequality to $F(e^{i\theta}) = e^{-iN\theta} f_k(e^{i\theta})$ and integrate over K. Then by Jensen's inequality, since $1/p > 1$,

$$\begin{aligned} \|P_N f\|_p &= \left(\int_K \int_{\mathbf{T}} |P_N f(e^{i\theta} k)|^p \, dk \, d\theta/2\pi \right)^{1/p} \\ &\leq \int_K \left(\int_{\mathbf{T}} |P_N f(e^{i\theta} k)|^p \, d\theta/2\pi \right)^{1/p} dk \\ &\leq C\|f\|_1. \end{aligned}$$

PROOF OF 2.1. Let $\mu \in M(K)$, $\operatorname{spec} \mu \subseteq E \cup \Delta$. Let $\{F_n\}_{n \in \mathbf{N}}$ be an approximate identity, consisting of trigonometric polynomials. Let $N \in \mathbf{N}$ be as in the statement of 2.1 and let $P = P_N$. Then $P[F_n * \mu] \in T_E(K)$ for all n and if $0 < p < 1$ then by 2.2, $\|P[F_n * \mu]\|_p \leq C\|F_n * \mu\|_1 \leq C\|\mu\|$ for all n. Since E is a set of type $\Lambda(1)$ the sequence $\{P[F_n * \mu] \colon n \in \mathbf{N}\}$ is norm bounded in $M(K)$. Hence there exists a subsequence converging weak-$*$ to a measure ν. Obviously, $\operatorname{spec} \nu \subseteq E$ and $\hat{\nu}(\tau) = \hat{\mu}(\tau)$ for τ in E, since $\hat{F}_n(\tau) \to \mathrm{Id}_{H(\tau)}$ as $n \to \infty$. Hence $\operatorname{spec}(\mu - \nu) \subseteq \Delta$ and therefore $\mu - \nu \ll dk$. On the other hand, each $\Lambda(1)$-subset of $\hat{K}$ is a Riesz subset (this follows from [1, Theorem 2.7]). Hence $\nu \ll dk$ and so $\mu \ll dk$.

As we will see in the next section, *central* $\Lambda(p)$-sets are more abundant than ordinary ones. It seems appropriate therefore to formulate a "central" version of 2.1. Call a subset Δ of $\hat{K}$ *central Riesz* if $\operatorname{spec} \mu \subseteq \Delta$ implies $\mu \ll dk$ for all *central* measures μ.

2.3 THEOREM. *Suppose that, in Theorem 2.1, $\Delta \subseteq \{\tau \in \hat{K} \colon n(\tau) < N\}$ is central Riesz and that $E \subseteq \{\tau \in \hat{K} \colon n(\tau) \geq N\}$ is a central $\Lambda(1)$-set which is also central Riesz. Then $E \cup \Delta$ is central Riesz.*

The proof of this theorem is the same as that of 2.1, provided one uses an approximate identity consisting of central trigonometric polynomials, which is possible.

REMARK. It is not clear whether "central $\Lambda(1)$" implies "central Riesz". The central $\Lambda(p)$ sets of Proposition 3.3 below will automatically be Riesz sets by [1, Remark 3.4].

3. Examples of (central) $\Lambda(p)$-sets

Let K again be a compact group whose center contains a circle group. One might hope that the generous supply of $\Lambda(p)$-sets in the dual of $\mathbf{T}$ will give rise to some interesting $\Lambda(p)$-sets in $\hat{K}$.

3.1 PROPOSITION. *Let $E \subseteq \hat{K}$ be such that for some $p \geq 2$:*
(i) *$n(E) := \{n(\tau): \tau \in E\}$ is a $\Lambda(p)$-subset of $\mathbf{Z} = \hat{\mathbf{T}}$;*
(ii) *the sets $E(m) := \{\tau \in E: n(\tau) = m\}$ $(m \in \mathbf{Z})$ are uniformly $\Lambda(p)$ (that is,* (1.2) *holds with the same constant C for all $E(m)$).*
Then E is a $\Lambda(p)$-subset of $\hat{K}$.

REMARK. Condition (ii) is obviously necessary for E to be a $\Lambda(p)$-set; condition (i) is not (cf. 3.4(c) below).

PROOF. We have to treat the cases $p > 2$ and $p = 2$ separately. First suppose that $p > 2$. Let $f \in T_E(K)$. By (2.1) and (2.2) the slice function f_k is in $T_{n(E)}(\mathbf{T})$ for all k in K. Since $n(E)$ is $\Lambda(p)$ and $p > 2$ there exists a constant $C = C(E,p)$ such that

$$\int_{\mathbf{T}} |f(e^{i\theta}k)|^p \, d\theta/2\pi \leq C \left(\sum_m |\Pi_m f(k)|^2 \right)^{p/2}.$$

Integration over K yields

$$\|f\|_p^2 \leq C \left\{ \int_K \left(\sum_m |\Pi_m f(k)|^2 \right)^{p/2} dk \right\}^{2/p} \leq C \sum_m \left(\int_K |\Pi_m f(k)|^p \, dk \right)^{2/p}, \tag{3.1}$$

by Minkowski's inequality for $p/2 > 1$. For each m, $\Pi_m f \in T_{E(m)}(K)$ and since the $E(m)$'s are uniformly $\Lambda(p)$, (3.1) implies that $\|f\|_p^2 \leq C\|f\|_2^2$ which proves that E is $\Lambda(p)$.

We now turn to the case $p = 2$. Arguing as above, we see that since $n(E)$ is $\Lambda(2)$ there exist a $q < 2$ and a constant C such that for all $f \in T_E(K)$

$$\left(\int_K \left(\sum_m |\Pi_m f(k)|^2 \right)^{q/2} dk \right)^{2/q} \leq C\|f\|_q^2. \tag{3.2}$$

https://doi.org/10.1017/S1446788700035187 Published online by Cambridge University Press

By Minkowski's inequality for $q/2 < 1$ (cf. Hardy, Littlewood and Pólya [5, Theorem 198]) the left hand side of (3.2) is larger than or equal to

$$\sum_m \left(\int_K |\Pi_m f(k)|^q \, dk \right)^{2/q} . \tag{3.3}$$

Since the $E(m)$'s are uniformly $\Lambda(2)$, the expression (3.3) is larger or equal to a constant times $\sum_m \|\Pi_m f\|_2^2 = \|f\|_2^2$.

3.2 Corollary. *Let $E \subseteq \hat{K}$ be such that for some $p \geq 2$:*

(i) *$n(E) := \{n(\tau) : \tau \in E\} \subseteq \mathbb{Z}$ is a $\Lambda(p)$ subset of $\mathbb{Z}$;*

(ii) *there exists an $M \in \mathbb{N}$, such that for all $m \in \mathbb{Z}$, $\#\{\tau \in E : n(\tau) = m\} \leq M$;*

(iii) *there exists an $N \in \mathbb{N}$, such that for all $\tau \in E$, $d_\tau \leq N$.*

Then E is a $\Lambda(p)$-subset of $\hat{K}$.

Proof. Let F be a finite subset of $\hat{K}$ and let V be the subspace of $T(K)$ spanned by the matrix elements of the representations in F. Then it is not difficult to show that for all $p \geq 1$ there exist constants C_1, C_2, depending only on p, $\#F$ and $\max\{d_\tau : \tau \in F\}$ such that $C_1\|f\|_p \leq \|f\|_2 \leq C_2\|f\|_p$, for all $f \in V$. From this it follows immediately that under conditions (ii) and (iii) on E, the $E(m)$'s are uniformly $\Lambda(p)$ for all $p > 1$.

If K is a compact connected Lie group, as we will suppose from now on, and if we restrict ourselves to *central* $\Lambda(p)$ sets, then condition (iii) is superfluous. The argument which we use below is due to Dooley [4], where it is used in another context. We will need Weyl's integration formula, (cf. Wallach [12]).

Let $\mathbf{T}^r$ be a maximal torus in K ($r = \operatorname{rank} K$). Then for $f \in C(K)$, f central,

$$w \int_K f(k) \, dk = \int_{\mathbf{T}^r} f(t) |q(t)|^2 \, dt, \tag{3.4}$$

where $q(t)$ is a certain trigonometric polynomial on $\mathbf{T}^r$ and w is the order of the Weyl group. Clerc [3] has proved that

$$\int_{\mathbf{T}^r} |q(t)|^{-\alpha} \, dt < \infty \quad \text{if } \alpha < \varepsilon_K := 2r/(\dim K - r). \tag{3.5}$$

We will also need the following observation, which is a direct consequence of the Weyl character formula (cf. Dooley [4]):

$$\|q\chi_\tau\|_{L^\infty(\mathbf{T}^r)} \leq w, \qquad \text{for all } \tau \in \hat{K}. \tag{3.6}$$

For $K = U(n)$ or $SU(n)$, (3.5) and (3.6) were already noted and used by Rider [9].

3.3 PROPOSITION. *Let K be a compact connected Lie group, $\mathbf{T} \subseteq Z(K)$ (that is, K has non discrete center). Let $2 < p < 2 + \varepsilon_K$ and let $E \subseteq \hat{K}$ be such that*

(i) *$n(E) = \{n(\tau): \tau \in E\}$ is a $\Lambda(p)$-subset of $\mathbf{Z}$,*

(ii) *there exists an $M \in \mathbf{Z}$, such that for all $k \in \mathbf{Z}$, $\#\{\tau \in E: n(\tau) = k\} \leq M$. Then E is a central $\Lambda(p)$-subset of $\hat{K}$.*

PROOF. Let χ_τ denote the character of $\tau \in \hat{K}$. Let $f \in T_E(K)$ be central, $f(k) = \sum c_\tau \chi_\tau(k)$ with $c_\tau = \int f(k)\chi_\tau(k^{-1})\,dk$. Arguing as in the proof of 3.1 we see that

$$\|f\|_p^p \leq C \int_K \left(\sum_m \left| \sum_{n(\tau)=m} c_\tau \chi_\tau(k) \right|^2 \right)^{p/2} dk.$$

Now because of 3.3(ii),

$$\left| \sum_{n(\tau)=m} c_\tau \chi_\tau(k) \right|^2 \leq M \sum_{n(\tau)=m} |c_\tau \chi_\tau(k)|^2.$$

Hence by (3.4), (3.5), (3.6) and the fact that $p - 2 < \varepsilon_K$,

$$\begin{aligned} w\|f\|_p^p &\leq C \int_{\mathbf{T}^r} \left(\sum_\tau |c_\tau \chi_\tau(t)|^2 \right)^{p/2} |q(t)|^2\,dt \\ &\leq C \left(\sum_\tau |c_\tau|^2 \cdot \|\chi_\tau(t) q(t)\|_\infty^2 \right)^{p/2} \int_{\mathbf{T}^r} |q(t)|^{2-p}\,dt \\ &\leq C \left(\sum_\tau |c_\tau|^2 \right)^{p/2} = C\|f\|_2^p. \end{aligned}$$

3.4 EXAMPLES. (a) The following modification of Example (37.21)(b) from Hewitt and Ross [6] will show that condition (ii) of 3.2 and 3.3 is necessary. We have to recall some facts from the representation theory of $SU(2)$.

For each $m \in \{0, 1/2, 1, 3/2, \dots\}$ there exists an irreducible representation τ_m of $SU(2)$; τ_m can be realized, for example, on the space of polynomials in two variables which are homogeneous of degree $2m+1$ (cf. Hewitt and Ross [6, Section (29.13)ff] for details). The τ_m's are pairwise inequivalent and each irreducible representation of $SU(2)$ is equivalent to some τ_m.

Now take $K = SU(2) \times \mathbf{T}$. Then $\hat{K} = \{\tau_{m,n}: 2m \in \mathbf{N},\ n \in \mathbf{Z}\}$, where $\tau_{m,n}(g, e^{i\theta}) = e^{in\theta}\tau_m(g)$, $g \in SU(2)$, $e^{i\theta} \in \mathbf{T}$. Note that $Z(K) = \{1\} \times \mathbf{T}$ and that $n(\tau_{m,n}) = n$. Let $\chi_{m,n}$ denote the character of $\tau_{m,n}$. By the decomposition of the tensor product: $\tau_m \otimes \tau_m = \bigoplus\{\tau_l \colon 0 \le l \le 2m\}$ (cf. [6, (29.26.i)]),

$$(\chi_{m,n})^2 = \sum_{l=0}^{2m} \chi_{l,2n}.$$

Hence $\|\chi_{m,n}\|_4^4 = 2m + 1$ and if $E \subseteq \hat{K}$ is such that $(\#\{\tau_{m,n} \in E \colon n = k\})_{k \in \mathbf{Z}}$ is unbounded, E can not be central $\Lambda(4)$ since $\|\chi_{m,n}\|_2 = 1$ for all m, n.

As it stands, this is not a counterexample to 3.3 minus (ii), since $\varepsilon_K = 2$. To correct this, take $K = \mathbf{T}^2 \times SU(2)$ instead.

(b) That condition (iii) of 3.2 is necessary follows from the main result of Price [8]: if $(\tau_m)_{11}$ denotes the $(1,1)$-matrix element of τ_m w.r.t. the standard basis of $H(\tau_m)$ as in [6], loc. cit., then for all $p > 0$, $\|(\tau_m)_{11}\|_p \sim m^{-1/p}$ as $m \to \infty$.

To make a counterexample to 3.2 minus condition (iii), again take $K = SU(2) \times \mathbf{T}$. If $\{n(1), n(2), \dots\}$ is an infinite $\Lambda(p)$-subset of $\mathbf{T}$ ($p \ge 2$), let $E = \{\tau_{m,n(2m+1)} \colon 2m \in \mathbf{N}\}$. Then E satisfies 3.2(i) and (ii) but fails to be a $\Lambda(p)$-subset of $\hat{K}$.

(c) Finally, 3.1(i), 3.2(i) and 3.3(i) are in general *not* necessary: take $K = \mathbf{T}^2$ and identify $\hat{K}$ with $\mathbf{Z}^2$ in the usual way. Let $\{n(m) \colon m \in \mathbf{Z}\}$ be an (infinite) $\Lambda(p)$-subset of $\mathbf{Z}$ for some $p \ge 2$. If we use the embedding $e^{i\theta} \to (e^{i\theta}, 1)$ of $\mathbf{T}$ into $\mathbf{T}^2$ the map $\tau \to n(\tau) \colon \hat{K} = \mathbf{Z}^2 \to \mathbf{Z}$ is the projection on the first coordinate. Hence the set $E := \{(n(m), m) \colon m \in \mathbf{Z}\} \subseteq \mathbf{Z}^2$ is $\Lambda(p)$ by 3.2. But if we use the embedding $e^{i\theta} \to (1, e^{i\theta})$, the associated map $\tau \to n(\tau)$ is the projection on the second coordinate and $n(E) = \mathbf{Z}$ then, which obviously is not $\Lambda(p)$.

References

[1] R. G. M. Brummelhuis, 'An F. and M. Riesz theorem for bounded symmetric domains', *Ann. Inst. Fourier* **37** (2) (1987), 139–150.

[2] C. Cecchini, 'Lacunary Fourier series on compact Lie groups', *J. Funct. Anal.* **11** (1972), 191–203.

[3] J. L. Clerc, 'Sommes de Riesz et multiplicateurs sur un groupe de Lie compact', *Ann. Inst. Fourier* **24** (1) (1974), 149–172.

[4] A. H. Dooley, 'Random series for central functions on compact Lie groups', *Illinois J. Math.* **24** (4), 545–553.

[5] G. H. Hardy, J. E. Littlewood and G. Pólya, *Inequalities*, (Cambridge University Press, 1934).

[6] E. Hewitt and K. A. Ross, *Abstract harmonic analysis*, vol. 2 (Springer, New York, 1972).

[7] P. Koosis, *Introduction to H_p spaces*, (London Mathematical Society Lecture Note Series 40, Cambridge University Press, 1980).

[8] J. F. Price, 'Non a sono insiemi infiniti di tipo $\Lambda(p)$ per $SU(2)$', *Boll. Un. Mat. Ital.* **4** (4) (1971), 879–881.

[9] D. Rider, 'Norms of characters and central $\Lambda(p)$ sets for $U(n)$', (Conference on harmonic analysis, College Park, Maryland, 1971, SLNM 266, 287–294).
[10] D. Rider, '$SU(n)$ has no infinite local Λ_p sets', *Boll. Un. Mat. Ital.* **12** (4) (1975), 155–160.
[11] W. Rudin, 'Trigonometric series with gaps', *J. Math. Mech.* **9** (2) (1960), 203–227.
[12] N. R. Wallach, *Harmonic analysis on homogeneous spaces*, (Marcel Dekker, New York, 1973).

Department of Mathematics
University of Amsterdam
Plantage Muidergracht 24
1018 TV Amsterdam
The Netherlands