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**DIVISION A**
**COMMISSION 7**

# CELESTIAL MECHANICS AND DYNAMICAL ASTRONOMY

*MÉCANIQUE CÉLESTE ET ASTRONOMIE DYNAMIQUE*

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## LEGACY REPORT : 2005-2015

## 1. Introduction

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In order to mark a distinction with the traditional triennial reports, for this legacy issue we have asked our present and past OC members, as well as a few other outstanding members of the Celestial Mechanics community, to write a short essay on "recent highlights and the future of Celestial Mechanics". Below we collect the contributions of the people who responded to our invitation. As it is natural, each of them interpreted their task differently. Some produced a dissertation on broad and general aspects, others focused on a specific topic of their interest. Some considered that their role was to provide a detailed review, with a list of key references, others preferred to mention the topics for which progress has been significant but without quoting any references, implicitly considering that this progress was possible thanks to the collective efforts of many scientists, and not just a few. This is great, as we appreciate the diversity of attitudes and opinions.

Despite this diversity, a common view stands out of all the essays reported below. Celestial Mechanics is today an *applied science*, devoted to understand the dynamics of specific objects. *Complexity* is the keyword. In fact, because the goal is to understand the dynamical behavior of a real object –or system– in detail, the models have to be as complete and realistic as possible, often in synergy with physical models.

This state of fact made us realize, over the last triennium, that it was reductive to consider Commission 7 "Celestial Mechanics and Dynamical Astronomy" solely as part of Division A "Fundamental Astronomy". Due to the expansion of our field, both in methods and in applications, we sought and obtained the co-affiliation to Division F "Planetary Systems and Bioastronomy". The same remains true for the renewed commission, now labeled C-A4, which is an inter-division commission under the parent divisions A and F. We think that the new structure reflects better the modern role of Celestial Mechanics and the interests of its community.

Even though Celestial Mechanics has evolved in its scopes and tools, we remain faithful to our ideals: it is not sufficient to describe or reproduce a phenomenon; the ultimate goal is to understand *why* it occurs. Numerical simulations are a tremendously powerful tool that allows to investigate complex problems which would be, in most cases, impossible to tackle analytically; however, the theoretical insight is needed to interpret and understand what the simulations show. Moreover, each problem requires its own particular approach and optimal set of tools. The essays below also reflect this diversity. Some dynamical problems require sophisticated analytical modeling, while others are better investigated by numerical means. Thus, modern day researchers in our field must be equally literate in both.

In this respect, our community faces a danger. Current Ph.D. students in Dynamical Astronomy know very well the objects of their studies and the observational constraints, but only a few have a knowledge of the theory of dynamical systems. Thus, one of the leading goals of the new commission C-A4 will be to fight the growing illiteracy in theoretical dynamics by promoting schools devoted to the subject, university courses etc. We need to act quickly, before that the theoretical know-how is lost with the retirement of the last generation of experts. Also, the Celestial Mechanics commission has to preserve a privileged relationships with mathematicians and promote their interactions with the astronomers.

Celestial Mechanics has a bright present and an even brighter future as a key field in modern astrophysics and planetary science in particular. But this success comes together with a radical change of what we intend by "Celestial Mechanics". This change is occurring very rapidly. We need to operate in such a way that the field does not lose contact with its roots if we want it to keep flourishing.

## 2. Recent Highlights and the Future of Celestial Mechanics

### 2.1. *Planet Dynamics*

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In the last decade, the field of Celestial Mechanics experienced a great development and renewed interest that can only be compared to the beginning of the spatial era in the 60's. One reason is the increase of computational power that allows us to run simulations of a large number of bodies in feasible amounts of time. As a result, we can now study many problems with the complete equations of motion, and not only with approximated methods. Another reason is the boom in the discovery of exoplanets. This has brought new problems to the field, since many planetary systems are found in weird configurations completely different from the Solar System, requiring for an explanation, and for which the classic tools are no longer appropriate.

One of the most interesting results was the final confirmation that the Solar System is unstable (Laskar & Gastineau 2009). Direct numerical simulations of the evolution of the Solar System over 5 Gyr, including contributions from the Moon and general relativity were carried a set of 2,501 orbits with initial conditions that are in agreement with our present knowledge of the parameters of the Solar System. It was found that for about 1% of the trajectories the inner system can be completely destabilized after 3.34 Gyr from now, with possible collisions of Mercury, Mars or Venus with the Earth. Subsequent works have shown that this behavior occurs due to the proximity of a resonance between the precession frequencies of Mercury and Jupiter (Boué *et al.* 2012; Batygin *et al.* 2015). It was also shown that the asteroid belt is strongly chaotic and their motion is unpredictable over 400 kyr (Laskar *et al.* 2011). As a result, it will never be possible to recover the precise evolution of the Earth's eccentricity beyond 60 Myr.

A considerable research effort has also been invested in an attempt to reconstruct the past history of the Solar System. The planets could have emerged from the dispersing circumsolar disk on precisely circular orbits in a common plane, and then giant planets migration owing to their interaction with a disk of planetesimals lead to dynamical instabilities that explain the present planetary orbits (Tsiganis *et al.* 2005; Morbidelli *et al.* 2007). The semi-major axes of the planets evolve discontinuously during encounters, with Jupiter's semi-major axis changing by as much as 0.5 AU (Morbidelli *et al.* 2009; Levison *et al.* 2011). As the outer solar system reconfigures, the inner planets and the asteroid belt follow the suit (Brasser *et al.* 2009; Walsh *et al.* 2012; Lykawka & Ito 2013). It was found that a late giant planet migration scenario that initially had five giant planets rather than four had a higher probability of satisfying the orbital constraints of the terrestrial planets (Nesvorný & Morbidelli 2012; Brasser *et al.* 2013). Planetary encounters also successfully explained the retrograde orbit of Triton (Agnor & Hamilton 2006), and the irregular satellites of the giant planets (Nesvorný *et al.* 2007).

Once the planetary orbits in the Solar System stabilize, the planetary spins and the orbits of the satellites can still continue to evolve until the present days due to tidal interactions. The exact process on how this evolution occurs is not very clear, because tidal friction is almost imperceptible in short time-scales and it is thus difficult to compare the existing models with the observations. However, the increasing precision in astrometric observations and the accumulation of data allowed to put some constraints in tidal dissipation (Efroimsky & Lainey 2007; Lainey *et al.* 2009). As a result, more accurate and reliable tidal models have been proposed (Efroimsky 2012; Ferraz-Mello 2013; Correia *et al.* 2014). All these new models converge in one point: for giant planets the dissipation increases with the tidal frequency, while for rocky planets it decreases.

The emergence of a large number of exoplanets, many of them in multi-planetary systems has boosted the community interest in these bodies. First of all, Celestial Mechanics can help in the detection of these bodies, or even find previously unseen planets (Rivera *et al.* 2005; Nesvorný *et al.* 2012). By studying the global short-term dynamics, it is possible to ascertain whether a set of orbital parameters is well determined or not. This approach was particularly successful for highly populated systems (Lovis *et al.* 2011), or for planets in mean motion resonances (Correia *et al.* 2005), since the gravitational interactions are stronger. In addition, it is possible to derive additional constraints for the orbits, such as the mutual inclinations (Correia *et al.* 2010), their inner structure (Mardling 2007), or the strength of tidal dissipation (Laskar *et al.* 2012).

Finally, the puzzling configurations of many exoplanets challenged the existing theories for the formation and evolution of planetary systems. N-body codes, adapted to include migration and eccentricity damping due to the gas disc via analytic prescriptions, and hydrodynamics codes that explicitly evolves a 2D protoplanetary disc model with embedded protoplanets have been developed (for a review see Mordasini *et al.* 2015). Some of these simulations help to explain the formation of mean motion resonances (Cresswell & Nelson 2006), or the mass versus semi-major axis observed distributions (Alibert *et al.* 2013). Secular models have also been developed to deal with long-term planetary evolution. One major success was the formation of hot-Jupiters due to secular interactions with a distant binary companion (Fabrycky & Tremaine 2007; Correia *et al.* 2011) or due to planet-planet scattering in the disk (Beaugé & Nesvorný 2012). Analytic simplified models have also successfully explained some peculiar features, such as the accumulation of planets near mean motion resonances observed by the Kepler space telescope (Lithwick & Wu 2012; Delisle *et al.* 2012).

The continuous improvement on computer power will allow to design more complete models, and to perform statistical studies that will certainly help to solve many of the questions still open for the Solar System, and also for other stellar systems. A new generation of instruments and spatial missions, such as, ESPRESSO, CHEOPS, SPIROU, PLATO, TESS, will continue to feed the community with large amounts of data on exoplanets. It is therefore expected that new planetary systems and improved data for the already existing ones will occupy researchers for a long time in this field. Finally, the large amount of spacecrafts that recently reached they targets, such as Rosetta (first visit of a comet), Dawn (first visit to the asteroids' belt), or New Horizons (first visit to Pluto and the Kuiper belt), will provide new data and constraints for the origin of the Solar System.

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## 2.2. *Tidal Evolution Theories*

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The tidal theories developed in the past 10 years are characterized for the consideration of the actual rheology of the bodies. The works of Efroimsky and collaborators were the first to point out the fact that the quality factor introduced in Darwin's classical theory is inversely proportional to the tidal frequencies while the values of the Earth's $Q$ determined by the seismologists follows a direct power law $Q \propto \chi^\alpha$ (in his notations) with $\alpha > 0$. In 2007 Efroimsky and Lainey objectively proposed that $\alpha = 0.3 \pm 0.1$. We note that, accurately calculated, the torque is proportional to $\sin 2\epsilon$ ($\epsilon$ is the tidal lag), that is, to $(Q + 1/Q)^{-1}$. Then, no singularity affects the torque when $Q \to 0$. The approach followed by Efroimsky and collaborators and applied to several cases by Makarov is Darwinian and uses the very accurate equations derived by Kaula, plugging in these equations the lag derived from a preliminary study of the rheology of the materials constituent of the considered body. In the case of stiff bodies, they propose to use the Andrade model as paradigm of the viscoelastic behavior, which, in some sense behaves as a Maxwell body to which a nonlinear hereditary component is added. Darwinian theories were also considered beyond the usual restriction to second harmonics, necessary when the two interacting bodies are too close one to another (Taylor).

A new rheophysical approach to calculate the tidal forces and torques, the creep tide theory, was introduced by Ferraz-Mello, 2013. In this approach, the body tends always to creep towards the hydrostatic equilibrium by the only action of the gravitational forces acting on it (self-gravitation and tidal potential) and does it with a rate inversely proportional to its viscosity. The adopted creep law is Newtonian (linear), and at every instant the stress is assumed to be proportional to the distance from the equilibrium. The coefficient of proportionality is the relaxation factor $\gamma$ (which is the critical frequency for which the torque is maximum). One consequence of this approach is that the rotation of one body is damped by tides to the neighborhood of a stationary state, as in classical Darwin's theory. However, the final state now depends on the viscosity of the body. In the near inviscid limit (i.e. $\gamma \to \infty$), the body tends to a final rotational state which is the same given by the classical Darwinian theories: it has a speed higher than the orbital mean motion and the excess of rotation speed is given by $6ne^2$. However, when the viscosity is large and $\gamma \ll n$ (e.g. in stiff bodies), no matter if the eccentricity is large or small, the final stationary rotation is a small oscillation about a synchronous or resonant rotation.

The creep equation, however, only gives the anelastic tide.To obtain the actual shape of the tidally deformed body it is necessary to add the anelastic tide given by the creep to the elastic component of the tide. A rheophysical model including simultaneously the elastic and the anelastic tides was proposed by Correia *et al.*, using a Maxwell model. The results are virtually equivalent to those of the creep tide theory. The developed Maxwell model is also a creep tide theory just with a slightly different creep law.

New other approaches were proposed to study tidal effects in fluid bodies such as stars and envelopes of giant planets and also on the inelastic planetary regions contribution

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to the tidal dissipation, be it in the mantle of Earth-like planets or in the cores of giant planets, all of them based on a deep study of the internal structure of the bodies (Sotin, Mathis, Remus, Zahn, etc.). These theories use an Eulerian model to derive the elastic-gravitational deformation of the body to get a description of the internal structure of the bodies and to the physical processes through which the energy conversion occurs in their interiors. The viscous component is added by transforming the elastic equation via the principle of correspondence. The complex Love numbers carrying the information on the potential of the forces generated by the tidal deformation of the body and the dissipation are calculated and used to contain the tidal torques. In the same direction we have to cite the theories of Ogilvie and collaborators, which pay special attention to the tidal forcing, propagation and dissipation of linear inertial waves in a rotating fluid body and to the damping mechanisms responsible for the dissipation.

At last, correct formulas adapted to the study of exoplanets were established and applied to newly discovered planetary systems (Mardling, Dobbs-Dixon, Greenberg, Jackson, Ferraz-Mello, Rodrigues, etc.). The more recent models even take into account the fact that solar-like stars loose angular momentum through the stellar magnetic wind. These new models were used to study the tidal evolution of the exoplanets and the influence of tides in the rotation of stars hosting massive close-in planets.

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## 2.3. *The State of Celestial Mechanics: A Historical Perspective*

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Following its golden era during the Apollo missions, celestial mechanics encountered a few episodes of slow and, at times, gradually declining progress. With our solar system presenting the only plausible ground for using celestial mechanics to explain the formation and dynamical evolution of a planetary system, progress was limited primarily to explaining the observed characteristics of the solar system. Notable advances included explaining the properties of the rings of giant planets, the orbital architecture of their satellites, the role of tidal effects in orbit-orbit resonances, and the effect of secular resonances on the dynamical evolution of planets such as Mercury and Mars.

Thanks to observational and computational advances, celestial mechanics witnessed major progress during the 80s. In general, this progress can be divided into three major categories: explaining the dynamical evolution and architecture of the asteroid belt, developing theories of planet-disk interaction and planetary migration, and developing modes of terrestrial planet formation.

Within the context of our solar system, two of the most significant achievements of celestial mechanics at that time were the explanation of Kirkwood Gaps using chaotic motions of small bodies in areas of overlapping resonances, and the application of secular resonances of giant planets to explain the observed features in the proper eccentricities and inclinations of asteroids. The development of these two theories was not only significant due to their capabilities in explaining observed features of the asteroid belt, they were also fundamental to our understanding of the formation of terrestrial planets as they showed that the formation and evolution of the inner solar system is strongly affected by the perturbation of giant planets. In other words, these theories demonstrated that the solar system is a tightly bound entity and any model of its formation and evolution has to account for the formation and evolution of all its constituents, simultaneously. In this new picture of the solar system, the Asteroid Belt behaves as the medium through which the perturbation of giant planets is transmitted to the region where terrestrial planets are formed.

In addition to explaining the dynamical features of the solar system, advances in computational techniques extended the progress of celestial mechanics to more general aspects of planet formation. Celestial mechanicians were now able to more accurately simulate the interactions between solid objects with one another and with a gaseous nebula. Two notable advances in this area included the development of the theory of dust/planet trap in pressure bumps that formed the basis for the modern theories of planetesimal formation, and the development of the models of planet-disk interaction that gave birth to planet migration theory.

In our solar system, initial studies of planet-disk interaction were motivated primarily by 1) the observation of cometary objects and their distribution throughout the solar system, and 2) the initiatives in modeling the formation of planetary bodies. Models of the interaction of giant planets with a disk of planetesimals demonstrated that a planet can migrate due to the exchange of angular momentum with the disk during which its gravitational perturbation will scatter planetesimals from the outer solar system into its inner regions. In the context of planet formation, models of planet-disk interactions showed that the interaction between a planetary body and a gaseous disk can cause the planet to migrate to close-in orbits, creating planetary architectures different from those seen in our solar system.

The 80s also witnessed the rise of what is known as the classical model of terrestrial planet formation. Although at that time, the details of the progression of dust particles to planetary objects were still unknown, scientists had widely accepted that the last stage of the formation of terrestrial planets of our solar system involved collisions between some tens or hundreds of large, Moon- to Mars-sized bodies known as planetary embryos. The initial work on modeling this state of terrestrial planet formation started in early 80s and since then has served as the basis for developing many new theories including the models of the formation of the Moon, the origin of Earth's water, the origin of the parent bodies of differentiated meteorites, and the origin of Mars.

The next decade set the stage for two revolutionizing achievements in observational astronomy that changed the course of planetary science completely. The first achievement was the discovery of Kuiper Belt Objects (KBOs). The detection of the first KBO in 1992 opened a new chapter in solar system studies, and the subsequent discovery of many of these objects provided a rich ground for the application of celestial mechanics and theories of planetary dynamics. The dynamical architecture of KBOs posed a fundamental challenge to celestial mechanics and became the indisputable evidence to the post-formation migration of giant planets. The notion that giant planets did not form where they are changed our views of the formation and evolution of the solar system so strongly that for the next 20 years, many dynamicists used this notion to develop different models for the dynamical evolution of the solar system, compositional properties of asteroids, as well as the origin and dynamics of small bodies including Trojan asteroids and irregular satellites.

The second fundamental achievement in astronomy during the decade of 90s was the discovery of extrasolar planets. The detection of planets around other stars is undoubtedly one of the triumphs of modern astronomy. Not only have these planets proven that our solar system is not unique, they have also revealed many new physical and dynamical characteristics that are atypical of the planets in our solar system and cannot be explained by the theories of solar system formation and dynamics. The diversity of the exoplanetary system, both in mass and orbital architecture reinvigorated the fields of celestial mechanics and orbital dynamics, and has confronted celestial mechanicians with many new challenges.

The most significant contribution of extrasolar planets to celestial mechanics was to allow the universality of its applicability to materialize. The solar system was no longer the only planetary system where the science of celestial mechanics was needed. The new planetary systems, with their new (and surprising) dynamical properties presented numerous venues where celestial mechanics could advance. The detection of the first extrasolar planets, 51 Peg b, a giant planet in a very short-period orbit around a solar-type star, gave new breath to previous research on planet-disk interaction, and turned planet migration into a major industry. Today, despite uncertainty in its stopping mechanism, planet migration is a major, inseparable part of any self-consistent theory of planet formation.

The discovery of the first extrasolar multiple planet system around star Ups And, followed by the discovery of the first resonant planetary system around M star GJ 876 opened new directions for the advancement and application of celestial mechanics as well. Ups And is host to three giant planets in short period orbits and GJ 876 hosts three giant planets in a 1:2:4 Laplace resonance. The discovery of these planets revealed that not only do planets migrate, they will also interact with one another, may get captured in resonant orbits (a phenomenon that does not exists among planets in our solar system), scatter each other into new non-resonant orbits, or get ejected out of the planetary system.

Another fundamental progress in celestial mechanics during this decade was the development of the models of giant planet formation: shortly after the discovery of the first extrasolar planets, the widely accepted Core-Accretion model and its rival, the Disk Instability scenario made their debut. Although soon after their introduction, both models were confronted with major challenges, they have maintained their popularity, especially the Core Accretion model, and have found many applications in the context of the formation of extrasolar planets.

The new millennium started with a major development in our understanding of the origin of Earth's water. Simulations of the late stage of terrestrial planet formation in our solar system suggested that the delivery of water to Earth was not all post-formation. These simulations suggested that in fact, the majority of Earth's water was delivered to its accretion zone during the formation of Earth, by large water-carrying bodies originating from the outer asteroid belt. Subsequent high-resolution simulations of terrestrial planet formation supported this idea confirming that both planetesimals and planetary embryos have contributed to the water delivery to the accretion zone of Earth. Despite its lack of success in explaining the formation of Mars, this scenario is still widely accepted as the main mechanism of the formation of terrestrial planets and water delivery to the inner solar system, and is commonly used to model the formation of terrestrial planets around other stars.

As mentioned before, discovery of extrasolar planets drastically changed our views of planetary systems. One of these discoveries that had a fundamental contribution to the advancement of celestial mechanics was the detection of super-Earth planets (the first super-Earth, a 6.8 Earth-mass planet known as GJ 876 d, was found in 2005 to orbit the M star Gliese 876 in a 2-day orbit). With a mass ranging from 2-10 Earth-masses and radii no larger than 1.6 Earth-radii, and that no super-Earth exists in the solar system, these objects presented major obstacles to models of planet formation. The past 10 years have seen major developments in explaining the formation of these bodies. Current models favor collisional growth of planetesimals and planetary embryos, similar to the process of terrestrial planet formation in our solar system, as the dominant mode of the formation of small super-Earths (i.e., smaller than $\sim 5$ Earth-masses); on the other hand, larger super-Earths which are planets with masses ranging from 6 to 10 Earth-masses, are mainly considered to be failed cores of giant planets that have been scattered or migrated into other orbits.

Modern celestial mechanics is highly influenced by the discovery of extrasolar planets and their theoretical challenges. Soon after the detection of close-in planets as well as those on eccentric orbits, planet-planet scattering and planet migration theory found their way back in the solar system, raising questions as to whether similar processes could have occurred during and after the formation of planets around the Sun. In the past 10 years, planet-planet scattering has been used to develop models of the post-formation evolution of the outer solar system and their contribution to the compositional characteristics of asteroid and collisional history of the inner solar system bodies. Similarly, planet migration has been proposed as a mechanism for the formation of terrestrial planets that can account for the small mass of Mars. The interaction of solid objects with gaseous disks has resulted in a new field of research known as planetesimal formation, and has been used to develop models that can account for the formation of giant planets within the average lifetime of gas in a planet-forming nebula.

The state of celestial mechanics is strong, showing clear signs of progress in all aspects of planetary science, and a bright and promising future. The unprecedented success of the Kepler space telescope in discovering several thousand planetary bodies has provided many unexplored territories where celestial mechanics can have major contributions. The upcoming missions TESS, PLATO, and CHEOPS will provide even richer grounds for the application of celestial mechanics and the advancements in theories of planet formation and dynamics. Also, the constructions of large ground-based telescopes such as TMT, and the development of the space telescope JWST will allow for the detection of biosignatures, which will extend the applicability of celestial mechanics to even more exciting objects, the extrasolar habitable worlds.

Thirty years after its success during the Apollo missions, celestial mechanics has once again reached a golden era: it found extrasolar planets, a gold mine with unlimited supply.

### 2.4. *Quo Vadis, Celestial Mechanics?*

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Science has its own ways, seldom accurately predictable. It also has its rhythms, alternating between the fast progress, steady advancement, stagnation and even backsliding, depending on the advent or lack of new ideas and breakthroughs. It is, thus, not a straightforward task to grasp the status-of.the-art and possible directions of development of an entire branch of science, especially of one so rich in past activities and future challenges as Celestial Mechanics. It is also nearly impossible and even perhaps counterproductive to attempt at recipes for the young researchers or at imposing instructions on what to study and how to proceed with their work. Still, some advice in this same sense may, after all, be useful, at least to some of them.

Past decades have been marked by two major breakthroughs in dynamical studies, that is, by an improved understanding, recognition of importance, and application of chaotic phenomena, and by a full appreciation and implementation of non-gravitational effects. At present, however, we seem to be in a sort of an intermediate state of affairs, waiting for the next major breakthrough to take place, which will give a new impetus to the research in the field. The previous period has also seen a great many of more specialized and specific achievements, most of which appeared as challenges to dynamicists due to the results obtained in other research areas, in particular in observational astronomy. Here, I would like to mention only a few of these, which, I presume, may also give a fair picture of the general advance in the field. It goes without saying that this is a largely incomplete and biased list, so it has to be understood as a result of author's necessarily limited knowledge and unavoidable subjectivity.

A special place among the results of the preceding period belongs, in my opinion, to the advances in the long-term dynamics of our planetary system, which, on one hand, enabled an accurate tracing of motion of the planets of the solar system till the current predictability horizon and a reliable study of the astronomical forcing of climate changes on the Earth and other planets, and, on the other, shed light to the processes in the early solar system and promoted migration of giant planets as one of the principal evolutionary mechanisms that shaped what we nowadays observe of it. In the same period, we began to also acquire some basic knowledge on the dynamics of extrasolar planets, the innovative methods of orbit determination made possible to cope with ever increasing observational data sets, the improved theories of tidal effects were successfully used to assess the internal structure of a number of natural satellites, planetary rotations and spin-orbit coupling started to be better comprehended and modeled, etc.

Most of the researchers active in the field agree on the necessity of changes that are already taking place in a natural and self-managing fashion, but which should be further strengthened and focused in the next future. What I have in mind is a kind of strategic turn which transforms the field from the pure science that deals with the development of methods and tools regardless of their application, to the contemporary complex endeavor primarily concerned with application of ever better and far reaching methods to the real world, assuming a dominantly problems solving approach.

From the methodological point of view, the continuous work on the improvement of existing and development of new methods and approaches, both analytic and numerical, is always worthwhile and needed. But, it is also very important for the future to preserve and upgrade the synergy between these two basic kinds of methods. The former are directly rooted in the theory and bring understanding, while the latter provide the power to compute solutions and model the problems otherwise out of reach of even the most sophisticated analytics. It is the mixture of different tools, even possibly the already known ones, but problem adapted and smartly combined, that can most efficiently lead to the new results.

From the topical point of view, there is, of course, a plethora of possibilities for the future research. Among the problems and challenges which, in my opinion, deserve special attention in the coming years are: improvement of dynamical modeling of the solar system evolution, possibly intertwined with extrasolar systems studies; more reliable characterization of the extrasolar systems, which, together with new discoveries and more accurate observations, should make their dynamics better known and easier to compare among themselves and with our own system; pushing to more remote epochs the predictability horizon of accurate ephemerides for both, regular and chaotic orbits, and for all kinds of bodies planets, satellites, asteroids, comets; more in-depth understanding of transport along the secular resonances and diffusion in the mean motion ones; transport mechanisms throughout the solar system and in particular in the Earth's vicinity, interplay of physical and dynamical properties of the small bodies, and so on.

As one can easily appreciate form the above, the legacy of the past decades of research in Celestial Mechanics is tremendous. It provides a solid base for the next generation of researchers to continue the hard work, with the same dedication and with, hopefully, even more impressive results.

### 2.5. *Regular Numerical Methods for the Few-Body Problem*

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In the stellar dynamical simulations of star clusters it has been found necessary to use regularization methods for the motion of strongly interacting bodies.

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Regularization requires one of the following alternatives:

-Transformation(s) which makes the equations of motion regular and easy to integrate numerically. [Kustaanheimo and Stiefel (KS) transformation (1965)]

-An algorithm that gives regular results. Equations need not to be regularized. [Logarithmic Hamiltonian (logH) or Time Transformed Leapfrog (TTL), Mikkola & Tanikawa (1999), Preto and Tremaine (1999), Mikkola and Aarseth(2002)]

-In all the alternatives high precision can be obtained with the help of an extrapolation method [Gragg /Bulirsh-Stoer]

-Very old starting points: Sundman's time transformation $t' = r$ (for the two-body problem), Levi-Civita's two-dimensional coordinate transformation $x + iy = (Q_1 + iq_2)^2$.

Kustaanheimo-Stiefel (1965) transformation from four dimensional space to three dimensions made finally regularization possible for stellar dynamics simulations.

The most recent advances in this field started in 1999 when Mikkola and Tanikawa (1999) as well as Preto and Tremaine (1999) invented the logarithmic Hamiltonian, which together with the leapfrog algorithm, produces regular results, in fact correct trajectory for the two-body problem. This method is useful also for the N-Body problem since during close approaches the problem reduces essentially to a two-body problem. In this more general case the algorithms naturally is not exact, but still gives regular results. Because the leapfrog is time reversible an efficient use of extrapolation method is possible.

Somewhat earlier Mikkola and Aarseth (1992) introduced the chain concept. This e.g. reduces significantly the round-off effects in simulations. Ten years latex Mikkola and Aarseth published the time transformed leapfrog which is in some cases equivalent with the logarithmic Hamiltonian, but gives more possibilities for the time-transformation.

Mikkola and Merritt (2006) combined the chain coordinate system with the logH and TTL algorithms and produces the Algorithmic Regularization Chain code (ARC). Later (Mikkola and Merritt (2008)) the algorithm was used for simulating stellar dynamics around gigantic black holes using the Post Newtonian terms for both the Schwarzschild- as well as Kerr-holes. In this first attempt to include the possibility of velocity dependent perturbations the implicit midpoint method was used to obtain a time reversible algorithm for the sub-steps in the extrapolation method. Due the implicit method this algorithm can be time consuming especially for the PN-terms that are quite complicated.

Later, Hellström and Mikkola (2010), suggested a new explicit algorithm for this case. This, auxiliary velocity algorithm, uses two velocity variables which actually have the same physical meaning but allows to construct an algorithm that is essentially a leapfrog for the coordinate dependent force and the modified midpoint method for the velocity dependent part. This way the algorithm allows efficient explicit use of the extrapolation method.

The AR-method can often be well used in systems in which the singularity in the potential is of the $1/r$ type and the rest of the problem is either integrable of can be approximated by a symplectic/(time reversible) procedure. A list of such examples can be found in Mikkola and Tanikawa (2013).

It is still essential to stress that in the Algorithmic Regularization (AR) the equations of motion are not regular, but the combination AR+Leapfrog+Extrapolation method, gives the good regular numerical results.

The ARC-code is simpler than the KS-Chain code. There are no coordinate transformations and very large mass ratios are allowed, even zero-masses can be included. With the coordinates $\mathbf{X}$, velocities $\mathbf{V}$ and accelerations $\mathbf{F}$ the physical time equations $\dot{\mathbf{X}} = \mathbf{V}, \quad \dot{\mathbf{V}} = \mathbf{F}$ are transformed to the form $\mathbf{X}' = t'_v \mathbf{V}, \quad \mathbf{V}' = t'_x \mathbf{F}$, were the two time

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derivatives $t'_v$ and $t'_x$ are different, but due to constants of motion, numerically equal along the correct orbit. In case of a close approach these both become (numerically) proportional to the distance. On the other hand $t'_v$ depends only on velocities and energy and $t'_x$ depends only on coordinates and time.

However, for the zero (ore very small) masses the algorithm is not really regularized since a vanishingly small mass has no effect in the energy and all the regularization algorithms depend explicitly on the total energy of the system. Thus zero masses are essentially 'invisible'. However, due to the time transformation, the physical time steps are small when the massive bodies are experiencing close approaches and so the forces affecting the tiny bodies change slowly. This means that, even if in that case the accuracy depends mostly on the brute force of the extrapolation method, the numerical results are typically reliable.

As explained above, the case of very different masses is still difficult for the existing simple algorithms. Thus I hope that it is possible to invent a simple accurate method such that precision is completely independent of the mass ratios. This can not be done simply with the formulations used thus far. The experience has shown that the methods, for the sub-steps in extrapolation algorithms, must be at least time reversible, sometimes it seems that symplectic formulations (as the leapfrog) are most stable regular and accurate.

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## 2.6. *The Future of Solar System Dynamics Studies*

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During the last decades, much research has been directed toward understanding the solar system origins with the planetary orbits being considered as important clues. Mercury has the largest orbital eccentricity ($e = 0.17$) and the largest inclination ($i = 7°$) among the solar system planets. Jupiter's orbit is more circular and co-planar ($e = 0.05$ and

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$i = 0.4°$). In principle, these values could have surged from some turbulent and difficult-to-characterize processes during the earliest stages of planet formation. The recent studies suggest, instead, that planets could have emerged from the dispersing circumsolar disk on precisely circular orbits in a common plane, and the $e$ and $i$ values observed today were established later (Tsiganis *et al.*, 2005; Morbidelli *et al.*, 2007; Brasser *et al.*, 2013).

The results can be conveniently illustrated with a computer simulation. Its starting point is the early solar system with the terrestrial and outer planets on the circular and co-planar orbits. The outer planets are assumed to be closer to the Sun than they are now, with Neptune at 20 AU and Saturn's orbital period being only 1.5 times longer than Jupiter's (this ratio is nearly 2.5 today). These assumptions are motivated by the orbital evolution of planets during the previous stage, when they exchanged the orbital momentum with a protoplanetary gas nebula, and converged inward and toward each other (Masset & Snellgrove, 2001). The gas nebula dissipation was also expected to damp orbital eccentricities and inclinations. A disk of small icy bodies or planetesimals, with the total mass of $\sim$ 20 Earth masses, is placed beyond the orbit of Neptune. Its remains survived in the trans-Neptunian region to this day as the Kuiper belt.

A number of things happens as the system evolves. Planetesimals leak from the outer disk onto Neptune-crossing orbits and are subsequently scattered inward or outward during close encounters with Neptune. The ones scattered outward come back and are scattered inward, where they encounter Uranus and Saturn. These planets act in much the same way as Neptune, eventually handling bodies to Jupiter, which ejects them from the solar system. As planetesimals move from the outer disk inward, the conservation of orbital momentum dictates that Saturn, Uranus and Neptune must move outward. This process, known as the planetesimal driven migration (Fernandez & Ip, 1984; Malhotra, 1993), explains how the outer planets reached their present orbital radii with Neptune at 30 AU. However, the planetary eccentricities and inclinations remain small during the planetesimal driven migration.

Interestingly, when Neptune reaches roughly 28 AU in the specific simulation discussed here (Nervorný & Morbidelli, 2012), a dynamical instability develops with the inner ice giant evolving onto an orbit intersecting those of Jupiter and Saturn. The instability trigger is related to the gravitational resonances encountered by the migrating planets (Tsiganis *et al.*, 2005; Levison *et al.*, 2011). The subsequent planetary encounters have several consequences. First, they excite eccentricities and inclinations of the outer planets to values comparable to the present ones. Second, the semi-major axes of planets evolve discontinuously during encounters, with Jupiter's semi-major axis changing by as much as 0.5 AU; the so-called jumping Jupiter (Morbidelli *et al.* 2009; Brasser *et al.*, 2009). Third, the inner ice giant is ejected into interstellar space. It is not known how many ice giants formed in the solar system, but the instability calculation with one extra planet on an initial orbit between Saturn and Uranus gives the best results (Nesvorný & Morbidelli, 2012).

As the outer solar system re-configures, the inner planets follow the suit. If Jupiter slowly migrated due to the planetesimal driven migration, gravitational resonances between the terrestrial planets and Jupiter would have plenty of time to act. They would disrupt the terrestrial system orbits, eventually leading to planet-planet collisions (Agnor & Lin, 2012). Jumping Jupiter solves this problem, because the resonant effects are reduced when Jupiter's orbit changes discontinuously. Nevertheless, the eccentricities and inclinations of the terrestrial planets become excited. Most notably, in the successful simulation highlighted in Fig. 1, Mercury's eccentricity and inclination reach their present values. Not everything is perfect, however. For example, the orbital inclination of Mars ends up slightly lower than its present value (4°). This may imply that Mars had some

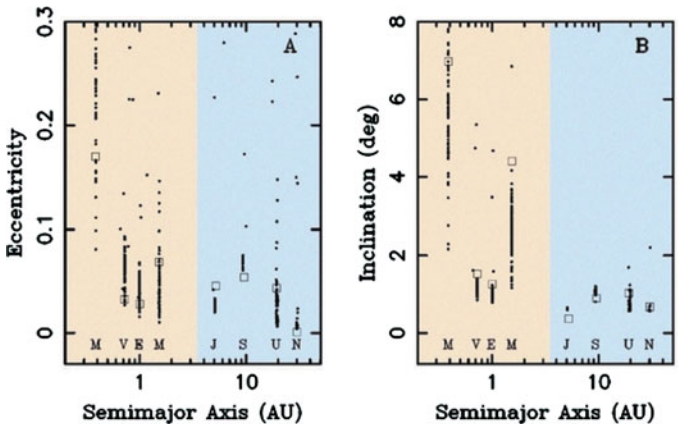


**Figure 1.** A computer simulation that starts with the circular and co-planar orbits of all planets. An outer disk of small planetesimals causes the migration and instability of the giant planets. The final planetary orbits (dots) obtained in a hundred of computer simulations, each starting from slightly different initial conditions, match the general properties of the present orbital architecture (squares).

orbital inclination initially, or that the specific evolution discussed here is still missing some important component.

Much of the future research will be directed toward the goal of improving the results shown in Fig. 1. A fundamental difficulty with these efforts is that the orbital evolution during the instability is chaotic and must therefore be studied statistically. The small bodies, such as the asteroids and Kuiper belt objects, place important constraints on the evolution history of planets. While the jumping-Jupiter model with an extra ice giant owns much of its success to matching the basic properties of these reservoirs, getting things right in detail may be difficult. All these issues open a wide range of challenges for the next generation of solar system dynamics studies.

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### 2.7. *The Stability of Multiple Planet and Satellite Systems*

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Because it displays remarkable complexity, the stability of a handful of low mass bodies in orbit about a star, interacting through gravity alone, has for centuries been a challenging and rewarding problem to study. With the exception of the inner Uranian

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satellite system, closely packed planar multibody systems had been explored only out of intellectual curiosity. However, with the recent discovery of a new multiple body satellite system (Pluto's; Showalter & Hamilton 2015) and numerous multiple exoplanet systems (Fabrycky *et al.* 2014), the stability of multiple body systems may be important in understanding the evolution and formation of most satellite and planetary systems.

Numerical integrations find that a system of two planets on initially zero-inclination and eccentricity orbits about a star never experience mutual close encounters if the initial semimajor axis separation is sufficiently large (Marchal & Bozis 1982; Gladman 1993, Mardling 2008, Giuppone *et al.* 2013). The large change in stability timescale as a function of semi-major axis separation is nicely explained (Wisdom 1980; Mardling 2008; Deck *et al.* 2013) using the 'resonance overlap criterion' for the onset of chaotic behavior (Chirikov 1959). Poincaré maps or surfaces of section of low-dimensional systems illustrate that there can be a dichotomy: the trajectories are either integrable or chaotic and we interpret the chaotic trajectories as unstable. The resonance overlap criterion is used to predict the location of the boundary where there is a large change in stability or lifetime.

Whereas stability is sharply delineated for two-planet systems, closely packed multiple planet and satellite systems instead display a range of stability timescales (e.g., French & Showalter 2012). A closely packed multiple body system is integrated until one body crosses the orbit of an other body, and the time of integration denoted the 'crossing time'. The crossing time scales like a power law with body mass and separation (Chambers *et al.* 1996, Duncan *et al.* 1997).

Celestial mechanics is unique among non-trivial low dimensional dynamical systems in the richness of developed perturbative techniques allowing us to estimate or calculate resonance locations and strengths. With this powerful calculation machinery, the resonance overlap criterion has been applied in increasingly complex settings including the three-body problem, and multiple planet and satellite systems (e.g., Wisdom 1980, Mardling 2008, Quillen *et al.* 2011, Deck *et al.* 2013, Quillen *et al.* 2014, Ramos *et al.* 2015, Showalter & Hamilton 2015). While many works have used the overlap criterion to predict or delineate a dichotomy in behavior, Quillen 2011 attempted to use it instead to account for the power-law behavior of integrated crossing times in closely-packed planar multiple body systems. She proposed that the power law behavior of crossing times is due to ubiquitous 3-body resonances and the strong dependence of crossing time on spacing and body mass due to the strong dependence of the three body resonance strengths on these quantities.

Subsequent work (Quillen & French 2014) showed that three-body resonant chains, pairs of bodies in pairs of two-body resonances, are usually stronger and so more important than the three-body resonance comprised of zero-th order terms considered in 2011. These works represent a first attempt to account for the trends numerical measured in crossing times. Even though the type of resonances responsible for the onset of chaotic behavior can be identified in multiple body systems (Migaszewski *et al.* 2012, Showalter & Hamiton 2015, Batygin *et al.* 2015) the connection between resonance strengths and their overlaps (both which we can calculate) and the the long timescale behavior is not very good. Simple diffusive estimates, using calculated resonance frequencies and strengths, poorly capture the power-law behavior for the crossing times and are orders of magnitudes off when used to predict lifetimes for individual bodies as a function of mass and inter-body spacings.

Despite our ability to pinpoint resonances, calculate their strengths and frequencies, and delineate regions where they overlap, it is difficult to predict the behavior on long timescales of multiple body systems. Surfaces of section give the appearance of a

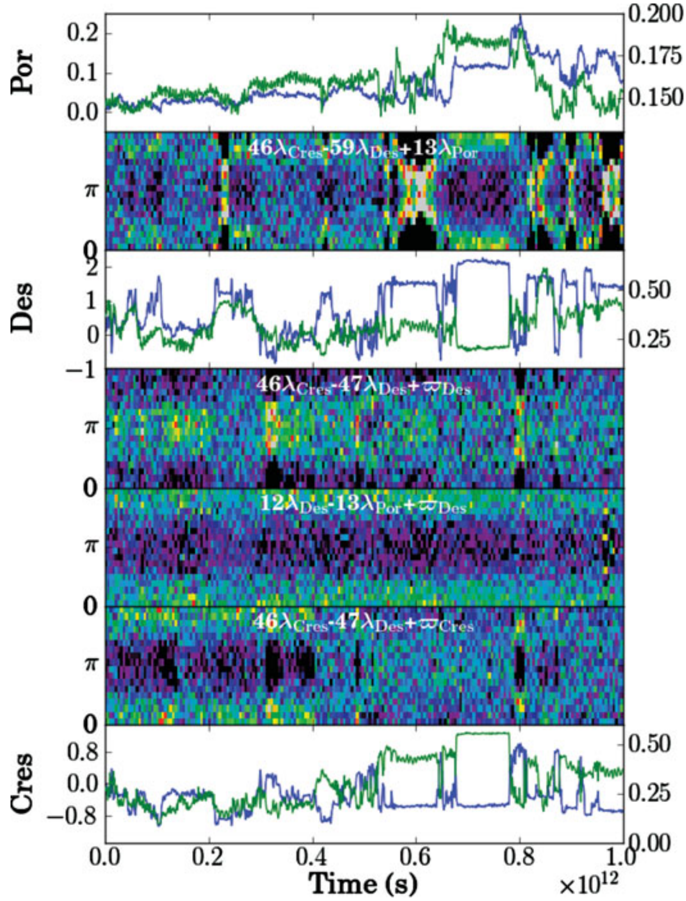


**Figure 2.** Semi-major axes and two and three-body resonant angles in an orbital integration of satellites Cressida, Desdemona and Portia in the Uranian satellite system by Quillen & French 2013. The system exhibits both intermittent behavior and long timescale wandering in eccentricity and semi-major axes of the three bodies. From the resonance angles, we infer that both two-body and three-body resonances are important. Even though we can calculate the resonances strengths and oscillation timescales, we lack procedures to predict the diffusive like behavior on long timescales of distributions of systems.

dichotomy of behavior in lower dimensional models that approximate the full system (e.g. Batygin *et al.* 2015). However a chaotic region in a surface of section can hide can-tori and their associated sticky orbits and strongly hyperbolic regions. Separatrix maps constructed to approximate dynamical systems illustrate some of this phenomena including intermittency and Levy flights (Shevchenko 2010). The dichotomy suggested by the resonance overlap model is not well approximated by a single diffusion coefficient in the overlap region. Intermittency and Levy flights in a random walk model strongly affect the long timescale diffusive-like behavior of a initial particle distribution. Our N-body dynamical system is not a random walk but we can discuss statistical properties of distributions of particles that originated with similar initial conditions.

We can think of our crossing timescale numerical integrations as a set of initial particle distributions confined to nearly circular orbits. If there are occasional large steps in particle trajectories, on long timescales the rare large events can dominate the statistical distributions. The particle distribution will eventually be determined by the probability and sizes of these rare but extreme events rather than the average diffusive-like behavior.Possibly a good direction to proceed is to try to leverage our ability to calculate resonance strengths to estimate the statistical properties of particle distributions on long timescales.

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### 2.8. *Celestial Mechanics of Rubble Pile Bodies*

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**Motivation and Past Research:** Recent observations of the asteroid population have strongly indicated that these bodies are rubble piles, comprised of many different components that rest on each other. The evidence for this understanding is broad, and rests on direct observations of asteroids at high resolution [3], population statistics on the spin rate as a function of size [9], the porosity of asteroids in general [1], and observational evidence that they undergo fission and mutually escape each other [10].

Motivated by these results, there have been significant investigations into the evolution of rubble pile asteroids as their rotational angular momentum is increased. One class of investigation involves $N$-body simulations accounting for mutual gravitation and surface forces, allowing the bodies to rest on each other [11, 12, 22]. These tools are valuable as computational laboratories but do not provide insight into the overall system energetics and the long-term evolution of the bodies or disrupted components. Continuum models have also been used to generate analytical insight into the deformation of rubble pile bodies, but these methods cannot deal with nonlinear deformations or track a body as it separates into multiple components [4, 5].

In a parallel track, the dynamical specification and simulation of bodies with non-spherical shape as they interact dynamically with each other has been a topic of interest and has made significant advances [7, 23, 8, 2]. However these studies are focused on

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describing specific motions and have not pursued the development of constraints on the general evolution of such systems.

To develop rigorous constraints on a rubble pile system's dynamical evolution, we have investigated the application of fundamental celestial mechanics techniques and concepts to such systems. Research has focused on developing conditions under which a rubble pile may reconfigure or fission as its angular momentum changes and, for those systems that enter a phase of mutual orbital dynamics, the stability of their final motions has been studied accounting for the full coupling between translational and rotational dynamics.

**Celestial Mechanics of Rubble Piles:** A first foray into such topics was developed in [13] where constraints and concepts from the $N$-body problem were generalized to the interaction of two general mass distributions, termed the Full 2-Body problem. This derivation enabled fundamental concepts from the classical point-mass $N$-body problem to be directly inherited into the Full $N$-body problem, while coupling rotational and translational motion together. A significant outcome of this study was the rigorous identification of stability results for full body problems. In particular, conditions for Hill stability (meaning that the system must remain bound) and impact stability (meaning that the system components cannot impact) were derived, and it was shown that these behaviors can co-exist in the same system.

Following from this result several studies of specific models and instances of the Full 2-body problem were made. These include the study of resting equilibria between an ellipsoid and a sphere, which provides conditions for the reconfiguration of component bodies and their fission [15]. Also investigated were the energetic stability of relative equilibria, robust to energy dissipation and having direct applications to identifying the final states that a full body system can evolve into [14, 17]. One significant result from these studies identified a direct relation between the Hill stability of a fissioned system and the mass ratio of its two fissioned components. Specifically, it was found that systems with a mass ratio less than $\sim 0.2$ would have a positive energy and thus could mutually escape – albeit through the extraction of rotational angular momentum from the larger component. Systems that fission with a mass ratio greater than this have negative energy and cannot escape. This result and the concept of asteroid fission was specifically tested in the observations of asteroid pairs and was largely found to provide a consistent explanation for the observed mass ratios between pairs and the spin rates of the remaining primary bodies [10]. Elements of this model have also been used to motivate specific simulations of asteroids spun to disruption in order to develop an overall theory for the creation, evolution and ultimate stability of binary asteroid systems [6].

A complementary direction was initiated in [16] which generalized the problem from the interaction of two bodies with finite density to multiple bodies. This initial result derived a general condition for how a collection of bodies resting on each other would preferentially fission, stated in terms of fundamental results from the $N$-body problem. The most significant result for this generalized problem was reported in [18] in which the $N$-body problem was formally restated in terms of the Full $N$-body problem. There were several significant results in that paper, including the proof that all Full $N$-body problems have stable, minimum energy configurations at all values of angular momentum (something which is not true in the classical $N$-body problem), the definition and use of a single scalar function for finding relative equilibria and evaluating their energetic stability, and the derivation of new results on relative equilibria in the 2 and 3 body problem. In particular, in the equal mass 3-body problem it was shown that there are 7 distinct relative equilibria, as compared to 2 in the equal mass classical 3 body problem. Furthermore, the sequence of bifurcation and stability of these relative equilibria was

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charted in detail. Most recently, in two short papers [19, 20] the concept of Hill stability was applied to the Full $N$-body problem, showing that sharp constraints can be found for how different systems can escape or disrupt. Such sharp results are impossible for the classical $N$-body problem with $N \geqslant 3$, but are easily established once finite density considerations are added. In [21] these results have been rederived from a more rigorous perspective and an analysis of relative equilibria and stability for the $N = 4$ body problem has been added.

Future work in this area is focused on generalizing the Full 3-body problem to account for different sized bodies, and the development of techniques to deal with the Full $N$-body problem without the use of simple spherical models. While it is not expected that the direct analytical study of $N \gg 1$ full body systems can be fruitful, the insights and results from the lower particle number cases can provide interesting directions for future study.

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### 2.9. *Dynamical Instabilities in Planetary Systems – The Legacy of the Nice Model*

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The preparation of this Legacy volume coincides with the 10-year anniversary of the publication of the *Nice model*[1]-[3] of solar system dynamical evolution. This is a short note on the legacy of the Nice model and the role it played in establishing *dynamical instabilities* in planetary systems as an integral part of evolution theories.

The Nice model offered an alternate view of the *planetesimal-driven migration* (PDM) phase, which was already believed to be responsible for shaping the outer solar system [4][5]. In particular, the Nice model aimed at bridging the gap between simulations of the preceding gas-driven migration (GDM) phase [6], which seemed to suggest that the orbits of Jupiter and Saturn, before the onset of the PDM phase, had a period ratio $P_S/P_J < 2$, and standard PDM models of the time, which implied mild planetary interactions during this phase (no resonances), but systematically resulted in circular planetary orbits, due to dynamical friction. At the same time, an attempt to define realistic initial conditions for the planetesimal disc was made, in order to correctly assess the duration of the PDM phase; an outer edge at $\sim 30$ AU was set, so that Neptune stops migrating at its current location, and an inner edge $\sim 3$ AU away from Neptune's orbit was set, to account for the depletion of this zone, during pre-PDM phases.

The results of the first batch of simulations was impressive! The planets started to migrate slowly, as in the 'classical' scenario, but an *instability* was induced to the system, as soon as Jupiter and Saturn crossed their 1:2 mean motion resonance. A simple analytical model was enough to demonstrate that, since the planetary orbits were diverging, resonance trapping could not occur and their eccentricities had to increase abruptly, when jumping across the libration zone. This eccentricity jump induced strong perturations on Uranus and Neptune and the system was driven into a short phase of *gravitational scatterring*, with Uranus and Neptune expanding their orbits and dispersing the distant disc within few My, whence PDM effectively ceased. At the end, all planets were stabilized on orbits very similar to their current ones, in terms of $(a, e, i)$. We then realized that the duration of the pre-instability phase is controlled by the location of the disc's inner edge, and so the instability epoch could be tuned to coincide with the beginning of the alleged *Late Heavy Bombardment* (LHB). In fact, the duration and total mass of planetesimals reaching the Earth during the instability were found to compare very well with the LHB characteristics. The complete dispersion of the Trojan regions found was certainly disturbing. However, we quickly realized that the distribution and total mass of the observed Trojans could be explained by the *chaotic capture* of planetesimals into Jupiter's 1:1 resonance, during the instability phase; this implied that Trojans originated in the planetesimal disc and therefore must be similar to KBOs.

The Nice model was certainly well-received, as it seemed to kill several birds in one stone; subsequent works successfully linked the formation of several small-body reservoirs – such as Neptune Trojans [7], irregular satellites [8], KBOs [9] and D/P-type main-belt asteroids [10] – to the instability phase, predicted by this model. On the other hand, the Nice model was rightfully criticized of depending sensitively on the assumed initial conditions. Let me state here though that this *does not* pertain to the very occurrence of an instability, as is sometimes wrongfully said. The system is forced to an unstable configuration by a generic mechanism that is independent of the exact initial set-up; it is simply a case of *adiabatic crossing of* (as opposed to, *capture into*) *resonance*; if the disc could not provide angular momentum to the planets, they would remain on their original (stable) orbits, as given by GDM.

The sensitivity issue was corrected in subsequent versions of the model where (a) the initial conditions for the planets were better linked to the end-state of GDM simulations [11], which suggested a *multi-resonant* planetary configuration as the most likely outcome, and (b) the gravitational self-stirring of the disc was included [12], and was found to slowly provide energy to the planets, until resonant phase-protection is broken, after a roughly constant $\sim 500$ My period. This version of the Nice model abandons resonance-crossing as a trigger, in favor of *adiabatic extraction from resonance.*

In following studies, it was realized that the migration of the planets towards their current orbits actually had to be dominated by mutual encounters; otherwise it would be too slow and the asteroid belt would look very different from the current one [13]; the stability of the terrestrial planets would also become problematic [14] and the final orbits of the giant planets would not have the correct secular evolution [15]. Thus, a sub-set of Nice-model evolutions (the *'jumping-Jupiter' model*) emerged as the preferred model; unfortunately this was a disturbingly low-probability outcome. However, it was later found that, if our system initially contained an additional Neptune-sized planet that escaped during the scatterring phase [16], the chances of having a successfull 'jumping-Jupiter' evolution would greatly improve.

Clearly, our understanding on solar system evolution has come a long way, since the publication of the original Nice model. The model is continuously under scrutiny (as it should!), and the debate on whether it can indeed explain observations, without violating important constraints, is always vivid. To this end, let me emphasize that the Nice model only describes the PDM phase. The 'jumping-Jupiter' version clearly implies that the inclination distribution of main-belt asteroids and their small cumulative mass most likely originated in previous stages; the "Grand Tack" model [17] successfully explained these features, linking them to the GDM phase.

Despite the active debate, I believe it is generally accepted that the Nice model played an important role in establishing dynamical instabilities in planetary systems, as an integral part of evolution theories. Today, the idea that planetary systems can become *temporarily* unstable, and even 'lose' planets along the way, does not seem strange; gravitational scattering and resonance trapping are both needed to explain the morphological variety of exosystems. In this respect, the Nice model simply postulates that our solar system was shaped by the same dynamical mechanisms that shaped all planetary systems. Whether the Nice model will still be considered viable after another decade, or whether it could be replaced by a completely different model, remains to be seen. In any case, our community has to continue the fruitful debate on this topic, with the purpose of advancing our understanding of solar system formation.

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