

HEWITT, E. AND ROSS, K. A., *Abstract harmonic analysis*, volume 1 (die Grundlehren der mathematischen Wissenschaften, Band 115: Springer-Verlag, Berlin-Göttingen-Heidelberg, 1963), viii+519 pp., 76 DM.

This is the first of two volumes, in which the authors set out to give an account of abstract harmonic analysis as at present understood—that is, the generalisations of Fourier series and integrals to the case of locally compact abelian groups, and of the classical theory of matrix representations of finite groups to representation of locally compact topological groups by operators on a Hilbert space. The present volume comprises (to quote from the authors' Preface) “. . . all of the structure of topological groups needed for harmonic analysis . . . integration on locally compact groups in detail . . . an introduction to the theory of group representations”. The second volume will treat “harmonic analysis on compact groups and locally compact abelian groups, in considerable detail”. The scope of volume one is indicated by the chapter headings: Preliminaries, Elements of the theory of topological groups, Integration on locally compact spaces, Invariant functionals, Convolutions and group representations, Characters and duality of locally compact abelian groups. There are three appendices, totalling 53 pages (Abelian groups, Topological linear spaces, and Introduction to normed algebras), and a 14-page bibliography. It would be a considerable over-simplification, but perhaps not entirely misleading, to say that the present volume goes as far as the Pontryagin duality theorem in the abelian case, and the Gelfand-Raikov theorem (on the sufficiency of the irreducible unitary representations) in the general case. The duality theorem is here treated by the original structure-theoretic method of Pontryagin and van Kampen; the alternative (and shorter) Fourier transform proof is promised for volume two. The core of the book is, of course, in the last two chapters; the rest is essentially preparatory, though there is much of independent interest to be found in the earlier chapters.

The treatment is designed to be suitable for beginners as well as experts; in this context a beginner is assumed to be a graduate with a good knowledge of algebra, topology and measure theory—equivalent, say, to substantial parts of van der Waerden, Kelley and Halmos. The exposition is careful and detailed, and indications are given of those sections that may be omitted on a first reading of the subject. An uninitiated reader is likely to find his difficulties of a global rather than a local nature; the total mass of material is rather formidable. Harmonic analysis is, in its essentials, a rather more elementary subject than appears from the present volume. It is possible to penetrate to the basic theorems without the use of quite such a formidable array of analytical apparatus; in particular, only a rather primitive version of measure theory is needed. However, with suitable guidance, this book could very well be used as a text by a beginning research student.

For the more experienced analyst, there is much of interest; the wealth of detail and the numerous indications of related work make the book very useful as a reference. The historical and bibliographical notes, and the “miscellaneous theorems and examples” that appear after most of the sections are particularly valuable.

The writing has evidently been done with great care, and the typography is (of course) immaculate. The reviewer detected no errors (printers' or other), and only one redundant hypothesis (p. 351, line 25; the assumed condition follows from the Banach-Steinhaus theorem). Notation and terminology are on the whole quite standard; the only startling departure being the use of the tilde instead of the asterisk for the adjoint operation. This may possibly have something to commend it, in view of the heavy burden of meaning already carried by the asterisk. In some cases a little care is needed to ascertain the precise conventions being used; for instance, if one begins half-way through the book it may not be immediately realised that all neighbourhoods are open.

This is clearly destined to be a standard reference for many years to come, and should be on the shelves of every library and on the desk of every harmonic analyst. It is to be hoped that the second volume will appear without undue delay, to complete a major addition to the literature of the subject.

J. H. WILLIAMSON

HAMERMESH, M., *Group Theory and its Application to Physical Problems* (Pergamon Press, 1962), 509 pp., 105s.

This is an excellent textbook of which about 400 pages are devoted to pure mathematics and about 100 pages to applications. The exposition is extremely clear throughout and there are many examples embodied in the text by which the reader can test his understanding of the theory. In the opinion of the reviewer, the book is rather easier to read than the other (excellent) books on the subject, such as those by Boerner and by Wigner, which have recently appeared, and is therefore admirably suited to the beginning research worker in theoretical physics. The chapter headings of the book are as follows: 1. Elements of group theory; 2. Symmetric groups; 3. Group representations; 4. Irreducible representations of the point symmetry groups; 5. Miscellaneous operations with group representations; 6. Physical applications; 7. The symmetric group; 8. Continuous groups; 9. Axial and spherical symmetry; 10. Linear groups in n -dimensional space. Irreducible tensors; 11. Applications to atomic and nuclear problems; 12. Ray representations. Little groups. It seems a pity that the author has not found it possible to discuss the Lorentz group, for a treatment of this topic given with the clarity of the rest of the book would have been most acceptable; however, the author maintains that to do this adequately would involve giving an account of quantum field theory.

The treatment in Chapters 1-7 is very complete but, in the remaining chapters, some results are quoted without proof. One lapse in rigour, which could easily be corrected, is that it is the converse of Lemma II on page 100, and not the lemma itself (as stated on page 101) which is useful as a test of irreducibility. The smooth style in which the book is written and the clear printing and layout make it a pleasure to read.

D. MARTIN

GERRETSON, J. C. H., *Lectures on Tensor Calculus and Differential Geometry* (Noordhoff, 1962), xii+204 pp., Dfl. 25.

The first four chapters of this book deal with linear and metric vector spaces, bilinear and quadratic forms, and tensors, these being defined as multilinear forms on a real vector space with an inner product; there is no mention of the dual space. In Chapter 5 a manifold is defined, and throughout the book a manifold is always regarded as being embedded in a Euclidean space. Chapter 6 deals with curves, and the usual topics such as the Frenet-Serret formulæ, involutes and evolutes are discussed. In Chapter 7 geodesic differentiation is defined and a treatment given of the Christoffel symbols, geodesic correspondence, Ricci coefficients of rotation, etc. Chapter 8 is devoted to the theory of hypersurfaces and concludes with a proof of the invariant character of the Gaussian curvature, while Chapter 9 deals with covariant differentiation, the Riemann tensor, the Weyl conformal curvature tensor, Einstein spaces etc. The book concludes with an account in Chapter 10 of Bonnet's problem concerning the existence of hypersurfaces with given first and second fundamental forms (compatible with the equations of Gauss and of Codazzi).

For a textbook devoted to only the local aspects of differential geometry, the choice of material is very good, but the treatment, in the opinion of the reviewer, is rather uninspiring and lacks some modern colour which might have been expected.

D. MARTIN