the corresponding property of a point and a circle; and the quantity $\tan\frac{1}{2}APQ\cot\frac{1}{2}BPQ$ may be called the potency of the line PQ with reference to the circle. The above theorem is also true *in plano*, and affords a simple proof that the six centres of similitude of three circles lie in sets of three on four straight lines. To the fact of the collinearity of the centres of two circles and their two centres of similitude corresponds the proposition that the two parallel great circles and the two radical axes are concurrent.

The following examples of properties of circles which correspond according to the above method of transformation may also be noted.

To the fact that the radius of a circle is of constant length, corresponds the fact that the angle which the tangent to a circle makes with the parallel great circle is constant; and the proposition that the tangents make equal angles with the chord of contact transforms into the proposition that the two tangents from the same point are equal.

Again, the centres of similitude of two circles divide the line joining the centres of the circles so that the ratio of the sines of the segments is equal to the ratio of the sines of the radii. This theorem gives on transformation the theorem that the radical axes divide the angle between the parallel circles so that the ratio of the sines of the parts is equal to the ratio of the sines of the angles which the tangents to the circles make with the parallel great circles. This theorem evidently cannot have any analogue in plane geometry, as must in general be the case with the polars of such theorems as refer to the centre of a circle, since the line in a plane corresponding to a parallel great circle is altogether at an infinite distance.

Fifth Meeting, March 13th 1885.

George Thom, Esq., M.A., Vice-President, in the Chair.

Gilbert's Method of Treating Tangents to Confocal Conicoids.

By George A. Gibson, M.A.

The method of treating tangents to confocal conicoids, of which I propose to give an account, is discussed in the number for December

1867 of the *Nouvelles Annales de Mathématiques.* The writer of the paper referred to is M. Ph. Gilbert, Professor at the University of Louvain. The results arrived at are in nearly every case already well known, but the method of reaching them is somewhat novel, and I have thought that it might interest the members of this society if I were to give a statement of the chief methods and results of Gilbert's paper. In one or two cases I have altered the proofs, and I have added two or three propositions that seemed to follow naturally from the equations dealt with. Gilbert deals only with central conicoids; but I have put in an equation for the paraboloids that corresponds to Gilbert's fundamental one.

Gilbert's method of dealing with tangents to confocal conicoids is based upon a certain expression for the angle between the normals at two points, one on each of two confocals. Let the confocals be given by the equation

$$\frac{x^2}{\lambda^2}+\frac{y^2}{\lambda^2-b^2}+\frac{z^2}{\lambda^2-c^2}=1$$

2λ being the primary axis of any one of the series. Any individual confocal may be referred to by its primary semi-axis.

Let M be a point on λ, its co-ordinates being (xyz),

N ,, θ, ,, ,, $(\xi\eta\zeta)$,

p_λ the perpendicular from centre on tangent plane to λ at M,

p_θ ,, ,, ,, θ at N,

δ the distance between M and N.

Let $(\delta\lambda)$, $(\delta\theta)$ be the angles between MN and the normals to λ and θ at M and N respectively, and $(\theta\lambda)$ the angle between the normals themselves, the normals being in all cases drawn *outwards*.

The direction cosines of normal at M will be $\left\{\frac{p_\lambda x}{\lambda^2},\ \frac{p_\lambda y}{\lambda^2-b^2},\ \frac{p_\lambda z}{\lambda^2-c^2}\right\}$;

,, ,, N ,, $\left\{\frac{p_\theta \xi}{\theta^2},\ \frac{p_\theta \eta}{\theta^2-b^2},\ \frac{p_\theta \zeta}{\theta^2-c^2}\right\}$;

$$\cos(\theta\lambda)=p_\lambda\cdot p_\theta\left\{\frac{x\xi}{\lambda^2\theta^2}+\frac{y\eta}{(\lambda^2-b^2)(\theta^2-b^2)}+\frac{z\zeta}{(\lambda^2-c^2)(\theta^2-c^2)}\right\};$$

$$\cos(\delta\lambda)=\frac{p_\lambda}{\delta}\left\{\frac{x(\xi-x)}{\lambda^2}+\frac{y(\eta-y)}{\lambda^2-b^2}+\frac{z(\zeta-z)}{\lambda^2-c^2}\right\}$$

$$=\frac{p_\lambda}{\delta}\left\{\frac{x\xi}{\lambda^2}+\frac{y\eta}{\lambda^2-b^2}+\frac{z\zeta}{\lambda^2-c^2}-1\right\};\ \text{since } xyz \text{ lies on } \lambda;$$

$$\therefore\quad \frac{\delta.\cos(\delta\lambda)}{p_\lambda}=\frac{x\xi}{\lambda^2}+\frac{y\eta}{\lambda^2-b^2}+\frac{z\zeta}{\lambda^2-c^2}-1.$$

Similarly, $\frac{\delta \cos(\delta\theta)}{p_\theta} = \frac{x\xi}{\theta^2} + \frac{y\eta}{\theta^2 - b^2} + \frac{z\zeta}{\theta^2 - c^2} - 1.$

$$\therefore \quad \frac{\delta.\cos(\delta\lambda)}{p_\lambda} - \frac{\delta.\cos(\delta\theta)}{p_\theta} = (\theta^2 - \lambda^2)\left\{\frac{x\xi}{\lambda^2\theta^2} + \frac{y\eta}{(\lambda^2 - b^2)(\theta^2 - b^2)} + \frac{z\zeta}{(\lambda^2 - c^2)(\theta^2 - c^2)}\right\}$$

$$= (\theta^2 - \lambda^2)\frac{\cos(\theta\lambda)}{p_\lambda \cdot p_\theta};$$

$$\therefore \quad \cos(\theta\lambda) = \frac{\delta}{\theta^2 - \lambda^2}\left\{p_\theta \cos(\delta\lambda) - p_\lambda \cos(\delta\theta)\right\} \ldots\ldots (A)$$

This is Gilbert's fundamental equation, and from it he deduces very readily many theorems on confocal conicoids.

1. Suppose M and N to coincide, *i.e.*, let δ be zero, then cos $(\theta\lambda)$ will be zero. Hence :—"Two confocal conicoids through the same point cut each other orthogonally."

2. If $\cos(\delta\lambda)$ and $\cos(\delta\theta)$ be each zero, *i.e.*, if the line MN touch both the conicoids, then $\cos(\theta\lambda)$ will still be zero. Hence :—"If two confocals touch the same straight line, and if we draw the tangent planes at the points of contact, these planes will cut at right angles." Or, as the theorem may be stated :—"If two confocals be viewed from any point, their apparent contours cut at right angles wherever they appear to intersect."

3. Suppose MN to touch θ at N; then $\cos(\delta\theta)$ vanishes, and

$$\cos(\theta\lambda) = \frac{\delta p_\theta}{\theta^2 - \lambda^2} \cdot \cos(\delta\lambda) \ldots\ldots (B)$$

This equation is really that which Gilbert first proves; but it is evidently included in equation (A).

4. If $(\lambda\mu\nu)$ be the primary semi-axes of the three confocals through M, and if MN touch θ at N, then by equation (B) we know that $\cos(\theta\lambda) = \frac{\delta p_\theta}{\theta^2 - \lambda^2} \cdot \cos(\delta\lambda)$; $\cos(\theta\mu) = \frac{\delta p_\theta}{\theta^2 - \mu^2} \cdot \cos(\delta\mu)$ and $\cos(\theta\nu)$ $= \frac{\delta p_\theta}{\theta^2 - \nu^2} \cdot \cos(\delta\nu)$.

Now, since the normals to the three confocals through M are mutually perpendicular, these may be taken as rectangular axes. With reference to these axes the direction cosines of MN are $\cos(\delta\lambda)$, $\cos(\delta\mu)$, $\cos(\delta\nu)$, and of the normal at N $\cos(\theta\lambda)$, $\cos(\theta\mu)$, $\cos(\theta\nu)$;

$$\therefore \quad \cos(\delta\theta) = \cos(\delta\lambda)\,.\,\cos(\theta\lambda) + \cos(\delta\mu)\,.\,\cos(\theta\mu) + \cos(\delta\nu)\,.\,\cos(\theta\nu)$$
$$= 0, \text{ since MN touches } \theta.$$

Putting in the values just given for $\cos(\theta\lambda)$, $\cos(\theta\mu)$, $\cos(\theta\nu)$, we get

$$\frac{\cos^2(\delta\lambda)}{\theta^2 - \lambda^2} + \frac{\cos^2(\delta\mu)}{\theta^2 - \mu^2} + \frac{\cos^2(\delta\nu)}{\theta^2 - \nu^2} = 0.$$

As every tangent line from M to conicoid θ fulfils this condition, the equation of the enveloping cone must be

$$\frac{x^2}{\theta^2 - \lambda^2} + \frac{y^2}{\theta^2 - \mu^2} + \frac{z^2}{\theta^2 - \nu^2} = 0.$$

The form of the equation shows that the normals to the three confocals are the principal axes of the cone.

5. If θ_1, θ_2 be the primary semiaxes of the two conicoids which touch a line through a point M, the direction cosines of which, referred to the three normals at M, are l, m, n, then the following will, by § 4, be an identical equation :—

$$l^2(\theta^2 - \mu^2)(\theta^2 - \nu^2) + m^2(\theta^2 - \nu^2)(\theta^2 - \lambda^2) + n^2(\theta^2 - \lambda^2)(\theta^2 - \mu^2)$$
$$= (\theta^2 - \theta_1^2)(\theta^2 - \theta_2^2)$$

$\therefore \quad l^2 = \dfrac{(\lambda^2 - \theta_1^2)(\lambda^2 - \theta_2^2)}{(\lambda^2 - \mu^2)(\lambda^2 - \nu^2)}$, with similar values for m and n.

Now, as a particular case, let $\mu = \nu = b$ (and $\therefore\ \theta_2 = b$), *i.e.*, take M on the focal hyperbola. We thus get $l^2 = \dfrac{\lambda^2 - \theta_1^2}{\lambda^2 - b^2}$. Hence the theorem— :

"The enveloping cone of an ellipsoid is one of revolution, when its vertex lies on the focal hyperbola, the axis of the cone being the tangent of the hyperbola at the point." The vertical angle of the cone is

$$2\tan^{-1}\sqrt{\left(\frac{\theta_1^2 - b^2}{\lambda^2 - \theta_1^2}\right)}$$

6. By equation (B) $\dfrac{\cos^2(\theta\lambda)}{p_\theta^2\delta^2} = \dfrac{\cos^2(\delta\lambda)}{(\theta^2 - \lambda^2)^2}$

and $\quad \cos^2(\theta\lambda) + \cos^2(\theta\mu) + \cos^2(\theta\nu) = 1$

$$\therefore \quad \frac{1}{p_\theta^2\delta^2} = \frac{\cos^2(\delta\lambda)}{(\lambda^2 - \theta^2)^2} + \frac{\cos^2(\delta\mu)}{(\mu^2 - \theta^2)^2} + \frac{\cos^2(\delta\nu)}{(\nu^2 - \theta^2)^2}$$

Suppose $\mu=\nu=b$, and we get, after substituting the values for the direction cosines given in the last article,

$$p_\theta\delta=\sqrt{(\lambda^2-\theta_1^2)(\theta_1^2-b^2)}.$$

In words—"The rectangle contained by the side of a cone of revolution enveloping an ellipsoid, intercepted between the vertex and point of contact, and the perpendicular from the centre on the tangent plane at that point is constant."

7. We may write the value for $\cos^2(\delta\lambda)$ in the form

$$\frac{\theta^2-\lambda^2}{p_\theta\delta}\,.\,\cos(\delta\lambda)\,.\,\cos(\theta\lambda),$$ and we therefore find that

$$(\theta^2-\lambda^2)\cos(\delta\lambda)\,.\,\cos(\theta\lambda)+\&c.\quad=p_\theta\delta.$$

that is, $\lambda^2\cos(\delta\lambda)\,.\,\cos(\theta\lambda)+\mu^2\cos(\delta\mu)\,.\,\cos(\theta\mu)+\nu^2\cos(\delta\nu)\,.\,\cos(\theta\nu)$
$$=-p_\theta\delta$$

In words—"If from a point M we draw a tangent to a conicoid meeting it in N, and if we take on the normals to the three confocals through M lengths equal to the respective primary semiaxes of these confocals, the sum of the products of the projections of these lengths on the tangent by their projections on the normal to the given conicoid at N is equal to the product of the distance between M and the point of contact N by the perpendicular from the centre on the tangent plane at N."

8. Still using equation (B) we see that

$$(\theta^2-\lambda^2)\cos^2(\theta\lambda)=p_\theta\delta\,.\,\cos(\delta\lambda)\,.\,\cos(\theta\lambda).$$

$$\begin{aligned}\therefore\quad &(\theta^2-\lambda^2)\cos^2(\theta\lambda)+(\theta^2-\mu^2)\cos^2(\theta\mu)+(\theta^2-\nu^2)\cos^2(\theta\nu)\\ &=\delta p_\theta\{\cos(\delta\lambda)\,.\,\cos(\theta\lambda)+\cos(\delta\mu)\,.\,\cos(\theta\mu)+\cos(\delta\nu)\cos(\theta\nu)\}\\ &=0.\end{aligned}$$

$$\therefore\quad \lambda^2\cos^2(\theta\lambda)+\mu^2\cos^2(\theta\mu)+\nu^2\cos^2(\theta\nu)=\theta^2.$$

This equation proves a theorem of Chasles, viz :—If on a fixed plane P we take any point M and measure on the normals to the three confocals through M lengths equal to their respective primary semi-axes, the sum of the squares of the projections of these lengths on the normal to the plane P will be constant for all positions of M on the plane. This equation also gives the primary axis of the conicoid which touches a given plane, as the equation of § 4 gives the axes of the conicoids which touch a given line.

9. Suppose a normal drawn from M (λ,μ,ν) to a conicoid λ' meeting it in N. Denote by μ', ν' the primary semi-axes of the other two confocals through N. Then since the normal to λ' is a tangent to μ' and ν' the following equation will connect the direction cosines of the normal:—

$$\frac{\cos^2(\lambda'\lambda)}{(\mu'^2-\lambda^2)(\nu'^2-\lambda^2)}+\frac{\cos^2(\lambda'\mu)}{(\mu'^2-\mu^2)(\nu'^2-\mu^2)}+\frac{\cos^2(\lambda'\nu)}{(\mu'^2-\nu^2)(\nu'^2-\nu^2)}=0.$$

10. Returning to equation (A), we may suppose M and N to lie on the same conicoid, *i.e.*, take $\theta=\lambda$ and $\therefore \frac{p_\lambda}{\cos(\delta\lambda)}=\frac{p_\theta}{\cos(\delta\theta)}$ where p_λ, p_θ denote the perpendiculars from the centre on the tangent planes at M, N respectively. If a plane be drawn through the centre parallel to the tangent plane at M meeting MN in M′, then MM′ will be equal to $\frac{p_\lambda}{\cos(\delta\lambda)}$. Hence the theorem:—"If we take any chord MN of a conicoid, and through the centre draw planes parallel to the tangent planes at M, N meeting the chord in M′, N′ respectively, MM′ will be equal to NN′." Also, it may readily be deduced from this theorem that "The plane through the centre and the line of intersection of the tangent planes at M, N will bisect M′N′."

11. If in equation (A) $\cos(\theta\lambda)=0$, the equation $\frac{p_\lambda}{\cos(\delta\lambda)}=\frac{p_\theta}{\cos(\delta\theta)}$ still holds.

In this case the interpretation of the equation is:—"If on two confocal conicoids two points M, N be taken such that the normals at these points are perpendicular to each other, and if through the common centre planes be drawn parallel to the tangent planes at M, N cutting MN in M′,N′ respectively, then MM′ will be equal to NN′." From this it readily follows that, "The plane through the centre and the line of intersection of the tangent planes at M, N will bisect MN."

12. By § 5, we know that $\cos^2(\delta\lambda)=\frac{(\lambda^2-\theta_1^2)(\lambda^2-\theta_2^2)}{(\lambda^2-\mu^2)(\lambda^2-\nu^2)}$ where $\lambda\mu\nu$ are the primary semi-axes of the three confocals through a point M, and θ_1 θ_2 those of the two confocals touched by a line through M. If then a plane be drawn through the centre parallel to the tangent

plane at M, the square of the part intercepted on the line between M and this plane through the centre will be $p_\lambda^2\frac{(\lambda^2-\mu^2)(\lambda^2-\nu^2)}{(\lambda^2-\theta_1^2)(\lambda^2-\theta_2^2)}$ that is, $\frac{\lambda^2(\lambda^2-b^2)(\lambda^2-c^2)}{(\lambda^2-\theta_1^2)(\lambda^2-\theta_2^2)}$, a value constant for all points on λ. Thus, "If from any point of a central conicoid a line be drawn touching two given confocals, the portion of this line intercepted between the point and the plane through the centre parallel to the tangent plane at the point will be constant." As a particular case, suppose the line to pass through the two focal conics and it is clear that "The part intercepted on a bifocal chord through a point M on a conicoid and the plane through the centre parallel to the tangent plane at M is equal to the primary semi-axis of the conicoid."

13. Equation (A) may be written in the form

$$\theta^2-\lambda^2=p_\theta.\frac{\delta\cos(\delta\lambda)}{\cos(\theta\lambda)}-p_\lambda.\frac{\delta\cos(\delta\theta)}{\cos(\theta\lambda)}\ldots\ldots(C)$$

Now if the normal at N meet the tangent plane at M in P, it is easily proved that $NP=\frac{\delta\cos(\delta\lambda)}{\cos(\theta\lambda)}$. Denote this intercept by ω_θ, which will be positive or negative according as the normal is drawn *inwards* or *outwards* to meet the tangent plane. If ω_λ denote $\frac{\delta\cos(\delta\theta)}{\cos(\theta\lambda)}$ with the same convention as to sign, (C) may be written $\theta^2-\lambda^2=p_\theta\omega_\theta-p_\lambda\omega_\lambda$. Hence :—"If at any two points M, N on two confocal conicoids the normals be drawn, the products of the intercepts on these normals between the points of contact and the tangent planes by the perpendiculars from the centre on these tangent planes differ by a constant quantity; the constant being the difference of the squares of the primary semi-axes of the confocals."

As a particular case, suppose the tangent planes at M and N parallel; therefore $\omega_\theta=-\omega_\lambda=p_\theta-p_\lambda$ and $\therefore\ p_\theta^2-p_\lambda^2=\theta^2-\lambda^2$. Thus :—"If two parallel tangent planes be drawn to two confocals, the difference of the squares of the perpendiculars from the centre on these planes will be constant."

14. If $\lambda=\theta$, that is, if M and N be on the same conicoid, $p_\theta\omega_\theta=p_\lambda\omega_\lambda$. Therefore :—"If normals be drawn at two points M, N on a conicoid the products of the intercepts on these normals between the surface.

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and the tangent planes at N, M by the perpendiculars from the centre on the tangent planes at M, N are equal to each other."

15. Again, should the line MN touch the conicoid θ, ω_λ would be zero, and the equation would become $p_\theta \omega_\theta = \theta^2 - \lambda^2$. Hence :—"If from any point M on a conicoid a tangent be drawn meeting a confocal in N, the product of the perpendicular from the centre on the tangent plane at N by the intercept on the normal at N between the tangent planes at M and N is constant; the constant being the difference of the squares of the primary semi-axis of the confocals."

It is easy to get an equation analogous to (A) for confocal paraboloids. Let the series of paraboloids be given by

$$\frac{y^2}{b+\lambda} + \frac{z^2}{c+\lambda} = 2x + \lambda.$$

As before,

Let M be a point on λ, its co-ordinates being (xyz),

N " θ, " " $(\xi\eta\zeta)$,

p_λ perpendicular from $(-x,0,0)$ on tangent plane to λ at M,

p_θ " $(-\xi,0,0)$ " θ at N,

δ the distance between M, N.

Proceeding to find the values for $\cos(\delta\lambda)$, $\cos(\delta\theta)$, $\cos(\theta\lambda)$ we get

$$\frac{\lambda.\delta\cos(\delta\lambda)}{p_\lambda} = \left\{\lambda + \xi + x - \frac{\eta y}{b+\lambda} - \frac{\zeta z}{c+\lambda}\right\},$$

$$\frac{\theta.\delta\cos(\delta\theta)}{p_\theta} = \left\{\theta + \xi + x - \frac{\eta y}{b+\theta} - \frac{\zeta z}{c+\theta}\right\},$$

$$\cos(\theta\lambda) = \frac{p_\lambda p_\theta}{\lambda\theta}\left\{1 + \frac{\eta y}{(b+\theta)(b+\lambda)} + \frac{\zeta z}{(c+\theta)(c+\lambda)}\right\},$$

$$\cos(\theta\lambda) = \frac{\delta}{\lambda-\theta}\left\{\frac{p_\theta}{\theta}\cos\delta\lambda - \frac{p_\lambda}{\lambda}\cos\delta\theta\right\}.$$

Suppose $\delta = 0$; $\therefore$ $\cos(\theta\lambda) = 0$. Hence confocal paraboloids intersect orthogonally.

Suppose $\lambda\mu\nu$ to be the parameters of the three confocals through a given point M, θ that of another confocal, and we get as before for the equation of the cone referred to the three normals :—

$$\frac{x^2}{\lambda-\theta} + \frac{y^2}{\mu-\theta} + \frac{z^2}{\nu-\theta} = 0.$$

The following cases in confocal conics may be noted.

The equation to the two tangents from a point M (whose co-ordinates are $\lambda\mu$) to a conic θ is, since the equation

$$\cos(\theta\lambda) = \frac{\delta}{\lambda^2 - \theta^2}\left\{p_\lambda\cos(\delta\theta) - p_\theta\cos(\delta\lambda)\right\} \text{ holds,}$$

$$\frac{\cos^2\delta\lambda}{\lambda^2 - \theta^2} + \frac{\cos^2\delta\mu}{\mu^2 - \theta^2} = 0, \ \lambda > \mu.$$

Let ϕ denote the angle between one of these tangents and the tangent to the confocal ellipse through M,

$$\therefore \ \sin^2\phi = \cos^2(\delta\lambda), \ \cos^2\phi = \cos^2(\delta\mu) ;$$

$$\frac{\sin^2\phi}{\lambda^2 - \theta^2} = \frac{\cos^2\phi}{\theta^2 - \mu^2} = \frac{1}{\lambda^2 - \mu^2}.$$

Hence :—The two tangents to an ellipse from any point are equally inclined to the tangent to a confocal ellipse through the point.

We also have the value of θ^2 given by $\lambda^2\cos^2\phi + \mu^2\sin^2\phi = \theta^2$ which is the corresponding theorem to that of No. 8.

Corresponding to No. 10 is the theorem :—If MN be any chord of a conic, and if lines drawn through the centre parallel to the tangents at M and N meet the chord at M′ and N′ respectively, MM′ will be equal to NN′; and the line through the centre and the point of intersection of the tangents will bisect M′N′.

No. 11 gives the theorem :—If on two confocal conics two points M, N be taken such that the normals at these points are perpendicular to each other, and if through the common centre lines be drawn parallel to the tangents at M, N cutting MN in M′, N′ respectively, then MM′ will be equal to NN′; and the line joining the centre to the point of intersection of the tangents will bisect M′N′.

The theorem corresponding to that of No. 12 may be stated :—If from a point M on a central conic a tangent be drawn to a confocal conic, the intercept on the tangent between M and a central radius parallel to the tangent at M is constant. This holds when the confocal becomes the line-ellipse.

From No. 13 we get the theorem :—If at any two points MN on two confocal conics the normals be drawn, the products of the intercepts on these normals between the points of contact and the tangents by the perpendiculars from the centre on these tangents differ by a constant quantity—the constant being the difference of the squares of the primary semi-axes.

If the tangents at M and N be parallel, the squares of the perpendiculars from the centre on these tangents differ by a constant.

From No. 15 we get the theorem:—If from any point M on a conic, a tangent be drawn meeting a confocal in N, the product of the perpendicular from the centre on the tangent at N by the intercept on the normal at N between the tangents at M and N is constant.

Mr J. S. MACKAY gave a synopsis of Frans Schooten's "Geometry of the Rule," as it is contained in the second book of the *Exercitationes Mathematicae*, Leyden, 1657.

Mr P. ALEXANDER contributed a note on the two definite integrals

$$\int_0^\infty \sin nx dx \quad \text{and} \quad \int_0^\infty \cos nx dx.$$

Sixth Meeting, April 10th, 1885.

A. J. G. BARCLAY, Esq., M.A., President, in the Chair.

Note on the evaluation of functions of the Form 0^0.

By T. B. SPRAGUE, M.A., F.R.S.E.

Let $f(t)$, $\phi(t)$, be two functions of t, such that they both vanish with t, that is, $f(0)=0$, $\phi(0)=0$; and put

$$z=\{f(t)\}^{\Phi t}.$$

Then, in order to find the limiting value of z when $t=0$, we proceed as follows:—

$$\text{Log}z=\phi(t)\,.\log f(t)=\frac{\log f(t)}{\dfrac{1}{\phi(t)}}$$

This fraction takes the form $-\frac{\infty}{\infty}$ when $t=0$, and we therefore have