An Introduction to Differential Geometry, by T. J. Willmore. Oxford at the Clarendon Press, 1959. 317 pages. 35 shillings.

It is a matter of record that American Universities have been steadily dropping geometrical disciplines from their undergraduate curricula or employing such subjects as examples illustrative of either algebraic or analytic theorems. The reason for this is perhaps found in the prevalent opinion that the only significant geometrical results are essentially algebraic or analytic anyway and that the outstanding problems of geometry are of the "push-back-the-decimal-point" type. Most of the standard text books on geometrical subjects were written at least twenty-five years ago and tend to lend credence to these opinions. To the geometer, then, it is heartening to observe the recent appearance of a number of texts whose contents and style of presentation should counteract the above criticisms. In the opinion of the reviewer, Willmore's book is such a text.

The book was written for senior honours undergraduates or post graduate students. An agreeable blending of classical and modern techniques is used in the development of each topic. Many computational results which are commonly found in the body of a text are here relegated to the exercises which appear at the end of each chapter. Several comparatively lengthy proofs are included in appendices so that the geometrical train of thought may not be interrupted. Unsolved problems are frequently mentioned as well as references to other books for more extended coverage of specific topics. The reviewer feels that a student should be brought to the threshold of current research by a diligent perusal of this book.

The book is divided into two parts, each of which comprises four chapters. The first part deals with three dimensional Euclidean spaces. In the first chapter of this part we find a more than usually careful treatment of curves and their arc-lengths leading up to the Serret-Frenet formulae and the fundamental existence theorem. The second chapter covers the local intrinsic properties of a surface with the same care as was used in the previous chapter. The standard topics of surface theory occur here as well as various results concerning correspondences such as isometries, conformal and geodesic maps. No attempt is made to conceal the difficulties inherent in the precise approach to geodesics. Examples are quoted to indicate the types of circumstances that may arise and certain unproved theorems concerning surface neighbourhoods are mentioned. Chapter three contains local non-intrinsic properties of surfaces and is based on a discussion of the second fundamental form. The equations of Gauss, Weingarten and

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Mainardi-Codazzi are developed and the fundamental existence theorem for surfaces proved. Chapter four is devoted to the geometry of surfaces in the large. This rather surprising inclusion is remarkably well handled although it was obviously necessary to practice a good deal of restriction in the choice of topics and to state several results without proof. Compact and complete surfaces are defined and Hilbert's theorem on the non-existence of a complete analytic surface of constant negative curvature is proved. The problem of the "second" variation in the calculus of variation is used to discuss conjugate points of geodesics. For later purposes the intrinsic definition of a manifold is given as well as that of a two dimensional Riemannian manifold. Triangulation, the genus of a surface and its connection with the Euler characteristic and problems of embedding are mentioned briefly with appropriate references.

Part two is devoted to the geometry of n-dimensional spaces, beginning with a chapter on tensor algebra. Here a nice balance is maintained between purely algebraic considerations and component representations. The chapter ends with a discussion of Grassmann algebra and its applications. In the second chapter of this part we encounter general manifolds, intrinsically defined. This is followed by a discussion of the possible methods of defining tangent vectors ending with the linear mapping approach. The work of the preceding chapter is then applied to obtain the properties of tensor fields. In the sections on affine connections and covariant differentiation which follow, we find, as well as the usual work, brief references to fibre-bundles and possible extensions of the concept of connection. The third chapter deals with Riemannian geometries in which the metric may not be positive definite. The Christoffel symbols occur as the unique symmetric metric connection parameters of such a space. A discussion of curvature, geodesics and special spaces now appears. Several sections are then devoted to the consideration of parallel distributions (fields of r-dimensional "planes") and recurrent tensors. The latter sections of this chapter are devoted to a brief exposition of É. Cartan's approach to Riemannian geometry and a statement of certain results (such as Hodge's theorem) of global geometry. The last short chapter contains a revision of the surface theory of E_3 in tensorial form.

Although the overall impression left by this book is certainly favourable, there are a number of criticisms which come to mind. In general the author's style is such that details are often dealt with in a rather cavalier fashion. This allows him to cover a good deal of ground but it is sometimes trying for the reader. Singular points of curves in E_3 are not considered even though these occur in a natural way when curves are projected onto the plane determined by its normal and binormal. The treatment of the fundamental existence theorem for surfaces in E_3 and the derivation of the Weingarten equations etc.

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could perhaps have been made less repetitive. The proof that various characterizations of a complete surface are equivalent (chapter IV, §6) is somewhat vague and in one spot (top of p. 135) definitely misleading. There are too few exercises appended to the chapters of the second part and those that do appear cover only a small part of the material in the text. Several of these contain misprints or incomplete formulations. The discussion of exterior differentiation is extremely short, considering its importance for later topics. In view of the calculated conciseness of most of the presentations it could possibly be argued that too much space is taken up with a discussion of parallel fields of planes and distributions (almost as much as the whole final chapter).

In conclusion, the reviewer feels that this book deserves to be expanded in certain parts and that minor details should be clarified but that it is the best book of its kind available to English readers.

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A Modern View of Geometry, by L. M. Blumenthal. Freeman, San Francisco, 1961. xii + 191 pages. $2.25.

Like B. Segre, the author takes the word modern (as applied to geometry) to mean "over a field that is not necessarily commutative." The first six of the eight chapters constitute a carefully prepared account of the rigorous introduction of coordinates in the manner developed by Marshall Hall, Skornyakov, and Bruck. The historical introduction includes Gauss's remark, "I consider the young geometer Bolyai a genius of the first rank," and Hilbert's evaluation of the invention of non-Euclidean geometry as "the most suggestive and notable achievement of the last century." A discussion of infinite sets and truth tables leads naturally to the idea of a system of axioms (or "postulates", as the author prefers to call them). This idea is illustrated by the finite planes PG(2, 2) and EG(2, 3). The author remarks that "the period from 1880 to 1910 saw the publication of 1,385 articles devoted to the foundations of geometry." He cites absolute geometry as "a good example of postulational system that is very rich in consequences and [yet] incomplete" (that is, not categorical).

In Chapter V, he considers the possibility of introducing, into a "rudimentary affine plane," coordinates x and y in terms of which a line has a linear equation. He finds a necessary and sufficient condition to be the "first Desargues property" (i.e., Desargues's theorem for triangles that are congruent by translation). For the coordinates to belong to a field (not necessarily commutative), a necessary and sufficient condition is the "third Desargues property" (i.e., Desargues's theorem for homothetic triangles). The author

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