SECULAR PERTURBATIONS FOR ASTEROIDS BELONGING TO FAMILIES

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The first-order theory of secular perturbations for an asteroid indicates that the secular motions of the longitudes of the perihelion and of the ascending node have an equal absolute value with opposite signs, that is, $\dot{\Pi} = -\dot{\Theta} = b$, where b is a function of the semimajor axis and has the disturbing mass as a factor, if the eccentricity and the inclination are sufficiently small so that their squares are of the order of the disturbing mass and if there is no commensurable relation between the mean motions of the asteroid and the disturbing planet. Thus $\Pi + \Theta$ is a stable quantity according to the first-order theory, therefore, the origin of families of asteroids has been discussed by using the data for distribution of $\Pi + \Theta$ as well as those for dispersions of other orbital elements of asteroids belonging to a family.

However, $\Pi + \Theta$ is not so stable even if short-periodic terms are subtracted, since the eccentricity and the inclination are not generally so small and the second-order terms are not so small as one estimates by a formal expression (Kozai, 1953).

The secular terms of the second-order of the disturbing mass and of the fourth degree of the eccentricity and the inclination can be included in the disturbing function if there is no commensurable relation by either von Zeipel's or Lie-Hori's method easily. Although the differential equations including these terms are no more linear, they can be solved approximately (Kozai, 1953; Yuasa, 1973).

The results show that the mean secular motion of $\Pi + \Theta$ take the following form:

$$\dot{\Pi} + \dot{\Theta} = a_1 v^2 + 2a_2 v\mu + a_3 \mu^2 + m' a_4,$$

where μ and v are, respectively, the proper eccentricity and the proper inclination of the asteroid, m' is the disturbing mass, and a_j $(j = 1, \ldots, 4)$ are functions of the semimajor axis and have a factor of m'. Therefore, for a fixed value of the semimajor axis the equation $\dot{\Pi} + \dot{\Theta} = 0$ can be expressed by a quadratic curve in (μ, v)-plane. If the values of μ and v of asteroids belonging to a family are plotted as well as the quadratic curve, it is found that there is a tendency that the points are not so far from the curve, that is, $\dot{\Pi} + \dot{\Theta}$ takes a small value for an asteroid which belongs to any family.

There is also a very interesting relation between $\Pi_1 + \Theta_1$ and their time derivatives. In Figure 1 the values of $\Pi_1 + \Theta_1$ which are free from short-periodic perturbations are plotted against their time derivatives for asteroids of Flora family ($\alpha = a/a' = 0.43$). If points in left-side between 0 and 2π are moved vertically by 2π upwards and those in right-side are moved downwards by 2π, it is found that all the points line on a straight line. The slope of this line indicates that $T = 9.98 \times 10^5$ yr ago $\Pi_1 + \Theta_1$ took the same value for the asteroids. We cannot easily claim that this is the age of Flora family, since there is still ambiguity to determine this number.

Y. Kozai (ed.), Stability of the Solar System and of Small Stellar Systems, 81–82.

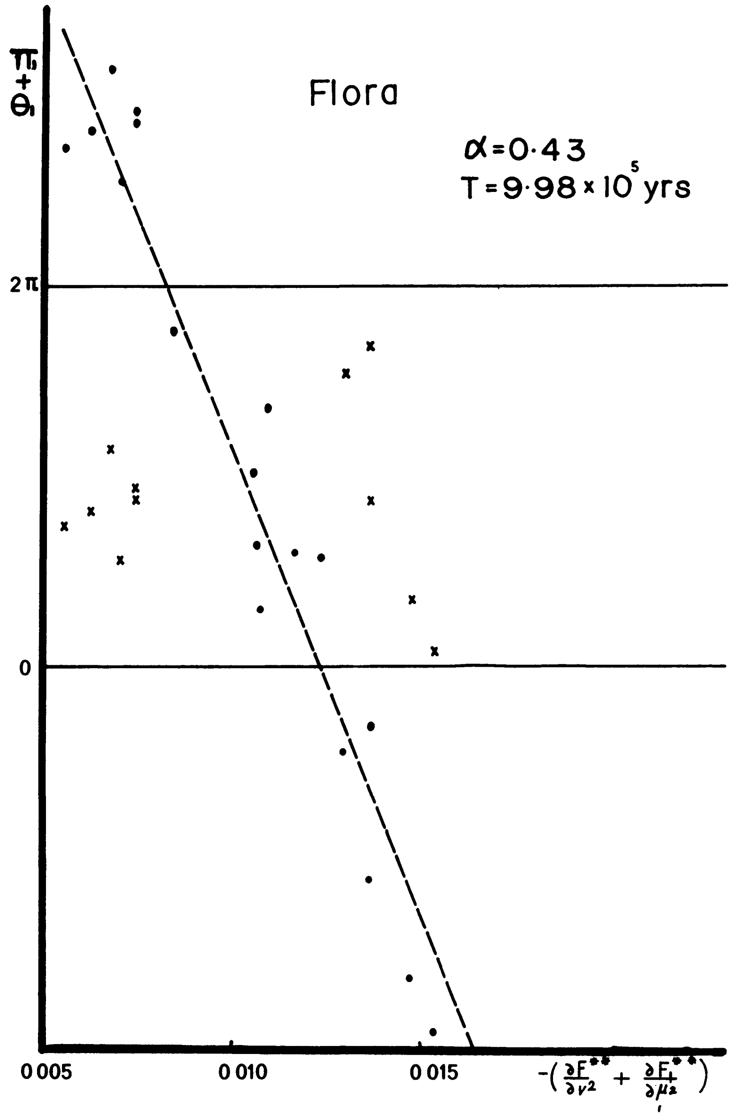

Fig. 1. $\Pi_1 + \Theta_1$ against their time derivatives for asteroids of Flora family. Points × are moved vertically by 2π to

For Coronis, Eos and Themis families $T = 3.7 \times 10^6$, 6.3×10^5 and 4.6×10^5 yr, respectively, are derived.

References

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https://doi.org/10.1017/S007418090007042X Published online by Cambridge University Press